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STOCHASTIC MODEL UPDATING OF BALLAST STIFFNESS IN COLD CONDITIONS

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ABSTRACT

For high speed railway bridges it is mandatory to perform checks on the acceleration to ensure ballast stability and passenger comfort. The usual way to perform these checks is by simulations using dynamic finite element models. Given the complexity that this models can reach usually mode superposition techniques are used to speed up computations, which depend critically on the accuracy of the estimated eigenmodes and eigenfrequencies. Some assumptions done for static analysis are not safe for dynamic considerations, since they affect the modal shapes and frequencies, so more parameters need to be known accurately for dynamic analyses. Further some properties might change during the lifetime of the structure, so all their potential values need to be known and studied.
For the Skidträsk Bridge, that forms part of the Swedish high-speed Bothnia Line, it was observed that the eigenfrequencies shown a step-like increase when the temperatures dropped below 0 C. This increase is important enough to invalidate simulations that do not consider this effect. In low temperature conditions not only the mean value of the measured frequencies increased, also the variance increased considerably. It was hypothesized that this change in the eigenfrequencies was due to higher ballast stiffness in colder conditions. Given the large spread of the measured eigenfrequencies the ballast stiffness parameters were assumed to be stochastic variables with an unknown distribution, rather than fixed values. A Bayesian Updating scheme was implemented to determine this distribution from measurements. Data gathered during 9 months of monitoring was used in conjunction with a finite element model and a meta-model in the procedure resulting in an estimation of the ballast stiffness in cold condition that matches the increased mean and variance in the eigenfrequencies.

KEYWORDS

Markov chain Monte Carlo, stochastic model updating, seasonal effect, railway bridge.

INTRODUCTION

In general a numerical model of a structure will not be an exact representation due to modeling errors (Oden *et al.* 2005; Sanayei *et al.* 2001) (unknown boundary conditions, erroneously assumed homogeneous material, imperfections in construction…). Using model updating techniques selected parameters in the model can be then updated to match data measured on the structure (Friswell and Motterhead 1995; Brownjohn *et al.* 2003; Jaishi and Ren, 2006). The problem with this "deterministic" updating scheme is that the data measured on the structure is seldom perfectly consistent. Variations arise from measurement errors, variability in nominal identical structures, changes in the environmental conditions, etc. Instead of obtaining a single "most likely" value for the updated parameters stochastic model updating returns the probability distribution of these parameters, representing thus the natural variability on the measured data (Vanel *et al.* 2000; Hua *et al.* 2008). The field stochastic model updating is recognized to have been started by Collin *et al.* (Collins *et al.* 2012) in their seminal work and has since become a well-established discipline with a large number of active researchers. This method is especially useful when modeling structures that will be mass-produced (vehicles and machine part inter alia), since it can account for the variability in the production process. Bridges, on the other hand, are one of-a-kind structures, and therefore stochastic model updating is less often applied to bridge structural models (there have been a number of application for deterioration and reliability models and creeping-shrinking (Papadimitriou *et al.* 2001; Enright and Frangopol 1999; Estes and Frangopol 2003). Some construction materials used in bridge construction can nonetheless show significant variation during their lifetime making it useful to have a probabilistic characterization of its properties.

Typically when railway bridges are designed for low speeds, only static analyses are performed. The magnitude and position of the loads to consider are given in the building regulations. A number of simplifying assumptions are usually made for these calculations. For example, when the modulus of elasticity or other stiffness properties of a material or component is unknown, it is safe for a static analysis to assume a very low value. However, these assumptions need not to be safe in dynamic analyses, which are mandatory for bridges designed for high-speed trains in EUROCODE (CEN, 1994). Dynamic analyses are mostly performed in the form of a Finite Element (FE) dynamic simulation. The usual objective of such a dynamic simulation is to check for the levels of accelerations of the bridge deck. Limits are imposed in the codes for the vertical accelerations in order to ensure track stability and passenger comfort. Results from this kind of analyses are only accurate if the eigenmodes and eigenfrequencies of the structure are accurately represented by the FE model. Both the eigenfrequencies and the eigenmodes can be unrepresentative of the real structure if too low (or otherwise inaccurate) a stiffness is assumed for a material, as it is sometimes done for static analyses.
Furthermore, some of these parameters can be expected to change with time, either due to aging or seasonal effects. In general, there is no simple way of deciding how an increase in the stiffness of an element will affect the accelerations. Therefore, to ensure the safety and comfort levels in bridges, simulations should be made taking into account the natural variability of certain parameters. It then follows that, for bridges for high speed trains, it is of importance to have a precise estimation of the values that these parameters might assume during the lifetime of the structure.
In this particular study we look at the variability of ballast stiffness in extreme low temperatures (from -10 °C to -30 °C) that are common in colder climates such as the one in northern Sweden. The bridge under consideration is part of a high-speed line, so the limits imposed by the concerned regulations regarding the vertical deck accelerations apply.
The contribution of the ballast to the overall stiffness of the bridge is usually disregarded in static analyses. This assumption is conservative in that type of analysis. Even for dynamic considerations, the stiffness of the ballast is generally very low and almost insignificant, especially at temperatures above zero. It was observed, though, that the eigenfrequencies of a bridge could vary considerable from winter to summer (18% for the first eigenfrequency). This change is far more than what could be expected by thermal expansion in a simply supported beam (Xia *et al.* 2012) and the change was a step-shaped one rather than linear. It was therefore attributed to an increased stiffness in the ballast due to water freezing and acting as a bonding between the ballast particles.
The goal of this study is to infer some stiffness properties of the stiff ballast layer from measurements carried out on the bridge. This information can be necessary for accurate dynamic analyses under some conditions, as will be shown. The main parameters measured were the two first eigenfrequencies of the structure and their dependence on air temperature. During the monitoring period, an important variability of the eigenfrequencies was observed in the measurements, even for a fixed temperature. The stiffness properties of the ballast were thus not expected to be completely deterministic for a given temperature, but to have a stochastic variation. Among the possible explanations of this fact there is for example the inherent uncertainty of the bonding properties of frozen water and also in the fact that for a given air temperature the ballast temperature can vary significantly due to sun light exposure and other factors. Thus, rather than a specific value of the stiffness properties of the ballast, a probability distribution of values was sought. We attempt to find the multivariate distribution of the ballast stiffness properties that corresponds to and explains the observed distribution of eigenfrequencies.
The process used to obtain this distribution (graphically depicted in Figure 1) resulted in an estimation of the distribution of ballast stiffness parameters that results in a frequency distribution that matches the observed one.

