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MODAL ANALYSIS OF A CONCRETE HIGHWAY BRIDGE - STRUCTURAL CALCULATIONS AND VIBRATION-BASED RESULTS

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ABSTRACT

In the field of civil infrastructure, Structural Health Monitoring systems are implemented more and more frequently with the aim to safeguard the safety and service-life of structures such as bridges and tunnels. Changes in the integrity of the material and/or structural properties of this class of structures is known to adversely affect their performance, which can also be observed from the structures' dynamic response, such as the natural frequencies, damping ratios and mode shapes. The procedure to obtain these response parameters is known as *modal analysis*. Two methods for obtaining these parameters are compared in this paper, one based on a careful analysis of measured vibration sensor data, and another one is based on a structural calculation using a Finite Element Method (FEM).

The dynamic modal characteristics of a structure can be obtained by using vibration-based damage identification techniques such as Stochastic Subspace Identification (SSI). The SSI technique is capable of extracting modal parameters from *output-only* measurements, i.e. using raw data collected by simply monitoring a structure under its normal load. In this paper the application of SSI for the "Hollandse Brug", which is a six-lane Dutch concrete highway bridge under in-service conditions, is described. The bridge is equipped with a sensor network of 145 sensors, including 34 vibration sensors (geo-phones). This paper demonstrates how the key modal parameters can be extracted by applying SSI to the sensor readings.

To assess the quality of the sensor-based results, the modal parameters derived using SSI are compared with the results obtained from Finite Element Method (FEM) calculations. For the FEM calculation, the computer program Scia Engineer is used. The results of the two methods agree well, which shows that the SSI technique under output-only and in-service conditions is an effective tool for modal analysis, and thus a valuable method to be used in structural health monitoring systems.

KEYWORDS

Stochastic subspace identification, stabilization diagram, output-only modal analysis, structural health monitoring, concrete bridge.

INTRODUCTION

The work reported in this paper is conducted as part of the InfraWatch program, which is a cooperative research project between the data analysis group of Leiden University and the materials & Environment group of Delft University of Technology. The aim of the project is to develop methods that can be used to assess the structural health of concrete bridges, by means of both data-driven and structural analysis of high-resolution sensor recordings measured under in-service conditions. The test-case of the project is a major Dutch highway bridge that was recently refurbished, and fitted with an extensive sensor network for determining strain, vibration and weather parameters. Because modal parameters are informative and stable indicators for structural health, determining these modal parameters for bridges is an important issue in the InfraWatch project. Tracking modal parameters over the course of the life span of a bridge will offer a handle to monitor the progress of degradation. InfraWatch is part of a larger nationally funded program, known as IS2C (Solution for Sustainable Construction), which includes a range of topics related to the aging of infrastructures, such as proof loading,

Alkali-Silica Reaction (ASR) and Chloride penetration. The current paper reports the bridge data used, the data analysis, the structural analysis and ends with a modal analysis of the two approaches, being the data driven approach and the FEM approach.

DESCRIPTION OF THE BRIDGE

The bridge in this project, shown as Figure 1, is called Hollandse Brug, which is a concrete bridge, built in the late sixties, and opened in 1969. This bridge forms the motorway connection between Amsterdam and the north-east of the Netherlands. The bridge is composed of 7 spans, with a total length of 354 meters. As shown in Table 1, each span contains 9 pre-stressed prefab girders, which are connected with in situ concrete and reinforcement steel in transverse direction. In addition, two in situ post-tensioned cross girders are present to reduce rotation and torsion of the girders.

Figure 1. The Hollandse Brug

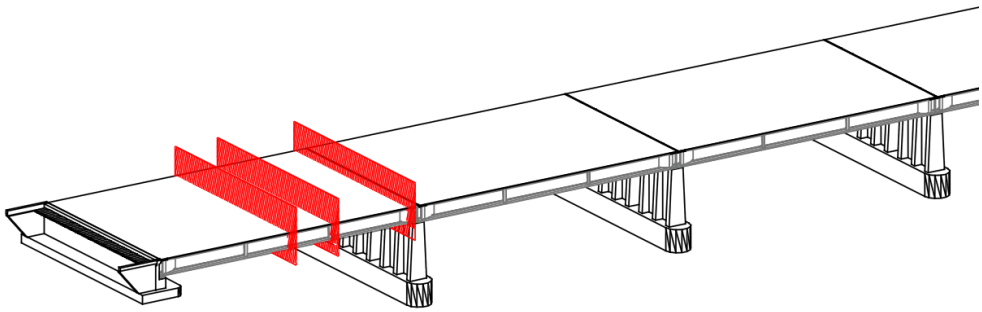

Figure 2. The layout of the sensor network on the Hollandse Brug

Dilatation joints are installed between the girders in longitudinal direction. Due to this connection, the girders can deform freely, and imposed deformations do not influence the internal stresses. Due to these deformation properties, the girders cannot transmit internal forces, and the bridge must be calculated in separated spans.

