The 6th International Conference on
Structural Health Monitoring of Intelligent Infrastructure
Hong Kong | 9-11 December 2013

PREDICTION OF THE FAILURE TIME OF CIVIL INFRASTRUCTURE SYSTEMS USING EMPIRICAL FAILURE MODEL

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ABSTRACT

This paper presents a short-term structural failure time prediction method based on stochastic model updating. A stochastic system degradation model termed empirical failure model is used to estimate the imminent failure time of monitored infrastructure system including structures and landslide. In the stochastic empirical failure model, failure time variable is assumed to follow the normal distribution and the damage index is either directly measured or derived from sensor data. One salient feature of this empirical failure model is that it includes the failure time in an explicit form and thus it can be directly updated with measured data. Sensor data is used as the input to update the random parameters of the stochastic system degradation model using the Bayesian theorem and MCMC simulation. Two case studies illustrating potential application of the empirical failure model for fatigue failure of steel structures and landslide are presented. Displacement and rotation data measured with wireless sensors are used to update the empirical failure model parameters. Results of this study show that the proposed method is reasonably accurate in failure time prediction of civil infrastructure systems.

KEYWORDS

Civil infrastructure, failure, sensor, prediction, prognosis, probabilistic structural health monitoring

INTRODUCTION

Sensors have been widely used for structural health monitoring in the past decade. How to efficiently extract information essential to decision making from huge amount of sensor data remains to be a challenging task (Frangopol *et al.* 2008). Probabilistic structural health prognosis method uses sensor data for system reliability updating, which can then be utilized for risk analysis and maintenance scheduling of engineering systems.

For fatigue crack development, usually three stages can be distinguished, as shown in Figure 1. Stage I is the initiation stage. At this stage, the crack propagation is very slow and difficult to predict. Its propagation rate depends on the microstructure of the material. Stage II is the rapid propagation stage. Paris Law governs the crack propagation rate and linear relation exists between the logarithm of crack growth rate and stress intensity factor (SIF) range in this stage. Stage III is the final stage when the crack growth becomes unstable as maximum stress intensity factor (K_{max}) approaches fracture toughness (K_c). Considering the different features of crack growth associated with these three stages, different deterioration models may be proposed for each stage. The linear elastic fracture mechanics (LEFM) model has been used for the modeling of crack growth at stage II. When fatigue crack enters its final growth stage, the LEFM model is no longer valid. However, for fatigue hazard, many structural failures happen at the final stage in which the crack size becomes very large and its growth becomes unstable, potentially leading to collapse of the structure. Therefore, it is of importance to develop a fatigue growth model which can be used to describe the crack growth pattern at the fast crack growth regime. In this paper, an empirical failure model is proposed for this purpose. This model is derived from the material failure equation proposed by Voight (1989), which has been applied to volcano eruption and landslide prediction by many researchers (e.g., Kilburn and Voight 1998). Parameters of the empirical failure model are discussed. Examples are presented to illustrate its use for real fatigue failure prediction and landslide prediction.
The empirical failure model is useful not only to assure civil infrastructural safety, but also for cost effective life cycle management such as maintenance, repair and replacement. Although it is demonstrated with two applications in fatigue and landslide, a variety of other applications such as pre-stressed bridge girder, dams, can be analyzed with this predictive model.

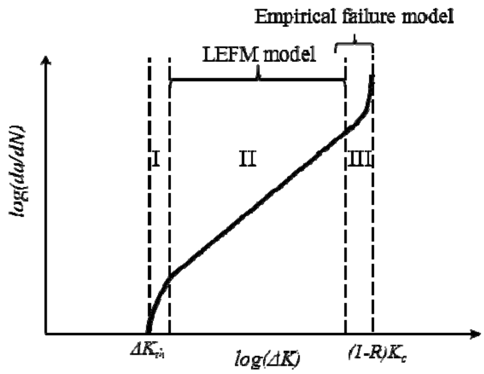

Figure 1. Fatigue crack growth stage

PROBABILISTIC STRUCTURAL DEGRADATION MODEL

A stochastic structural degradation model is used to estimate the life of the related engineering system. Sensor data is used as the input to update the random parameters of the structural degradation model using the Bayesian theorem and MCMC (Monte Carlo Markov Chain) simulation. For the purpose of accurate prognosis, uncertainties in the modeling and measurement (i.e., sensor data) are explicitly modeled as a time-evolving stochastic process.