THE BRIDGE

The Skidträsk Bridge is a one span, simply supported, composite railway bridge situated in northern Sweden. The Skidträsk Bridge is located between the towns of Jörn and Bastuträsk and carries a single track over a 36 meter span. The bridge is composed of two identical, I-beams with variable cross section, a concrete slab and a ballast layer with sleepers and tracks. The bridge is skewed, so the beams run parallel (separated by a distance of 2800 mm) but with a longitudinal offset of 1616 mm.
The I-beams vary in height in such a way that they are shallower at the mid-span. This is a purely aesthetical decision, since the bridge is simply supported and, consequently, the larger bending moments are located at the mid-span. To compensate for this both the thickness of the flanges and of the web, as well as the width of the flanges, increase from the ends and achieve a maximum value at the middle. By virtue of the increased width and thickness, the moment of inertia of the beams reaches its maximum at the mid-span even with the much reduced beam height. The two beams are connected by four transversal stiffeners.
The reinforced concrete slab is U-shaped and of 330 mm of thickness. The concrete used had a design compressive strength of 30 GPa. The ballast layer is 600 mm thick. On top of it the track are laid on concrete pre-stressed sleepers.

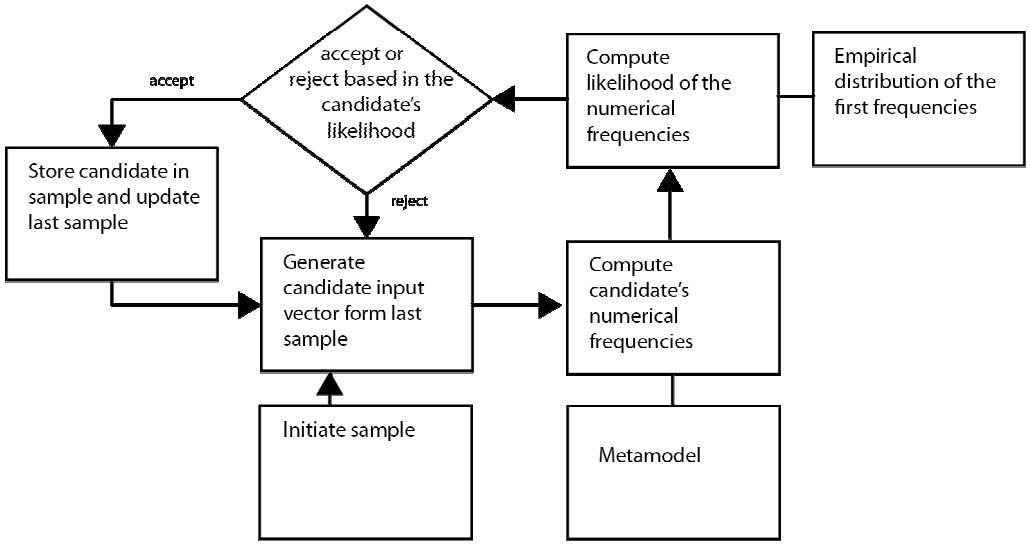

Figure 1. Flow chart of the updating method used in this study. It includes the acquisition and analysis of data (upper-right section), the development of the model and metamodel (lower-right corner) and the updating procedure (left section) that builds on the results of the two previous steps

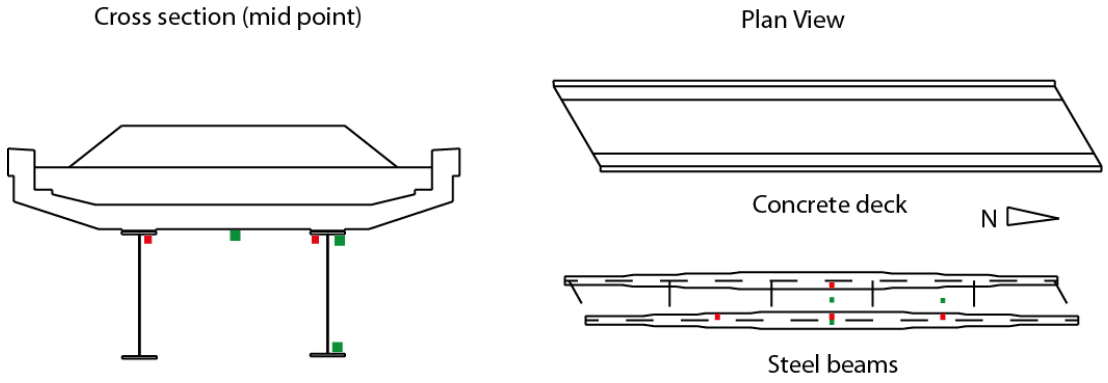

Figure 2. Concrete deck and steel beams in plan and cross section view with the location of the sensors marked (The green dots represent strain gauges and red dots represent accelerometers)

