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STUDY ON INTELLIGENT PLATFORM OF MODEL UPDATING FOR THE HEALTH MONITORING SYSTEM

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ABSTRACT

The main motivation of this paper is engaged to provide an automation and smart software platform to improve this time-consuming and laborious model updating process. The paper investigated the key technologies of model updating based on intelligent optimization algorithms, viz. accuracy judgment means of basic finite element model, parameter choice theory based on sensitivity analysis, commonly used objective function and its construction method, PSO and other intelligent optimization algorithms. Moreover, an intelligent model updating prototype software framework is developed with the commercial software ANSYS and MATLAB. Parameterized finite element modeling techniques is proposed to adapt different bridge types and different model updating requirement. The objective function library is built to enable different updating targets. Two case study are conducted to verify the feasibilities of techniques involved in the proposed software platform.

KEYWORDS

Structural health monitoring, model updating; software implementation, intelligent optimization algorithms, parameterized finite element modeling.

INTRODUCTION

Making use of the monitoring information from structure health monitoring system to update the finite element model of structure real-time and online, not only can update the structure benchmark analysis model, but also can track the changes of target physical parameters and characteristic index in any location continuously. Automation and intelligentization in model update is the key and bottleneck problem to achieve these goals, and also is one of the key steps in intelligentization of structure health monitoring system. After 20 years of research, the existing health monitoring system at home and abroad has entered the third stage, which emphasis on processing and utilization of the data to implement the data-based online early warning and assessment of health status. At this stage, the model-based assessment method is just regard as a supplementary means that is used in a offline way, and has not become the highlight of online algorithm. Model updating is also researched just for providing the benchmark model for this method.

In recent years, the theory and technology of model update continues to progress, especially in computational model updating (CMU) and model validation research, some of the techniques in updating parameter selection, the processing techniques of uncertainty in updating results from continuous monitoring information, the alternative technologies with small amount of calculation of complex finite element calculation in modified iteration step. Model updating software also got rapid progress, some commercial finite element software have also expanded its functions in model updating, such as the Belgian DDS's FEMtools, the French professor Balmes's structural dynamics toolbox SDTools, German Schedlinski's SysVal, etc.. All of the software is provided with some model updating function, but now these software's updating methods are single, poor universality, poor optimization ability. It is difficult to ensure the optimization of updating results, to be completely programmed and automation because of more human intervention needed. In spite of this, the technology progress still constantly stimulate the health monitoring system researchers to begin including model update in the scope of their research in the next generation of structural health monitoring system, to pursue intellectualization, online and real time in updating process, to achieve any of the physical and mechanical field information's deduction of the monitoring target, to expand evaluating sources and scope of information, and to make shortcomings, cause by limited gauging points, up.

To this end, this paper studies the online intelligent model update technique based on monitoring driving, presents the model update prototype software framework based on the commercial finite element software,

ANSYS, and the scientific computing software, Matlab, and proposes an intelligent algorithms library, which includes PSO, to drive most of the possible update tasks. On the platform, by means of the discretionary combination of update tasks, objective functions and intelligent algorithm, it can be achieved efficiently and flexibly that the automatic model update, without manual intervention, caters to any types of bridge on structure or component level, so as to pave the way for integrating the function of structure finite element model updating into existing structure health monitoring system. Finally, this paper achieves the key links of technique that are proposed in the paper, by introducing the result data from vibration test of a continuous beam and the monitoring system implementation of a cable-stayed bridge, proves the feasibility of the proposed technique.