Table 1. Parameters of Hollandse Brug (one span only)

Parameters	Value	Unit
Weight of the girders	2830	Kg/m
Number of girders	9	-
Weight of the bridge deck	500	Kg/m^2
Width of bridge deck	34	m
Total bridge deck weight	43000	Kg/m
Elastic modulus	38500	MN/m^2
Total bending stiffness	$650*10^9$	Nm^2
Girder length	50.55	m

In the last decades, the condition of the Hollandse Brug decreased dramatically, and after an inspection of TNO (the Dutch National Research Institute) in 2007, the bridge was considered ‘unsafe’. Heavy traffic was blocked from the bridge until a necessary renovation was finished. During renovation, the width of the bridge was also increased with extra girders. Due to these girders, an extra traffic lane in both directions could be realized. In addition to the renovation and the extra girders, a sensor network was installed underneath the first span of the bridge. The monitoring network consists of 34 geo-phones, 91 strain gauges, and 20 temperature sensors. These

sensors were installed at three cross-sections along the width of the bridge. The placement of sensors is shown in Figures 2-3. Furthermore, a weather station and a camera were installed.

Figure 3. Sensor locations of one of the three cross-sections

DATA DRIVEN CALCULATION

Modal analysis is a procedure that extracts model parameters (dynamic characteristics) of a structure from its measured response data. Modal analysis was originally used for Experimental Modal Analysis (EMA), primarily applied to aerospace and mechanical structures, where the structures are excited by controlled dynamic forces. The responses to these forces are then recorded, and the modal parameters are obtained based on both input and output measurements (Renders 2012). Due to improvements in computing capacity, technological advances and developments in sensors and data acquisition systems, these analysis techniques can also be applied in Structural Health Monitoring (SHM) systems for civil infrastructures. In SHM, modal analysis is often applied as a form of Operational Modal Analysis (OMA) (Zhang et al. 2012). The major difference between OMA and EMA is that the input forces of OMA are unknown, and only the output measurements are available. Considering a highway bridge under normal in-service conditions, the input forces may include various vehicles and environmental effects, such as wind and temperature changes, influences which are difficult to measure or quantify. Unfortunately, various techniques upon which EMA relies are invalid for OMA.

Driven by the demand for assessing the health of civil structures, a number of powerful techniques for OMA have been developed. Some common techniques are the Peak-Picking method, the Auto Regressive-Moving Average Vector model (Bodeux and Golinval 2001), the natural excitation technique (Next) (James 1993; James 1995), the Random Decrement Technique (Ibrahim 2001), the Frequency Domain Decomposition (Brincker et al. 2001) and the Stochastic Subspace Identification (SSI) (Van Overschee and De Moor 1996, Peeters and De Roeck 1999). The SSI algorithm is known as one of the most robust methods for OMA measurements, and has already been successfully applied to infrastructures under operational conditions, such as bridges (Peeters and De Roeck 2000a, Thai et al. 2007), towers (Foti et al., 2012; Peeters and De Roeck 2000b), buildings (De Roeck et al. 2000; Bakir 2011).