INSTRUMENTATION

The Skidträsk Bridge was originally instrumented in 2005 with four accelerometers, a thermocouple and two longitudinal strain gauges. The accelerometers where located one at each quarter point (on the east beam), and two at the mid-span point (in both the east and west beams). All the accelerometers measured vertical acceleration. The strain gauges were located at the upper and lower flanges of the east beam at mid-span. In 2006 this instrumentation set was enhanced with two strain gauges that measured transversal strains. The transversal strain gauges were located at the upper flange at the mid-span and quarter point of the east beam. The influence line of these sensors is much more localized that that of the strain gauges that measure longitudinally, therefore they can be used to identify the position and speed of trains as they cross the bridge. The layout of the sensors is outlined in Figure 2.

MEASUREMENTS

The instrumentation setup described above was used to gather data from train passages. Measurements of individual train passages were saved with a sampling rate of 600 Hz and filtered at 100 Hz with an anti-alias filter. Measurements were carried out for 9 months, from November 2005 to July 2006 and resulted in signals corresponding to thousands of train passages stored from all seasons and spanning temperatures of -30 °C to 30 °C. From these measurements the first bending and torsional eigenfrequencies were extracted. The bridge is skewed but, ignoring this skewness, it is almost perfectly symmetric. Therefore the first bending eigenmode can

be considered to be approximately symmetric along the longitudinal axis of the bridge and the first torsional mode can be considered to be practically anti-symmetric along the same axis. Therefore, since two accelerometers were placed at opposite sides of the mid span, the acceleration signal can be easily separated into a bending (by adding them together) and a torsional (by subtracting them) component. An example is shown in Figure 3.
The separation of the acceleration signals into a vertical and a torsional component allow for an easy and clear identification of the two first eigenfrequencies corresponding to the first bending mode and to the first torsional mode. These two frequencies are, at least in summer conditions, quite close to each other, so they cannot easily be separated from one single signal. The identification was done by means of a simple FFT of the free vibrations. The free vibrations are easy to identify in the measurements from 2006 given that the transversal strain gauges give a clear indication of the position of the train on the bridge. For the measurements corresponding to 2005 (which included the lowest temperatures) a different approach had to be used. For these measurements the RMS level of the acceleration and strain measurements were used to indicate the moment when the train leaves the bridge. The large amount of information did not allow for a manual inspection of each and every signal so the set of identified eigenfrequencies is expected to cause a number of outliers, i.e. identified eigenfrequencies that do not correspond to the assumed mode.

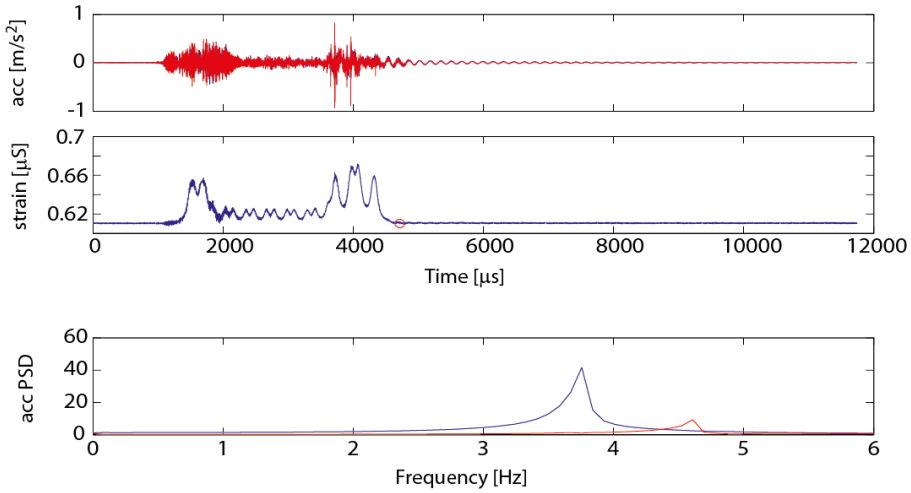

Figure 3. Top - two acceleration histories of a train passage (mid-span); middle - strain history of the same passage (the red dot marks the start of the free vibrations as identified by the program); bottom - bending and torsion accelerations on the frequency domain (only free vibrations)

Possible explanations are, for example, failure to properly identify the free vibration part of the signals. Also, the first torsional mode was excited very little during certain train passages, so it is possible, given that the bridge is not perfectly symmetric, that the energy content of the signal assumed to represent to the torsional acceleration could be higher at frequencies corresponding to the first bending mode.
These analyses revealed that the eigenfrequencies of the bridge varied with temperature. Specifically the detected eigenfrequencies were clustered around a value of 3.8 Hz for temperatures above 10 °C and around a completely different value of 4.5 Hz for temperatures below -10 °C. A transition zone was found in between -10 and 10 °C (see Figure 4).