COMPONENTS OF INTELLIGENT PLATFORM OF MODEL UPDATING FOR HEALTH MONITORING SYSTEM

At present, the most of the large finite element software, that are used commonly in engineering, lack of support for model updating, moreover, some finite element software, that is developed specifically for model updating, does not do well in conventional finite element analysis, is difficult to deal with complex engineering structure, and is insufficient in optimization ability. Basing on sensitivity-analysis-based theory of parameters model updating, combining programming ability of numerical calculation software MATLAB with finite element modeling ability of commercial finite element software ANSYS, developing the intelligent platform of model updating for health monitoring system, to realize the function of automation model updating for actual engineering structure, is the urgent need of health monitoring system at present. By extracting of the general steps of the dynamic-based parametric bridge model updating process, this paper gives a set of software architecture with certain commonality, and realizes some of the key technologies, so as to explore the way for subsequent large online bridges updating software.

Commonality of the suggested software framework is mainly reflected in: firstly, the optimization algorithm and target function is modular, according to different problems, different optimization algorithm and target function can be selected; secondly, it has certain extensibility, optimization algorithm and target function can be added independently based on need according to certain standards.

Data Exchange , Interface and Process Control Between MATLAB and ANSYS

ANSYS Parametric Design Language (APDL), shipped with ANSYS, can be used for general process control, but if you want to realize specific complex physical model it will be more difficult, moreover, very slow. In the suggested framework, MATLAB is responsible for optimization and control process, ANSYS is responsible for finite element calculation, the data generated by MATLAB's optimization module are passed to the ANSYS for structure analysis, and then ANSYS returned the updating data to MATLAB's optimization module, the circulation repeat again and again, until the results are convergent. In the process, to achieve collaborative work between MATLAB and ANSYS, there need to be smooth transmission of three kinds of information, the first is call instruction, the second is data interaction, the third is interaction of running state.

There is no ready-made interface between MATLAB and ANSYS, but MATLAB provides some universal methods for calling external programs, such as, "!", DOS function, the system function and so on. "!" and DOS function can call the shell program to execute the command that Windows system command the DOS to do, the system function can perform operating system command and return the results.

There is also no direct data exchange system between MATLAB and ANSYS, thus the exchange of data must be completed through files medium. This paper chooses text file as medium of data exchange. On the one hand, text file is plain but has the characteristic of high data storage capacity; on the other hand, both software provides enough functions to realize read and write of the text files.

How to realize the status information transfer between two pieces of software is the key of process control. This article achieves mutual notification of status information by means of setting up the state variables identification function (flag. M). It judges whether ANSYS is in operation through reading the status value in the MATLAB program: if ANSYS stops running, flag will be zero, notify MATLAB the current running is over and next step of calculation can run, otherwise MATLAB will wait. The format of using the system or the DOS functions to call ANSYS is as follows:

```
[status,result]=system('D:\Ansys\v110\ANSYS\bin\intel\ansys110 -b -i ansys/ansysconcle -o output.out')
```

After ANSYS calculation, the status returns value 0. So the returned value of status can be used to write flag file as below:

```
function flag(status)
if status==0
```

```
    out=objfunt4; % objfunt4 is the objective function
end
```

Parametric Finite Element Modelling

Parametric finite element technology refers to define the structure finite element model through a group of undetermined parameters, the parameters can be chosen between structural geometry size, physical and mechanical properties. During parametric model updating designing, the parameters, that need to be updated, constitute the undetermined parameters of the parameterized finite element model. ANSYS provides parameter design language (APDL) that is used to complete finite element analysis automatically. APDL's command is a kind of script command, it can provide users with parameters, vector, cycle and a series of functions, meanwhile it is also the batched analysis technology provided by ANSYS.

The basic steps of parametric finite element analysis method: firstly, abstract the characteristic parameters of described model according to the geometrical structure of the model, and simplify properly; secondly, establish the finite element analysis process including entity modeling, analysis and treating processes of results by command stream file in ANSYS; thirdly, replace the parameters in the model with abstracted characteristic parameters by APDL, to constitute finite element analysis process with variable parameters; finally according to the requirements of design and analysis, give specific characteristic value to the parameters, and carries on finite element analysis for the results. the main difference between parametric finite element analysis and general finite element analysis is that all of the preprocessing geometry model, material property and loading, boundary conditions and so on is parametric, and the parameters must be assigned values before calculating.