The SSI Method

Stochastic state space model

The SSI method is especially suited for operational modal parameter identification when only output measurements are available. In the text below, the core steps of the SSI method is discussed. A detailed explanation is beyond the scope of this paper and can be found in the reference (Van Overschee and De Moor 1996, Peeters and De Roeck 1999). The dynamic system of a vibration structure can be modeled by the following discrete-time state space model:

$$x_{k+1} = Ax_k + Bu_k + w_k \quad (1a)$$
$$y_k = Cx_k + Du_k + v_k \quad (1b)$$

where y_k is the measurement of l at discrete time instant k; x_k is the state vector; u_k is the input vector; A is the discrete state matrix, B is the discrete input(system control influence coefficient) matrix, C is a real output

influence coefficient matrix and D is the out control influence coefficient matrix; w_k is the process noise due to disturbances and modeling inaccuracies and models also the white noise input; v_k is the measurement noise due to sensor inaccuracy; Here the process noise w_k and measurement noise v_k are assumed to be zero-mean, white and with covariance matrices.

$$E\left[\begin{pmatrix} w_p \\ v_p \end{pmatrix}\begin{pmatrix} w_q^T & v_q^T \end{pmatrix}\right] = \begin{pmatrix} Q & S \\ S^T & R \end{pmatrix}\delta_{pq} \tag{2}$$

where E is the expected value operator and δ_{pq} is the Kronecker delta. The sequences w_k and v_k are assumed statistically independent of each other. In practice, the input vector u_k is not measured, and only the response of a structure is measured, so it is impossible to distinguish u_k from the process noise w_k and the measurement noise v_k. By implicitly modeling u_k with the noise terms w_k, v_k, the discrete-time stochastic state space model can be represented as:

$$x_{k+1} = Ax_k + w_k \tag{3a}$$
$$y_k = Cx_k + v_k \tag{3b}$$

Here the noise terms w_k, v_k still follows the white noise assumptions. One drawback of the stochastic state space model is that if the input contains some dominant frequency components except for the white noise, these frequency components will appear as poles[1] of the state matrix A.

Estimation of state matrices

Based on Eq. 3, there are several techniques that can be used for system identification through ambient measurements. The technique employed in this paper is called data-driven stochastic subspace identification. All the output measurements are organized in a block Hankel matrix $H \in R^{2li\times j}$ with $2i$ block rows and j columns. Every block consists of l rows. For statistical reasons, it is assumed that $j \to \infty$ The block Hankel matrix H can be represented as:

$$H = \frac{1}{\sqrt{j}}\begin{pmatrix} y_0 & y_1 & \cdots & y_{j-1} \\ y_1 & y_2 & \cdots & y_j \\ \cdots & \cdots & \cdots & \cdots \\ y_{i-1} & y_i & \cdots & y_{i+j-2} \\ y_i & y_{i+1} & \cdots & y_{i+j-1} \\ y_{i+1} & y_{i+2} & \cdots & y_{i+j} \\ \cdots & \cdots & \cdots & \cdots \\ y_{2i-1} & y_{2i} & \cdots & y_{2i+j-2} \end{pmatrix} = \begin{pmatrix} Y_{0|i-1} \\ \overline{Y_{i|2i-1}} \end{pmatrix} = \begin{pmatrix} Y_p \\ Y_f \end{pmatrix} \begin{matrix} \updownarrow li \\ \updownarrow li \end{matrix} \tag{4}$$

where $\frac{1}{\sqrt{j}}$ is the scaled factor, Y_p is stands for the past output matrix, Y_f represents the future output matrix.
The key element of the data-driven SSI is the projection of the row space of the future outputs into the row space of the past outputs. This projection can be defined as:

$$P_i = \frac{Y_f}{Y_p} = Y_f Y_p^T (Y_p Y_p^T)^{\dagger} Y_p \tag{5}$$

where $(\bullet)^{\dagger}$ represents the pseudo-inverse of a matrix.
The projection P_i can be factorized

$$P_i = \Gamma_i X_0 = \begin{pmatrix} C \\ CA \\ CA^2 \\ \cdots \\ CA^{i-1} \end{pmatrix} \begin{pmatrix} x_0 & x_1 & x_2 & \cdots & x_{i-1} \end{pmatrix} \tag{6}$$

where Γ_i is the observability matrix, and X_0 represents the Kalman filter state sequence at time lag zero.
With the help of the singular value decomposition (SVD), the projection P_i can be further decomposed as:

[1]Poles can be understood as peaks in a spectrum, which represent modes.

$$P_i = USV^T = \begin{pmatrix} U_1 & U_2 \end{pmatrix} \begin{pmatrix} S_1 & 0 \\ 0 & S_2 \end{pmatrix} \begin{pmatrix} V_1^T \\ V_2^T \end{pmatrix} \approx U_1 S_1 V_1 \quad (7)$$