A stochastic structural degradation model which models the variation of an engineering system over the time is adopted for sensor-driven structural health prognosis. The stochastic structural degradation model can be expressed as Eq. 1:

$$y(t) = F(t, \alpha) + \varepsilon(t) \tag{1}$$

where *y(t)* is the degradation index which can be directly measured or indirectly computed from sensor data (e.g., features of fused sensor data), and *t* denotes the time. F(t,α) is a function describing the evolution of the system. α is the model parameter vector containing quantities related to system properties and loading quantities such as strain, temperature, or any other excitations/load applied to the system. Due to the uncertainties commonly associated with the model parameters, they can be described as random variables following pre-specified probability distributions. $\varepsilon(t)$ is the error term that represents the combined model error and measurement error effect.

With sensor data from monitoring system, initial model parameter distributions can be updated with measured data in order to reduce the uncertainty in the prediction. Bayesian theory is used here for updating the model parameters using sensor data. In practice, the degradation model, F(t,α), may have a large number of random model parameters that would make computing $f(\alpha|Y)$ difficult because of multi-dimensional integration being involved. MCMC simulation, which is suitable for simulating high dimension probability density function, is selected here as the alternative approach to calculate the posterior distribution. The MCMC method draws samples from a pre-specified distribution by running a constructed Markov Chain that will converge after certain number of loops. The Metropolis-Hastings algorithm (Metropolis *et al.* 1953; Hastings 1970) is employed here for sample drawing from the posterior distribution. Details of the sensor driven prognosis method can be found in Li and Zhang (2013)

EMPIRICAL FAILURE MODEL

Many material failure phenomena are preceded by clear accelerating rates of strain, displacement and seismicity (Cornelius and Scott 1993). Voight (1989) proposed a relation between the acceleration in a geophysical precursor Ω (such as strain or number of earthquakes) and its rate for conditions of constant stress and temperatures, shown below as Eq. 2 (Bell *et al.* 2011).

$$\frac{d^2\Omega}{dt^2} = G\left(\frac{d\Omega}{dt}\right)^{\alpha} \tag{2}$$

Quantities that can be represented by Ω include strains for deforming alloys, metals, polymers, concrete, soil, rock, or ice. Fields of potential application of Eq. 2 include materials science, various branches of engineering, and the earth sciences. Eq. 2 may also apply (at least approximately) to predominantly rate-independent applications, such as some cases of fatigue (rate-independent repeated loading) (Voight 1989).

Rates of crack growth increase exponentially with crack length in many cases (Kilburn and Voight 1998). Eq. 2 can be implemented for such case of material failure (e.g. volcano eruption). As for the case of fatigue cracking, at its final stage with the presence of large "unstable" crack, the crack growth rate would increase sharply. According to the Forman Equation shown below (Forman *et al* 1967), clear acceleration of crack growth rate occurs at the final stage when ΔK approaches $(1-R)\cdot K_c$. K_c denotes the fracture toughness of the steel specimen. R is the stress ratio ($S_{\min}/S_{\max}$).

$$\log(da/dN) = \log(C) + m \cdot \log(\Delta K) - \log[(1-R) \cdot K_c - \Delta K] \quad (3)$$

Thus, potentially Eq. 2) can be applied to estimate the fatigue failure time, for which the crack length can be taken as the precursor signal.

Derivation of explicit model

Eq. 2 can be transformed into Eq. 4 to represent the final stage of cracking before catastrophic failure (Kilburn and Voight 1998).

$$\left(\frac{d\Omega}{dt}\right)^{-1} = \left(\frac{d\Omega}{dt}\right)_0^{-1} - \gamma(t - t_0) \quad (4)$$

In Eq. 4, Ω denotes the damage index. γ is a rate-related coefficient. t_0 is the initiation time. t_f is the total life counted from t_0. Without loss of generality, by letting $t_0 = 0$, Eq. 4 can be transformed into Eq. 5 below,

$$\Omega(t) = -\gamma \ln(t_f - t) + \gamma \ln t_f \quad (5)$$

Based on many curve fitting test of fatigue test data, Eq. 5 is further modified by making the second term at right hand side of the equation an independent parameter as shown in Eq. 6. Eq. 6 is chosen as the empirical failure model for fatigue failure.