According to the basic flow of the parametric finite element analysis, this paper divides general finite element model into the corresponding module, as shown in figure 1. Ansysconsole is the main control module of model, which is responsible for the finite element analysis process control; Text file Inf and Para are initial parameters files of parametric model, the Inf, records model's calculation type, basic parameters and so on, the Para records the trial data generated by optimization algorithm, MCF and MCS are the result files of finite element analysis, the former records modal frequency, the latter records modal shape. The above four files make up the the data module of model, are the key part of interaction with Matlab; Model, Loading, D&D, BC form the basic model file together, in which the Model is the initial Model, Loading simulates the load conditions, D&D can simulate the damage and performance degradation of structure, BC is model boundary condition, and also can add a specific module according to the fixed format; finally, Cal_m is the computing core of model, taking the task about model analysis.

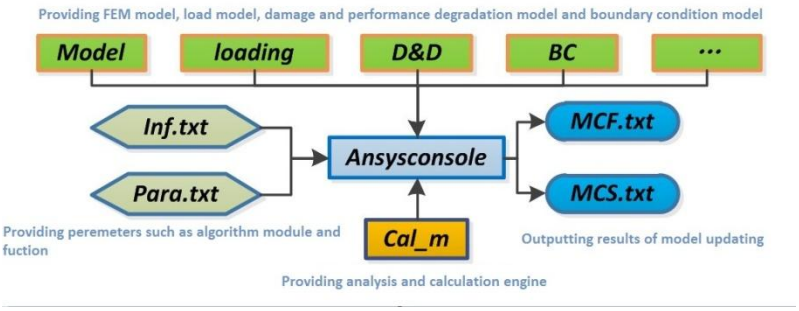

Figure 1. Schematic of parameterized FE model

The Objective Function Library

Objective function is a metric to describe the level of difference between theoretical model and experimental model features, consisting of geometrical and mechanical parameters of structure. Optimization processes are the error minimizing processes between experimental model and theoretical model. The objective functions used for model updating include static-based objective functions, dynamic-based objective function and the objective function combined with static and dynamic.

The means of calling objective function: because of the data generated by the intelligent algorithms are the initial parameters that need to be updated, and the data needed by objective function are the corresponding response of structure, an interface function, parafinder, must be established between the intelligent algorithm and objective function. Its main function are: passing the parameters provided by master program, passing the parameters provided by the optimization algorithm, preliminary processing the parameters provided by optimization algorithm, selecting objective function, returning objective function values to optimization algorithm. The means of calling objective function is shown as figure 2,

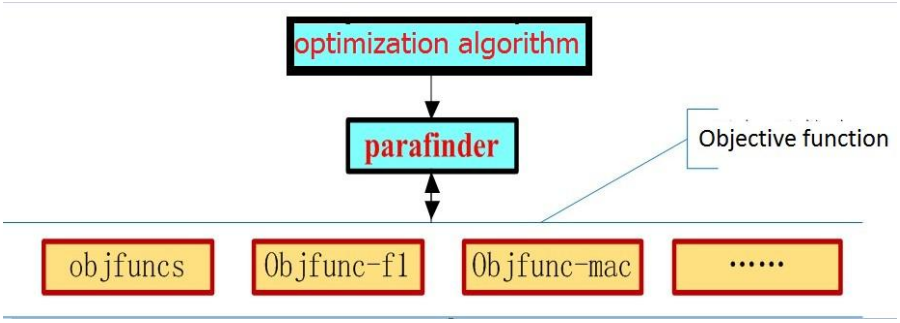

Figure 2. Invoking routine of target functions library

Objective function library: according to the parameters transferred from parafinder, realize the objective function calculation in different objective functions. Every objective function has its own name, its main fuctions include: obtaining the parameters passed by master program, obtaining the data produced by optimization algorithm, calculating the objective function value. According to the commonly used objective functions of bridge model updating, form the objective function library with the standard interface, as the objective function library table shown in table 1.