IDENTIFICATION PROBLEM

The working assumption for this study was that the higher eigenfrequency detected at cold temperatures was caused by the stiffening of the ballast due to the freezing. This assumption justifies to some extent the fact that the frequency seems to vary in a step-like fashion rather than linearly with the temperature. Water will freeze in between the ballast particles, giving it a higher stiffness and it is this stiffness contribution that increases the eigenfrequencies of the bridge as a whole. The transition zone for temperatures between -10 °C and 10 °C can thus be explained by the fact that at those temperatures the ballast can be in any intermediate step between completely “glued” together by ice (referred to as “stiff” or “cold” ballast) and completely loose, as in summer conditions. This is due to the fact that the actual temperature on the ballast can be expected to vary from the temperature measured on the thermocouple due to differences in incoming sun light and the different thermal inertias of the ballast and the thermocouple. Naturally, the question as to what the expected stiffness of the stiff ballast is then arises. As can be observed, the frequency in winter conditions present a much larger spread when compared to the value obtained in summer conditions. This fact stands to reason, since the amount of bonding

can hardly be expected to be consistently the same for a given temperature. With this in mind the stiffness properties of the stiff ballast were considered to be stochastic variable, the distribution of which was to be identified, rather than a fixed deterministic quantity.
In Figure 2, the bond strengths predicted using the proposed bond-slip models are compared with the results of the 253 pull tests in Lu *et al.*'s (2005) database. It can be seen that the proposed bond-slip models give results in close agreement with the test results and perform better than any other bond-slip models. The results of the precise model and the simplified model are almost the same, with the precise model performing very slightly better. Table 1 shows that the prediction of the proposed bi-linear model for the bond strength, which can be given as a closed-form expression (Lu *et al.* 2005), performs significantly better than all existing bond strength models except Chen and Teng's (2001) model. For the prediction of bond strength, Chen and Teng's (2001) model is still recommended for use in design due to its simple form and good accuracy.

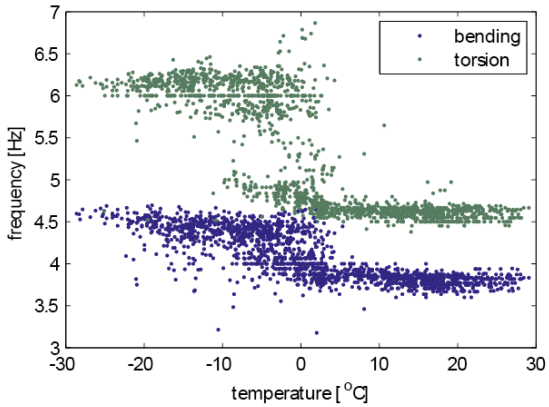

Figure 4. Measured first and second eigenfrequencies for a number of train passages (free decay) (Their dependence on the temperature is apparent)

BAYESIAN UPDATING

Bayesian updating is a technique to take into consideration new knowledge (also called evidence) about stochastic variables. If a certain variable, due to epistemic or statistic uncertainty, is given an initial probability distribution, subsequent measurements of that variable can improve the available knowledge about it and thus lead to a modified and more accurate probability distribution. This process can be performed in a rigorous and systematic way using Bayes theorem. The initial assigned probability distribution is called the prior distribution, while the updated distribution that takes the new evidence into consideration is called the posterior distribution (see Figure 5).
Bayes theorem states that the (posterior) probability p_{post} of a certain event x given a set of observed evidence Y_{obs} equals the likelihood of the evidence given the event, times the probability of the event disregarding the evidence times a normalizing constant (see equation below). This normalizing constant called Bayes Integral, is the likelihood of the evidence disregarding the event (i.e. the integral over all possible events). Bayes Integral is expensive to evaluate, because it generally involves multidimensional numeric integration, but there are methods to obtain the posterior distribution without having to evaluate this integral (Ntzoufras 2009).

$$p_{post}(x \mid Y_{obs}) = \frac{p_{prior}(x) L(Y_{obs} \mid x)}{\int_X p_{prior}(x) L(Y_{obs} \mid x) dx} \quad (1)$$

In our study we attempt to find the probability distribution of the stochastic properties of our model given our evidence (i.e. the measured eigenfrequencies). The variables considered stochastic in our model were the modulus of elasticity of the ballast and the shear stiffness of the interface between the ballast and the concrete. Both variables are updated together, resulting in one bivariate distribution that considers the interdependence of the two variables.
In other words, we want to determine the bivariate distribution of the (two) properties of the ballast that we chose to update that will result, when used as input in the FE model, in a frequency distribution that approximates the observed one. The observed frequency distribution is also bivariate, since the two first eigenfrequencies are considered. Naturally, when the first eigenfrequency is high it is expected to be due to a high ballast stiffness, which should lead to a high value of the second eigenfrequency also, so the distribution for the two first eigenfrequencies is obtained considering their interconnection with a so-called copula (Nelsen, 1999), as will be explained later.