$$\Omega(t) = -\gamma_1 \ln(t_f - t) + \gamma_2 \quad (6)$$

The above model is termed empirical failure model here. In Eq. 6, γ_1, γ_2 and t_f are all considered as random parameters. γ_1 and γ_2 are model parameters affected by material properties and external loading conditions. They are assumed to follow lognormal distribution. t_f is the fatigue failure time and is assumed to follows the normal distribution. $\Omega(t)$ is the damage index that is either directly measured or derived from sensor data. This empirical failure model includes the failure time t_f in an explicit form and thus t_f is directly updated with measurement data $\Omega(t)$. One advantage of applying the empirical failure model for prognosis is that the failure time t_f is explicitly treated as a parameter in the equation and its value can be directly obtained by updating its distribution using sensor data. In this way, no additional failure criterion needs to be defined (e.g., for the LEFM based prognosis, a critical fatigue size has to be defined first). In this study, the potential use of Eq. 6 for fatigue prognosis is examined using real fatigue test data and landslide monitoring data.

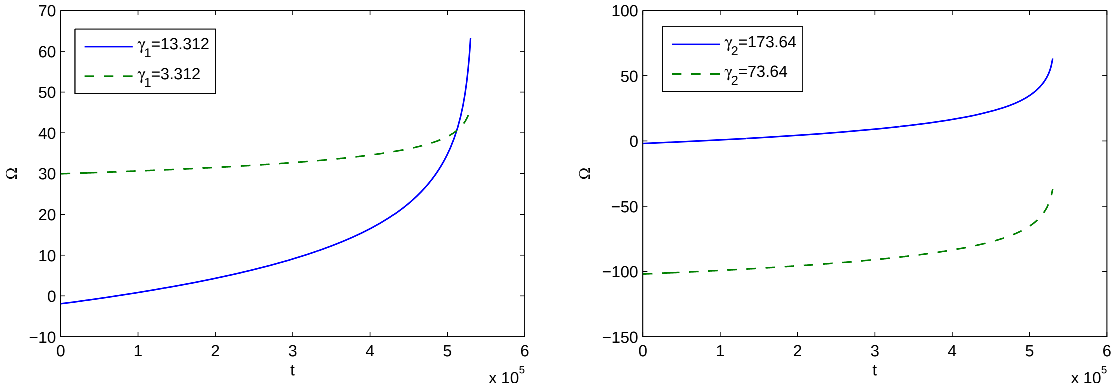

Figure 2. Empirical failure model with varying values of γ_1 and γ_2

In the empirical failure model described by Eq. 6, γ_1 determines the growth rate of the damage index Ω. γ_2 represents the intersection with y-axis. Figure 2 shows the effect of the two parameters on the curve. The initial values are γ_1 = -13.312, γ_2 = 173.64, t_f = 5.34x10^5. For each figure, the value of one of the three parameters is varied while the other two parameters are kept constant.

A case study is presented next to show the feasibility of using the empirical failure model for fatigue failure time prediction.

Case study

In order to examine how well the empirical failure model would fit real fatigue crack growth data, fatigue crack data from the experiments done by Virkler *et al.* (1979) is used for curve fitting. In their test (Virkler *et al.* 1979), 68 replicate fatigue tests with the same loading on specimens with the same design were implemented on 2024-T3 aluminium alloy under constant load amplitudes. Crack length trajectories were recorded for each test. All tests started with an initial crack length of 9.0 mm and tests were terminated at the crack length of 49.80 mm. Mean values of the crack length data at specific load cycles are calculated from these test data. This crack growth data set has also been used in statistical analysis of fatigue crack growth and prognosis by other researchers (Perrin *et al.* 2007; Guan *et al.* 2012). A set of the mean crack length data is used here. The crack length data is fitted with the empirical failure model. Figure 3 shows the fitted curve with the fitted model expressed as: $\Omega(t)=-13.312*\ln(5.34\times10^5-t)+173.64$. In Figure 3, the residuals of the fitted curve are also plotted. They are all quite small comparing with the measurements indicating that the model fits the data well. Moreover, the coefficient of determination (R^2) is selected as the index to evaluate how well the model describes the behaviour of the data. More details about R^2 application can be found in Steel and Torrie (1960). The closer R^2 value is to 1, the better the model describe the data. In this case, R^2 is calculated to be 0.9989. It suggests that the model fits the crack growth data very well.