The order n of the system can be determined by neglecting the smaller singular values in S_2, and the observability matrix Γ_i and Kalman filter state sequence X_0 can be estimated by:

$$\hat{\Gamma}_i = U_1 S_1^{1/2} \quad (8a)$$
$$\hat{X}_0 = S_1^{1/2} V_1^T \quad (8b)$$

The system parameter matrices A and C can be obtained based on the estimated observability matrix $\hat{\Gamma}_i$:

$$A = \hat{\Gamma}_{i1}^{\dagger} \hat{\Gamma}_{i2} \quad (9a)$$
$$C = \hat{\Gamma}_{li} \quad (9b)$$

where $\hat{\Gamma}_{i1}$ denotes Γ_i without the last l rows, $\hat{\Gamma}_{i2}$ represents $\hat{\Gamma}_i$ without the first l rows, and $\hat{\Gamma}_{li}$ stands for the first l rows of Γ_i.

Modal parameters

The modal parameters are derived from the system parameter matrices A and C:

$$A = \Psi[\mu_i]\Psi^{-1} \quad (10)$$
$$f_i = \frac{|\lambda_i|}{2\pi} \quad (11)$$
$$\xi_i = \frac{Re(\lambda_i)}{|\lambda_i|} \quad (12)$$
$$\Phi = C\Psi \quad (13)$$

where μ_i are the discrete time poles, $\lambda_i = \frac{\ln(\mu_i)}{\Delta T}$ are the continuous poles, f_i are the natural frequencies, ξ_i are the damping ratios, Φ is the mode shape matrix.

The stabilization diagram

It is assumed that all the input forces of the SSI procedure are white noise and the length of the recording is infinite. In practice, the measurements used for SSI are limited, and usually contain some other dominant frequency components. As shown in Eq. 7, the order of the system is obtained by ignoring the smaller singular values, which is usually higher than the actual system order. All of these factors may introduce spurious, numerical poles to the system. To criticize the physical and the spurious, numerical poles, the stability diagram is introduced. The basic idea of the stabilization diagram is to iterate the system order n from a lower value to the maximum order. It is assumed that the lowest order is unstable, so the modal parameters of current order with those of one order lower are compared. If the differences are under user defined limits, then this order is considered to be a stable order. The limits are defined as:

$$\left|\frac{f_k - f_{k-1}}{f_k}\right| < lim_f\% \quad (14)$$
$$\left|\frac{\xi_k - \xi_{k-1}}{\xi_k}\right| < lim_\xi\% \quad (15)$$
$$\left(1 - MAC(k, k-1)\right) < lim_{MAC}\% \quad (16)$$

where $k > 1$ denotes the model order, f is the frequency, ξ is the damping ratio, $\lim_f$ % is the frequency limit, $lim_\xi\%$ is the limit for the damping, $lim_{MAC}\%$ is the limit for the modal assurance criterion (MAC).

The MAC value ranges from 0 to 1. 0 means that there is no similarity between the compared mode shapes, and 1 means these two mode shapes are consistent. The MAC can be defined as:

$$MAC(k, k-1) = \frac{|\Phi_k^H \Phi_{k-1}|^2}{(\Phi_k^H \Phi_k)(\Phi_{k-1}^H \Phi_{k-1})} \quad (17)$$

STRUCTURAL CALCULATION

The mode properties can be calculated using measurement data, but also by using structural properties. Structural calculation has been used to model the behaviour of the Hollandse Brug as well. The calculations are mainly based on the preserved blue prints, which were made in the mid-sixties. These drawings contain information about the dimensions and properties of the bridge, including reinforcement and prestressing details. Properties which are not present in the drawings are based on a site visit or based on information obtained from bridges of the same age. The computer program Scia Engineer (Nemetschek_Scia 2011) is used for the structural calculation. This program is based on the finite element method (FEM), which includes line and surface elements. The two-dimensional elements contain additional properties for approximating the three-dimensional properties of the actual deck. In addition, it is possible to apply a certain vertical distance between elements to include the correct properties. The bridge is modelled as plate with a constant width, which is the deck, and internal edges, which are the girders of the bridge, as shown in Figure 4. The girders are located underneath the deck, so that the structural properties are calculated correctly. The moment of inertia of the bridge is calculated automatically by the program, and validated using manual calculation.