Table.1 Library of target functions library

The Objective Function Type	Features	Name	Function Name
Static-based		Static objective function	Objfuns
Static and dynamic based		Static and dynamic objective function	Objfunsd
Dynamic-based	Transmission characteristics	Frequency	Objfund-f1
			Objfund-f2
		Mode of vibration	Objfund-mac1
			Objfund-mac2
		Strain mode	Objfund-strain
		Frequency-response function	Objfund-response
	Transmission curvature	Modal curvature	Objfund-mcurvature
		Flexibility curvature	Objfund-fcurvature
	Characteristic parameter	Frequency band energy spectrum	Objfund-spectrum
		Subband energy spectrum	Objfund-subspectrum
	Complicated function	Joint frequency and mode shape	Objfund-fmac
		Flexibility matrix	Objfund-fmatrix
		Modal strain energy	Objfund-starinenergy

Intelligent Algorithms Library Design

After establishing objective function, the model updating problem has transformed into constrained optimization problem with multi-objective functions, in this section, the main problem need to be solved is to provide solver for model updating problem, namely the optimization algorithm. According to achievement that the current intelligent optimization algorithm research has achieved, establishing intelligent algorithm Matlab libraries, acts as solution tools for the model updating software. Below will introduce Particle Swarm Optimization (PSO) as an example to introduce establishment and use method of intelligent optimization algorithm class library.

PSO particle swarm intelligence optimization algorithm: PSO algorithm is inspired from population behavior characteristics and applied to solve optimization problems, each case in the algorithm represents a potential solution of the problem, each particle has a corresponding fitness value determined by the fitness function. Particle's velocity determines the direction and distance of particle movement, the velocity make dynamic adjustment on its own and other particles' mobile experience, so as to realize the individual optimization in solution space. Calling format of PSO algorithm is as follows:

```
d_shown=[]; %PSO drawing parameters 。
```

```
D= ; % The dimension of the input。
mv=60; % The largest particle velocity。
VR=[];% Hunting zone。
PSO = [10 300 20 3 3 0.9 0.9 100 1e-25 2000 1e-5 1 1 1e-5 10 10.0];% Basic parameters of PSO .
seed=;%Original value.
[optOUT, tr, te, bestpos]=pso_Trelea_vectorized('parafinder', D, d_shown, mv, VR, 0, PSO, 'goplotpso', seed);
```

Parafinder is interface function between PSO algorithm and objective function library.

Intelligent optimization algorithms library: according to the calling methods and the means of establishing interface function of PSO algorithm, programing standard library including other commonly used intelligent optimization algorithms, as shown in table 3.

Table 3 Library of intelligent optimal algorithms

Type	Name	Interface	Input	Output
Traditional algorithm	Sequential Quadratic Programming	byfmincon	a,b	X_N
Intelligent algorithm	Particle Swarm Optimization	Bypso	a,b,c,d,e	$A_{M\times N}$
	Genetic Algorithm	Byga	a,c,f,g,h,i	$A_{M\times N}$
	Artificial Fish-swarm Algorithm	Byaf	a,c,j,k,l,m	$A_{M\times N}$
	Ant Colony Algorithm	Byaca	a,s,n,o,p	$A_{M\times N}$
	Immune Algorithms	Byia	a,c,h,i,q,r,s	$A_{M\times N}$
	Annealing Algorithm	Bysa	t,u,v,w	$A_{M\times N}$

In the table, the parameters of the output X_N is the N dimensional trial vector, N is the number of parameters; $A_{M\times N}$ represents M * N trial matrix, M represents the number of population, N represents the number of parameters; the input parameters, a represents the maximum number of iterations, b represents convergence error, c represents the number of population, d represents algorithm type, e represents disturbed value, f represents individual length, g represents generation gap, h is crossover probability, i is mutation probability, j represents the largest feed testing number, k represents perception distance, l represents the crowding degree factor, m represents the moving step, n represents the importance factor of pheromone, o represents the importance factor of inspiration function, p represents the pheromone volatilization factor, q represents the volume of memory database, r represents the evaluation parameters of diversity, s represents the number of distribution center, t represents cooling rate, u represents the initial temperature, v represents terminate temperature, w represents the chain length.