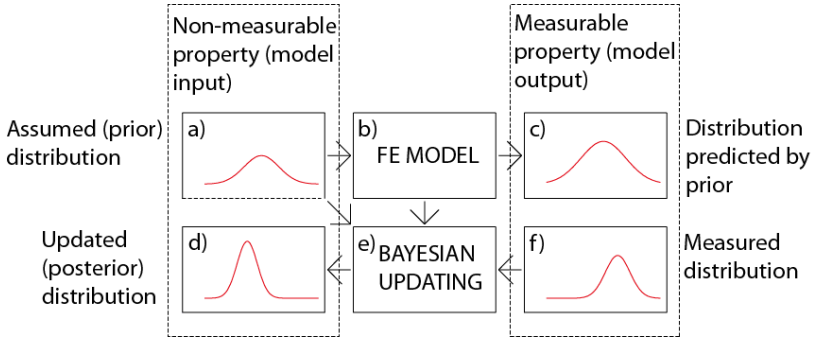

Figure 5. Schematic of the principle behind Bayesian updating - to obtain the distribution of a non-measurable property the information available about another measured property and a model of the structure are combined with an assumption on the sought property

MODEL AND METAMODEL

A final element model was used to estimate the eigenfrequencies in which a given set of ballast parameters will result. The steel beams were modeled as beam elements. The neutral layer of the beams was made to vary vertically, to correspond to the vertical curvature of the beams. On the other hand the slight horizontal curvature of the bridge was disregarded. The cross section of the beam elements varies continuously to match the actual cross-section of the beams. The steel beams where rigidly connected to the concrete slab that was modeled as a center horizontal slab and two "wings" forming an angle with the horizontal part of the slab. Shell elements where used for the concrete slab. The thickness of the slab was considered to be constant throughout the model, although there are some variations in thickness in the real structure. The concrete was assumed to be entirely in compression as it was designed to, so cracks were not considered. The reinforcement was considered by means of a slightly increased modulus of elasticity. The edge beams where modeled also as beam elements and connected rigidly to the sides of the outer concrete slabs.

The ballast was modeled as a layer (also with shell elements) above the horizontal part of the concrete slab and connected to it via linear springs in the horizontal direction. The ballast was extended at both sides of the bridge for 36 m (the length of the bridge itself). The extension of the ballast was coupled to a "soil" that had all it degrees of freedom locked. The connection between the ballast and the "soil" was of the same type than the connection of the ballast to the bridge. This was done to take into consideration the continuity of the ballast. Given that the ballast was assumed to have non-zero stiffness, its continuity was expected to contribute to the overall stiffness of the structure.

The track was also included in the model with beam elements rigidly connected to the ballast layer. It was also made continuous over the bridge ends, further contributing to the stiffness of the bridge (see Figure 6).

Obtaining the first two eigenfrequencies from this model took about 16 seconds. Thousands of calculations were needed to obtain a good estimation of the distribution of ballast properties that match the measured the distribution of eigenfrequencies. Therefore it was judged appropriate to generate a metamodel to speed up the process (Martin and Simpson, 2005).

The roll of the finite element model in this work was to return the two first eigenfrequencies given the two input variables (namely, the horizontal or shear stiffness of the ballast-to-concrete interface and the modulus of elasticity of the ballast).

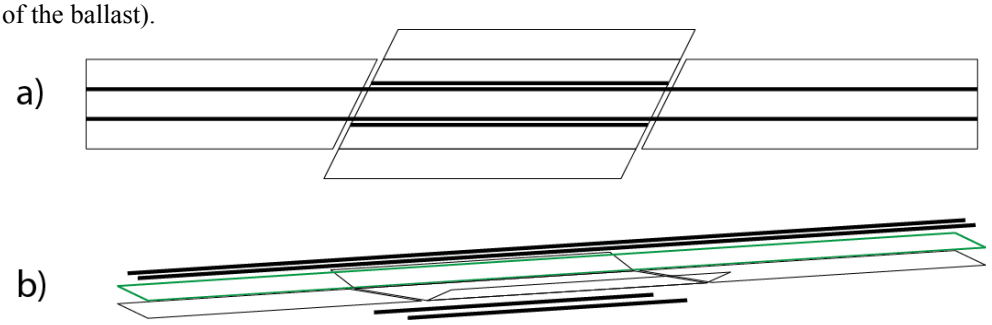

Figure 6. Schematic of the FE model - the ballast, tracks and soil extend one bridge length at each side of the bridge (the track is connected rigidly to the ballast which is connected elastically to the soil and bridge)

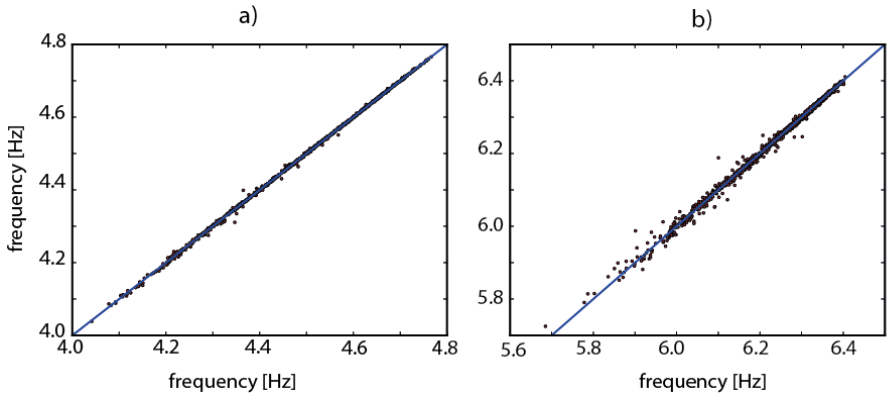

Figure 7. Results from the metamodel plotted against those from the real FE model for the first (a) and second (b) eigenfrequencies, for a large number of input parameters. The mean squared relative error is 4.0 10^{-4} for the first eigenfrequency and 5.6 10^{-3} for the second