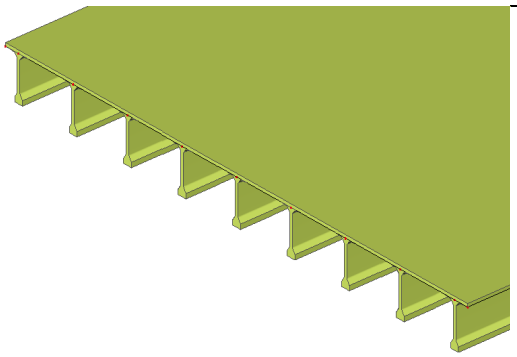

Figure 4. Model

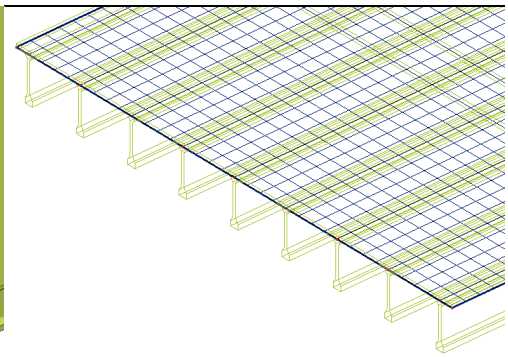

Figure 5. Elements FEM

The calculation method is based on the Mindlin-Reissner plate theory (Durban, Givoli, Simmonds 2002) (Steele, Balch 2009), which contains stresses and deformations due to moments, as well as shear. Mindlin-Reissner is a first order shear deformation theory that implies a linear displacement variation through the thickness of an element. The relative error can be reduced by using smaller elements. For an acceptable relative error, there must be at least twenty elements in longitudinal direction. For the calculation of the Hollandse Brug, average element sizes of 1.0 meters were chosen, which corresponds to one fourth of the distance between the girders and one fiftieth of the span (see Figure 5). Mode calculation is one of the possible calculation methods of the program. Natural frequencies are calculated based on the modulus of elasticity, sectional area, moment of inertia, and the unit mass of the materials.

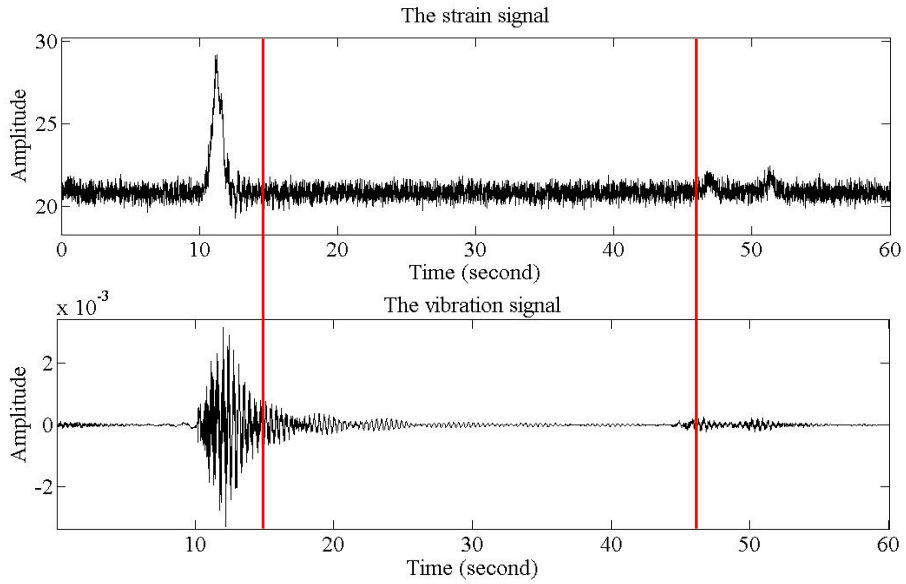

Figure 6. Free vibration period: the period between two red lines

MODAL ANALYSIS ON THE HOLLANDSE BRUG

Data Acquisition

The 145 sensor at the bridge have been measuring around the clock, at a sampling frequency of 100 Hz. The signals of geo-phones (vibration) sensors are chosen for modal analysis. A total of 12 vibration sensors located on the girders were selected, which were equally spaced in both longitudinal and transversal direction. The sensors are located on four different girders on three locations. To reduce the influence of inputs that cause non-natural frequencies, only data from the free vibration period is selected, which is the period that occurs immediately after a vehicle has passed, and before a next vehicle appears on the bridge. Details of how such periods can be identified in the data can be found in (Miao et al. 2013). Following this procedure, a free vibration period lasting for 34 seconds (3400 data points for each sensor) has been obtained.