Technical Proposal and Prototype Software of Model Updating Platform

After choosing needful updating parameters, it is necessary to enter the updating module, the parameters produced by optimization algorithm are analysed by finite element analysis to achieve the physical quantities which correspond to measured values, by calculating the objective function value to determine whether the model error has been minimum, and determine whether the iteration should continue, otherwise the updating process is over. The basic flowchart is shown in the figure below.

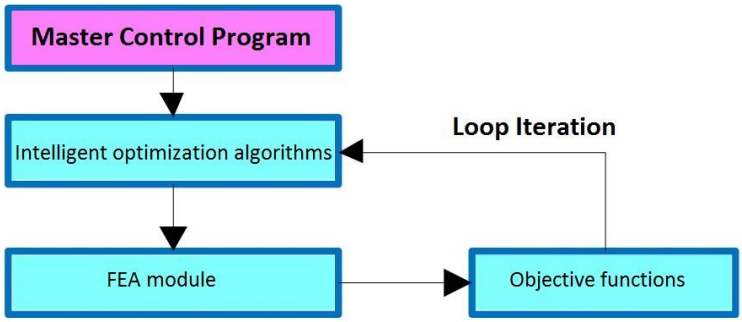

Figure 3. Flowchart of protol software of intelligent model updating

According to above processes, the prototype of the whole software framework is realized by using Matlab, in the prototype c program, ANSYS manipulation can be easily implemented in the Matlab environment, to realize the free combination of model, objective function and optimization algorithm, so as to realize the whole process of automation model updating.

VERIFICATION FOR THE VALIDITY OF PROTOTYPE SOFTWARE

Offline Verification for Prototype Software of Intelligent Updating Platform - A Continuous Beam Experiments

Test model is a three-span reinforced concrete continuous beam structure, the three spans are (1000 + 1700 + 1700) mm respectively, as shown in figure 4, in the bottom of continuous beam, there are five positions, A, B, C, D, E, installed with acceleration sensors. The test is single-point excitation method, and the sampling frequency is 512 Hz.

Before model updating, first parameters with larger sensitivity should be chosen as the parameters for updating. According to the basic situation of the structure, the chosen parameters are shown in the table below,

Table 4. Parameters to be updated

Num	Parameter	Initial value	Upper limit	Lower limit
1	E/Mpa	3.2×10^4	3.6×10^4	2×10^4
2	D/kg/m^3	2.5×10^3	2.6×10^3	2.2×10^3
3	K_1/N. m/rad	1000	1×10^6	0
4	K_2/N. m/rad	1000	1×10^6	0
5	K_3/N. m/rad	1000	1×10^6	0
6	K_4/N. m/rad	1000	1×10^6	0

In the table, E represents elastic modulus, D represents density, Ki represents the rotational stiffness of the ith bearing.