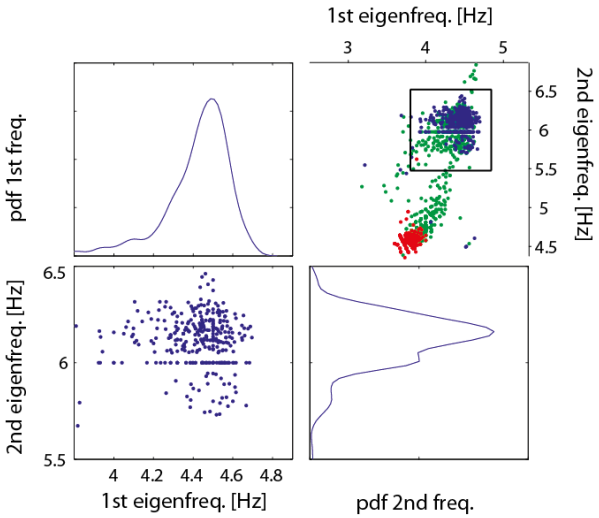

Figure 8. Upper-right corner: first and second eigenfrequencies for all the train passages measured (free decay); red indicates passages at temperatures 10 °C and higher; blue indicate temperatures -10 °C and lower; the cold frequencies in the highlighted area are zoomed-in in the lower-left corner; the other two panels show the experimental PDF's of the cold frequencies

In the chosen sampling method (described in the following section), each sample depends on the previous and on its likelihood. Therefore the process cannot be parallelized and the conversion from input variables to eigenfrequencies and then to likelihood has to be performed one after the other. To avoid this bottleneck, a large number of randomly generated input variables where evaluated using the FE model. Thanks to the parallelization this process could be done orders of magnitude faster, requiring hours instead of days. The distribution of these input values was chosen to densely cover the area of what was considered to be realistic values. Then a Gaussian Process was fitted to these input values and their associated eigenfrequencies by Kriging (Cressie 1993). In this way, new input values could be evaluated (or rather interpolated) from the Gaussian Process bypassing the FE model altogether. A comparison between the results obtained by the model and metamodel is shown in Figure 7.

TARGET AND PRIOR DISTRIBUTION

As mentioned before, the measured eigenfrequencies vary considerably from train passage to train passage, even for a fixed air temperature. This (bivariate) distribution is needed to compute the likelihood of a give input. To obtain it, the marginal probabilities of the first and second frequencies are estimated using kernel smoothing (Wand and Jones 1995). A normal copula was fitted to the data to take into consideration the fact that the first and second eigenfrequency are observed and expected to have important interdependence. In that way the joint probability density function (PDF) and cumulative density function (CDF) of the eigenfrequencies f_1 and f_2 become, respectively:

$$P(f_1, f_2) = C(H_{m1}(f_1), H_{m2}(f_2)) \quad (2)$$

$$p(f_1, f_2) = c(H_{m1}(f_1), H_{m2}(f_2)) \cdot h_{m2}(f_2) \cdot h_{m2}(f_2) \quad (3)$$

Where P is the bivariate CDF of f_1 and f_2, p is the bivariate PDF of f_1 and f_2, C is the normal copula, c is its second mixed derivative, H_{m1} and H_{m2} are the marginal CDF of f_1 and f_2 respectively and h_{m1} and h_{m2} are the marginal PDF of f_1 and f_2, respectively.
Since no data was available in the stiffness of ballast in cold conditions it was preferred to make the prior distribution as uninformative as possible. For this a uniform distribution was chosen for the ballast stiffness properties. For the two properties, the lower limit was set to zero, since that is the value it is normally assumed for dynamic simulations. The modulus of elasticity of the ballast was expected to be much lower than that of the concrete, so 30 GPa was used as an upper limit for that variable. For the shear interface between ballast and concrete, the maximum value was set to be twice the estimation of this variable made in previous studies (Battini and Ülker-Kaustell 2011) on the same bridge, to ensure that all likely values were included. The scatter plot and the marginal distributions of the eigenfrequencies are shown in Figure 8.

MARKOV CHAIN MONTE CARLO SAMPLING

As mentioned above there are methods for updating a distribution according to Bayes Theorem without having to compute Bayes Integral. One well-established family of such methods is the Markov Chain Monte Carlo (MCMC) sampling methods. As the name indicates, in these algorithms a Markov Chain is generated which, asymptotically, can be shown to behave as the sought distribution. MCMC methods are designed to sample from distributions with a known algorithmic expression, but that are difficult to sample from analytically.
The modified version of the Metropolis-Hasting sampling algorithms (Metropolis *et al.* 1953; Hasting 1970) used in this study was first suggested by Tarantola (2005). The algorithm starts from an initial point X_i in the space to explore. This X_i can be set manually or chosen at random from the prior distribution and it constitutes the first sample of the collections of samples to generate. X_{last} denotes the last X generated. Initially X_{last} is X_i. The metamodel M is evaluated with input X_{last} and the eigenfrequencies F_{last} are obtained. The likelihood L_{last} of X_{last} is computed from the known distribution of eigenfrequencies D.

$$L_{last} = p(F_{last}) = p(M(X_{last})) \quad (4)$$

A candidate X_{can} is picked from a distribution that depends only in the previous sample X_{last}. Typically a normal distribution centered in X_{last} is used to generate the candidate X_{can}.