Modal Parameters Extraction

The first activity to extract modal parameters from measurements with the SSI method, is creating a Hankel matrix with 24 block rows (30 rows per block), and 3377 columns. One key parameter for SSI is the order of the system. Because of operational noise, it is impossible to obtain the system order precisely from the singular value of the Hankel matrix projection. If the system order is estimated with a lower value, some physical poles will be missed; otherwise, spurious numerical poles may appear. The stabilization diagram is useful to separate physical poles from spurious numerical poles. In the stabilization diagram, the stable criteria are set as 1% for natural frequencies, 30% for damping ratios and 1% for MAC. The system order is tested from a minimal order 2 to a maximum order of 30. As shown in Figure 7, the physical poles are represented as red stars and spurious poles are represented as black circles. We assume the initial status of each pole is instable, e.g. the two poles of mode order 2 are represented as blue circles. The background spectrum is derived from the discrete Fourier transform (DFT) on one of the selected 12 vibration signals. The high coherence between the peaks in DFT spectrum and physical poles obtained with the SSI method indicates that it is reasonable to employ the SSI method to analyze modal parameters. However, with the SSI method, we can obtain more poles, e.g. the one around 10 Hz, which is absent in the DFT method.

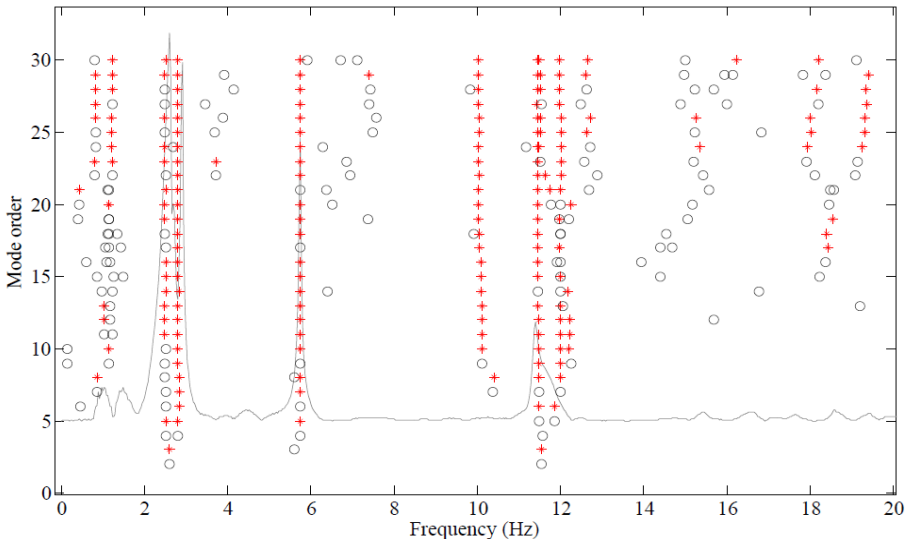

Figure 7. The stabilization diagram: The red stars represent stable physical poles; The black circles represent the spurious poles

In this paper, the modal parameters calculated with the FEM are based on the first and the second order mode shapes. The modal parameters for higher modes are obtained by making combinations of these two basic mode shapes.

RESULTS

Natural Frequency

The natural frequencies obtained from the two methods are displayed in Table 2. It can be observed that the first three frequencies show a good match, while the fourth and the fifth frequency contain differences up to 21 per cent.

Table 2. Modal parameters

Mode	Mode shape	Frequency (SSI calculation)	Frequency (FEM calculation)	Relative error
1st	Bending	2.51 Hz	2.63 Hz	4.8 %
2nd	Torsional	2.81 Hz	2.74 Hz	2.5 %
3rd	Bending & Torsional	5.74 Hz	6.25 Hz	8.9 %
4th	Bending	10.09 Hz	8.56 Hz	15.2%
5th	Torsional	11.47 Hz	9.05 Hz	21.1%
6th	Bending & Torsional	11.99Hz	11.68 Hz	2.6%

There could be three reasons for this error. The first is an FEM modeling error; the second order frequency is sensitive to non-equally distributed masses, like the cross girders. Due to uncertainty in mass and connection of these girders, FEM calculation errors increase. The second and the third reason could point to the SSI calculation. Due to the finite measurements, it is difficult to measure the exact cycle of a higher frequency. Furthermore, the amplitudes of the higher modes are marginal, which makes it difficult to generate the natural frequencies from the noise.