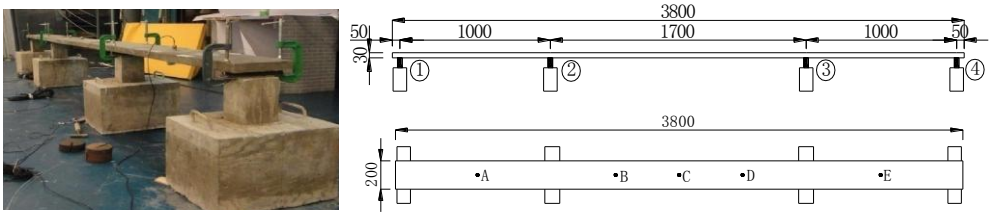

Figure 4. A 3-span continuous beam test

According to the case of model test measured physical quantities, choose the objective function given in equation (9), it is realized by m function, Objfund-fmac, in objective function library. After choosing PSO algorithm in the intelligent optimization algorithms library and configuring appropriate operating parameters, the m scripts automatically perform the task successfully: objective function has reached to convergence approximately, the convergence process is shown in figure 5; parameters searched are shown in table 5; comparison of the modal parameters before and after updating is given in table 6

Table.5 Parameters to be updated

Num	Parameters	Original values	Updating values
1	E /Mpa	3.2e4	2 .67e4
2	D/ kg/m^3	2.5e3	2.53e3
3	K_1/N. m/rad	1000	48320.33
4	K_2/N. m/rad	1000	2448.81
5	K_3/N. m/rad	1000	87109.14
6	K_4/N. m/rad	1000	233.8

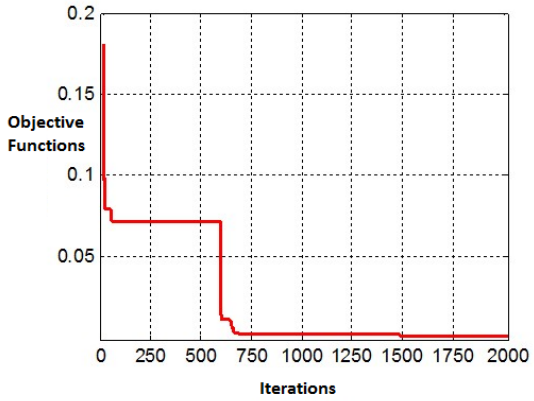

Figure 5. Convergence curve of objective function

Table.6 Frequencies and mode shape parameters after updating

Degree	Test Frequency（Hz）	Original Frequency（Hz）	Updating Frequency（Hz）	Error Before Updating（%）	Error After Updating（%）	MAC Before Updating	MAC After Updating
1	26.38	25.02	26.26	4.76	0.5	0.995	0.98
2	57.68	54.75	58.37	5.06	1.2	0.937	0.989
3	65.22	64.54	65.15	1.05	0.1	0.955	0.979
4	88.49	93.11	87.89	5.22	0.7	0.817	0.901

It can be concluded from table.6 that the updated frequency is very close to the measured frequency, except for the third order, the others' testing frequency error is below 1%; all four order MAC are above 0.9, which shows that the updated mode has good correlation with the measured modal and which further proves the effectiveness of the updating results. The updating process can be writed well in the Matlab script, the main content of the script is model choosing, objective function and optimization algorithm, configuring the input parameters of calling functions, calling ANSYS, monitoring ANSYS performance, finally outputting the calculated results file. Once launched, the whole process runs automatically, no need for manual duty. The example shows the feasibility of the automatic model updating software framework the paper proposed.

Online Verification for Prototype Software of Intelligent Updating Platform---- Cable Force Monitoring

In the above example, the feasibility of automation model updating has been verified offline with measured data. There will be more challenges and application value to updating structure online with going experimental data.

Therefore, choose the real-time cables monitoring devices on a cable-stayed bridge which is equipped with structure monitoring system, connect the prototype software environment in the article to the system, use the program that is developed by the author in the Matlab environment to read remote web-based bridge monitoring system, establish web connection to appointed cable acceleration channel, capture the appointed period of cable acceleration records automatically by compiled m script, identify cable vibration frequency automatically, call ANSYS program automatically for modal analysis of built finite element model, input the measured frequency and the calculated frequency into the objective function automatically, choose the particle swarm optimization algorithm in intelligent optimization algorithm library, update cable model online automatically, choose cable force and bending stiffness as the updating parameters, finally get the updated results.