$$X_{can} = X_{last} + N(0, \sigma) \quad (5)$$

where σ is some predefined variance. Then, the likelihood L_{can} of X_{can} is computed as above

$$L_{can} = p(F_{can}) = p(M(X_{can})) \quad (6)$$

and their ratio b is computed

$$b = \frac{L_{can}}{L_{last}} \quad (7)$$

The candidate X_{can} is accepted with certainty if b>1 (i.e. if the likelihood of the candidate is larger than that of the latest accepted sample) and, if b<1 X_{can} is accepted with probability b. For this a random number between 0 and 1 is generated from a uniform distribution. If this random number is less than b then X_{can} is accepted. If not X_{can} is rejected. If X_{can} is accepted X_{can} is added to the list of samples and X_{last} is updated as

$$X_{last} = X_{can} \quad (8)$$

and the process is repeated. If the candidate is rejected a new candidate is generated and the process is repeated.
Each new sample X requires at least one call to the model (more if some candidates are rejected) and they depend on the previous sample. Thus, the process cannot be parallelized. This justifies the use of a metamodel to avoid evaluating the computationally expensive FE model. To obtain a reliable metamodel takes less calls to FE the model than the MCMC algorithm. More importantly, these calls are independent and therefore can be parallelized, rendering the computation much faster. Both the model and the metamodel are described in the section below. There is a certain transient period before the samples converge to the desired distribution, so the first N samples obtained in this way are discarded. The discarded samples are called the burning period. The number of iterations required for the algorithm to converge, the burning period and the distribution from which the candidates are sampled are open parameters that the engineer should set, but there exist monitoring convergence algorithms (El Adlouni *et al.* 2006).

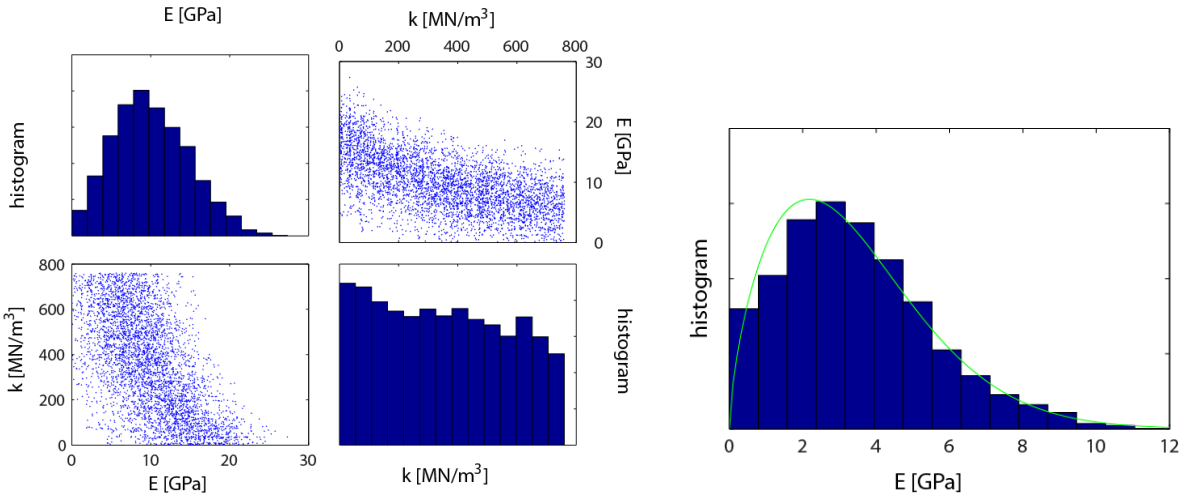

Figure 9: Left: posterior distribution of the updated parameters showing the marginal distribution and the scatter to make the dependence of the variables apparent (cold condition only); right: posterior distribution of the ballast elasticity modulus when the shear stiffness of the ballast-concrete interface is fixed at the estimated made in previous studies (cold condition only)

RESULTS

The procedure detailed above was applied for both warm (above 10 °C) and cold (below -10 °C) conditions. In the warm condition the ballast was not expected to significantly contribute to the stiffness. For this condition, the procedure returned a stiffness of 0. An explanation for this negative result can be the non-linear behavior of the bridge observed in previous studies (Batini *et al.* 2011). The first eigenfrequency of the bridge experiences a shift to the left when the amplitude increases. In this study the eigenfrequencies are determined by an FFT of the free-decay (containing both large and small amplitudes). Therefore, the measured frequency will be lower that the eigenfrequency for infinitesimal amplitude. The algorithm interprets this lower eigenfrequency as negative ballast stiffness, but since that is physically impossible, it returns a stiffness of zero. With no ballast stiffness, the shear stiffness of the ballast-to-concrete layer loses all significance.

In the cold condition, on the other hand, the expected ballast stiffness is well above zero (Figure 9). The original distribution of the ballast-to-concrete shear stiffness was hardly updated during the process. Only a small increment of the likelihood in the lower values could be observed, but in all, the posterior distribution resembled the prior uniform distribution. This means that the discussed parameter had little bearing in the eigenfrequencies of the model and thus on the likelihood of a given set of parameters. The slight tendency to very low shear interface stiffness caused relatively large estimates of the elasticity modulus. As can be seen in Figure 9 (right), for the lowest values of k, E is considerably larger. The shear stiffness k was not expected to be zero. In fact, when the value estimated in previous studies is used as a fixed value of k the distribution of the elasticity modulus shown in Figure 10 is obtained. This distribution shows an average value of 3.43 GPa and a most likely value of 2.85 GPa, which seem to be more physically realistic. In this case, the ballast's modulus of elasticity is closely fitted by a Weibull distribution with shape parameter k = 1.66 and scale parameter λ =3.82.

The results from the updating procedure seem to indicate that in the cold condition; only the modulus of elasticity of the ballast has a considerable importance and therefore is responsible for the frequency increase observed with temperature drop.