Mode Shapes

Figures 8-13 show the first six mode shapes for respectively the SSI calculation (left) and the FEM calculation (right). Because the sensor network just covers half of the bridge span, the mode shapes of the unmeasured half span are modeled using the existing measurements and structural knowledge. Clearly, the mode shapes derived from measurements are comparable.

Damping Ratios

The damping ratio depends on the material properties, the chemical composition, and the structural properties of the bridge. Additionally, the damping ratio depends on the size and amount of micro-cracks and the type of support and connection. Finally, the damping ratio is influenced by the construction work. Because the contribution of the damping ratio depends on a number of uncertain parameters, it is nearly impossible to calculate the damping ratio through structural analysis. Because of this, the damping ratio is not further discussed in this paper.

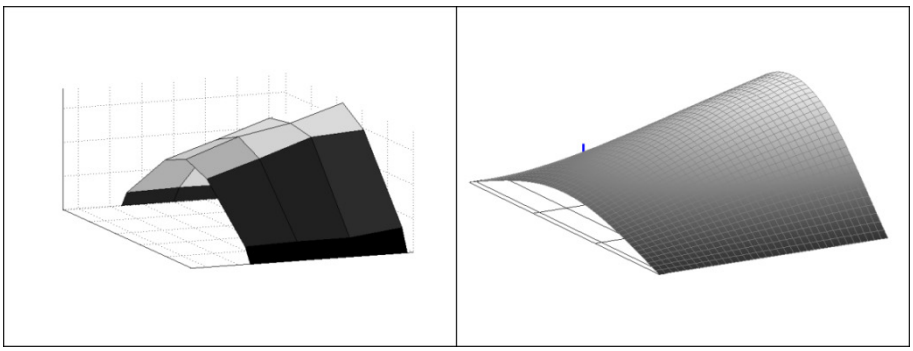

Figure 8. The first bending mode shapes: left: based on SSI; right: based on FEM

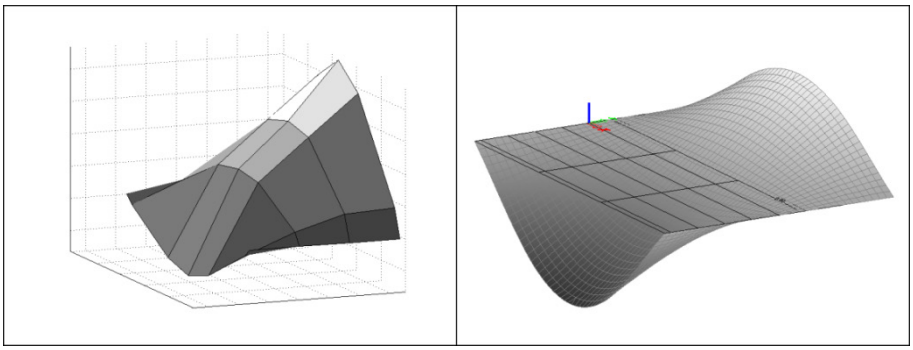

Figure 9. The second torsional model shapes: left: based on SSI; right: based on FEM

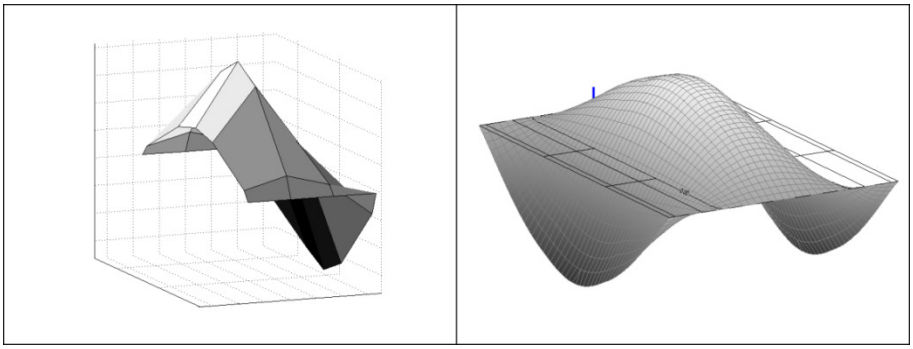

Figure 10. The third bending and torsional model shapes: left: based on SSI; right: based on FEM