According to the basic parameters of the cable from design data, these parameters are given corresponding input variable when calling parameterized finite element model. Choosing the PSO algorithm that is based on improved Clerk to update, after 20 iteration steps, the objective function is approximately convergent. The identification results of cable force and bending stiffness are shown in table 7, among them, the reference value of cable force is obtained with from frequency formula, the error between identified cable force and calculated cable force is 3.3%. EI0 represents the identified bending stiffness, EI1 and EI2 represents the bending stiffness when the steel cable is regard as fully bonding model and no binding model respectively. Comparison between the frequency sequence from model updating and the identified one is shown in table 8, decline curve of updating process objective function is given in figure 6.

Table.7 Cable forces and flexible stiffness identified

Cable forces（kN）		**Flexible Stiffness (kN·m2)**	
Updated value	2900	EI_0	486
Reference value	3000	EI_1	511
error（%）	3.3	EI_2	3.4

Table.8 Frequencies after updating

Order	**Measured frequencies（Hz）**	**Updated frequency（Hz）**	**Error（%）**
1	1.27	1.304	2.61
2	2.637	2.609	1.07
3	3.906	3.915	0.23
4	5.273	5.223	0.96
5	6.543	6.533	0.15
6	7.91	7.848	0.79
7	9.18	9.166	0.15

As shown in table 8, the frequency sequences calculated by the automatical model updating prototype software proposed in this paper and the measured frequency sequences are consistent approximately, maximum error of the first seven orders is 2.61%, and the bending stiffness estimation is also reasonable. Above results indicate that the proposed model updating prototype software performs also well and reliably in online environment, and has the value of further development.

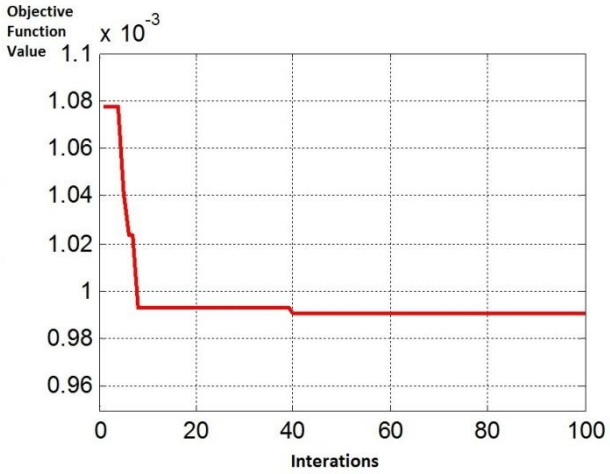

Figure 6. Convergence Curve of target function

CONCLUSION AND PROSPECT

This paper has studied intelligent model updating prototype software framework platform that is based on the commercial finite element software, ANSYS, and the scientific computing software, Matlab, suggested parameterized ANSYS finite element modeling scheme that is suitable for requirements of different types and different bridge structures model updating, built the standard objective function library that aims at different updating target. In order to realize the intellectualization and automation of the whole process of updating, the paper has constructed intelligent algorithms library, that includes PSO, for different types of updating task, the intelligent algorithms library is the drive motor of intelligent, automatic updating process. After careful planning, it can realize the free combination of bridge and component models, objective functions, intelligent algorithms, constitute a flexible, efficient and universal bridge structure intelligent updating prototype software platform in online monitoring environment. In order to illustrate the technical feasibility of the proposed scheme, it realizes the main technical links, proposed in the paper, with the data of a continuous beam vibrations test and a cable-stayed bridge monitoring system, the results proves that the established technical route of monitoring-based online intelligent model updating is feasible.

Further research will focus on the following aspects:

(1) refining requirements for bridge online model updating further, programing a web driven online model updating software environment with web GUI interface;
(2) solving the problem of great amount of calculation in single finite element calculation and slow calculation speed.

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