CONCLUSIONS

It was shown in this study that ballasted bridges can show a step-like variation of the eigenfrequencies with temperature. This change cannot be attributed to thermal variations of the elasticity modulus of steel and concrete, since it was observed to be non-linear and its magnitude is far larger than what is expected from this phenomenon. Water freezing and acting as a bond between ballast particles was the alternative explanation offered. This would explain at least the step-like shape of the frequency vs. temperature curve. The eigenfrequencies of the bridge showed important variance for any given temperature, but especially so for the cold conditions. This is compatible with the idea of water freezing being the cause of the frequency shift, since the bonding properties of ice can be expected to vary wildly with parameters other than air temperature (e.g. bridge temperature, sunlight, previous traffic). The changes observed in the eigenfrequencies are large enough to render numeric simulations inaccurate if not taken into consideration.

By means of a Bayesian Updating technique this measured variance in the eigenfrequencies was used to obtain the expected variance in the ballast stiffness parameters. Apparently the shear interface between the ballast and the concrete was of little importance and the elastic modulus of the ballast is the variable dominating the problem. A Weibull distribution was found to fit the estimated distribution closely.
This phenomenon can affect any ballasted bridge located in cold climates, but it only becomes a safety concern in bridges where the accelerations levels need to be investigated. Safety and comfort controls that fail to consider this possibility might be insufficient to guarantee the serviceability of the structure.

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REFERENCES

Battini, J.M. and Ülker-Kaustell, M. (2011). "A simple finite element to consider the non-linear influence of the ballast on vibrations of railway bridges", *Engineering Structures*, 33(9), 2597-2602.

Brownjohn, J.M.W., Moyo, P., Omenzetter, P. and Lu, Y. (2003). "Assessment of highway bridge upgrading by dynamic testing and finite-element model updating", *Journal of Bridge Engineering*, 8(3), 162-172.

CEN (1994). *ENV 1991-3 Eurocode 1: Basis of design and actions on structures, part 3: Traffic loads on bridges*.

Collins, J.D., Hart, G.C., Haselman, T.K. and Kennedy, B. (2012). "Statistical identification of structures", *AIAA journal*, 12(2). PAGES

Cressie, N. (1993). *Statistics for spatial data*. Wiley, New York, USA.

El Adlouni, S., Favre, A. and Bobee, B. (2006). "Comparison of methodologies to assess the convergence of Markov chain Monte Carlo methods", *Computational statistics and data analysis*, 50(10), 2685-2701.

Enright, M.P. and Frangopol, D.M. (1999). "Condition prediction of deteriorating concrete bridges using Bayesian updating", *Journal of Structural Engineering*, 125(10), 1118-1125.

Estes, A.C. and Frangopol, D.M. (2003). "Updating bridge reliability based on bridge management systems visual inspection results", *Journal of Bridge Engineering*, 8(6), 374-382.

Friswell, M. and Mottershead, J.E. (1995). *Finite element model updating in structural dynamics (Vol. 38)*, Springer, Netherlands.

Hasting, W. (1970). "Monte Carlo sampling methods using Markov chains and their application", *Biometrika*, 57(1), 97-109.

Hua, X.G., Ni, Y.Q., Chen, Z.Q. and Ko, J.M. (2008). "An improved perturbation method for stochastic finite element model updating", *International journal for numerical methods in engineering*, 73(13), 1845-1864.

Jaishi, B., and Ren, W. X. (2006). "Damage detection by finite element model updating using modal flexibility residual", *Journal of Sound and Vibration*, 290(1), 369-387.

Martin, J. and Simpson, T. (2005). "Use of Kriging models to approximate deterministic computer models", *AIAA journal,* 43(4) 853-863.

Metropolis, N., Rosenbluth, A., Rosenblith, M., Teller, A. and Teller, E. (1953). "Equation of state calculations by fast computing machines", *Journal of chemical Physics* 21(6), 1087-1092.

Nelsen, R. (1999). *An Introduction to Copulas*, Springer, New York, USA.

Ntzoufras, I. (2009). *Bayesian modeling using WinBUGS*, Jhon Wiley and Sons, New Jersey, USA.

Oden, J.T., Babuška, I., Nobile, F., Feng, Y. and Tempone, R. (2005). "Theory and methodology for estimation and control of errors due to modeling, approximation, and uncertainty", *Computer methods in applied mechanics and engineering*, 194(2), 195-204.

Papadimitriou, C., Beck, J.L. and Katafygiotis, L.S. (2001). "Updating robust reliability using structural test data", *Probabilistic Engineering Mechanics*, 16(2), 103-113.

Sanayei, M., Arya, B., Santini, E.M., and Wadia-Fascetti, S. (2001). "Significance of modeling error in structural parameter estimation", *Computer-Aided Civil and Infrastructure Engineering*, 16(1), 12-27.

Tarantola, A. (2005). *Inverse problem theory and methods for model parameter estimation*, SIAM, Philadelphia, USA.

Vanik, M.W., Beck, J.L. and Au, S. (2000). "Bayesian probabilistic approach to structural health monitoring" *Journal of Engineering Mechanics*, 126(7), 738-745.

Wand, M. and Jones, M. (1995). *Kernel Smoothing*, Chapman and Hall, London, UK.

Xia, Y., Chen, B., Weng, S., Ni, Y.Q. and Xu, Y.L. (2012). "Temperature effect on vibration properties of civil structures: a literature review and case studies", *Journal of Civil Health Monitoring* 2(1), 29-46.