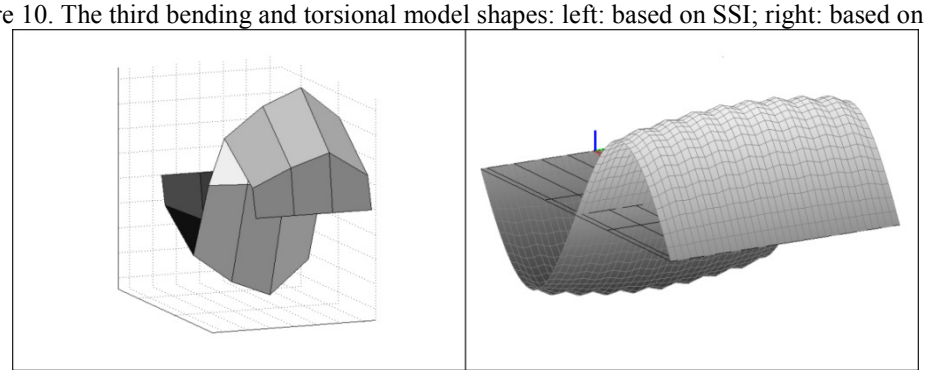

Figure 11. The fourth bending model shapes: left: based on SSI; right: based on FEM

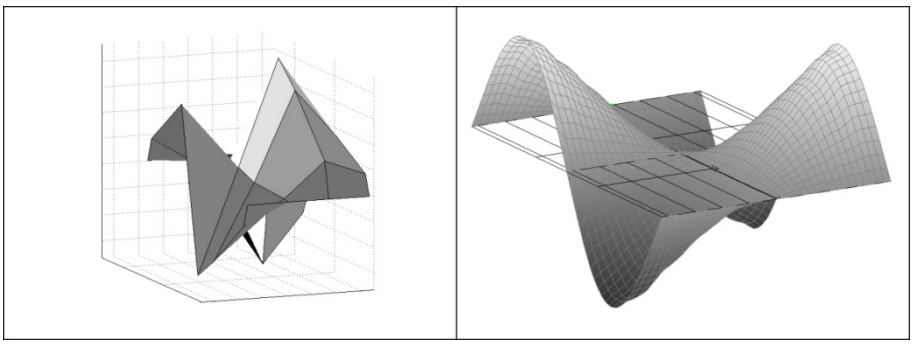

Figure 12. The fifth torsional model shapes: left: based on SSI; right: based on FEM

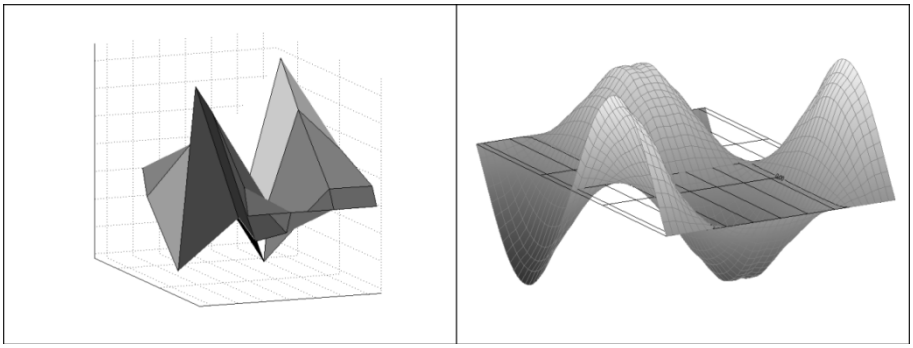

Figure 13 The sixth bending and torsional model shapes: left: based on SSI; right: based on FEM

CONCLUSIONS

The mode properties of a concrete highway bridge are calculated using two different calculation principles. The first method is an SSI calculation, which is based on the measurement data archived from the sensor network at the bridge. The second method is an FEM calculation, which is mainly based on design drawings with structural properties. The first-order natural frequencies show good agreement between both calculation methods. The second-order frequencies show some level of relative error. Uncertainties in modeling and measuring are the

main causes of this phenomenon. The error can be limited using more sensors, working at a higher frequency and a more realistic calculation model.

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