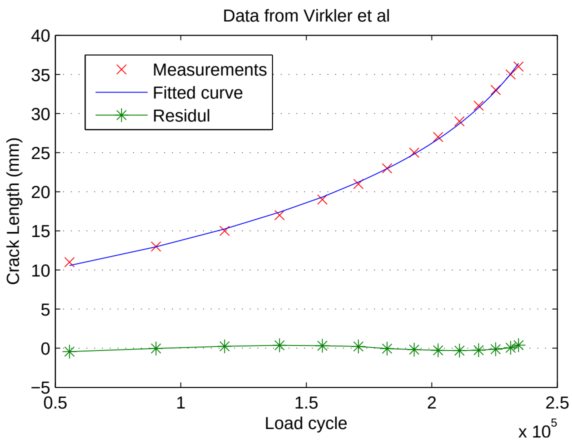

Figure 3. Curve fitting of fatigue test data from the Virkler *et al.* (1979) test

Fatigue failure time prediction using Empirical Failure Model

One feature of the empirical failure model is that the growth rate of the modeled degradation process should be monotonically increasing. However, in reality, the crack growth rate might slow down occasionally due to various reasons, for example, reduced fatigue load range. In such cases, multiple acceleration stages could happen during the fatigue crack propagation process. Under such condition, it is preferred that the data set is divided into several segments and the empirical failure model is updated for the stages with monotonically increasing rate only. In order to automate this in prognosis practice, a data selection method based on the coefficient of determination (R^2) is proposed in this study.

For data set with monotonically increasing rate, generally increasing number of data points would improve the goodness of fit for the updated model. Therefore, the R^2 value would get closer to 1 when more data points become available. Small fluctuation might exist in R^2 values due to the uncertainty and noise in the measured data and the degradation process. Yet no significant drop of R^2 value should happen. However, if deceleration in the measured data is observed, the goodness of fit of the empirical model to the data set would also deteriorate which will be reflected as the sudden drop of R^2 value. In this study, a data selection procedure based on this observation is proposed. A flow chart illustrating this procedure is shown in Figure 4. With each data point taken in and the Bayesian updating executed, the R^2 value is examined to see if a new acceleration stage starts. If a relatively large drop of R^2 value occurs, it is concluded that a new acceleration stage begins and all previous data points are discarded. The selection of a small constant value δ in the criterion is to avoid misinterpretation of data fluctuation (e.g. caused by noise) into new acceleration stage.

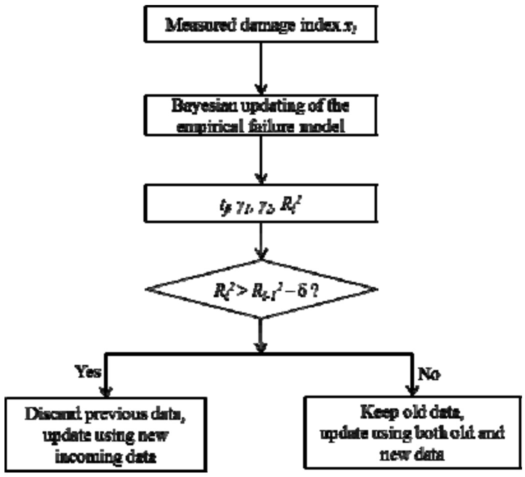

Figure 4. Flowchart of R^2 based data selection method

The prognosis for WTJ5 is performed again, but augmented with the R^2 data selection method. The constant value of δ is set to be 0.03 based on some tests. For different application, this value might vary. At N = 3.0169x10^5, there is a sudden drop for R^2 value. Therefore, according to the procedure presented in the flowchart shown in Figure 6.8, all data before N = 3.0169x10^5 are discarded. Updating is implemented using refreshed data points collected after that point. The variation of R^2 in the updating process is shown in Figure 5a. It is seen that the R^2 value gradually increases and approaches 1. The T_f result is shown in Figure 5a.

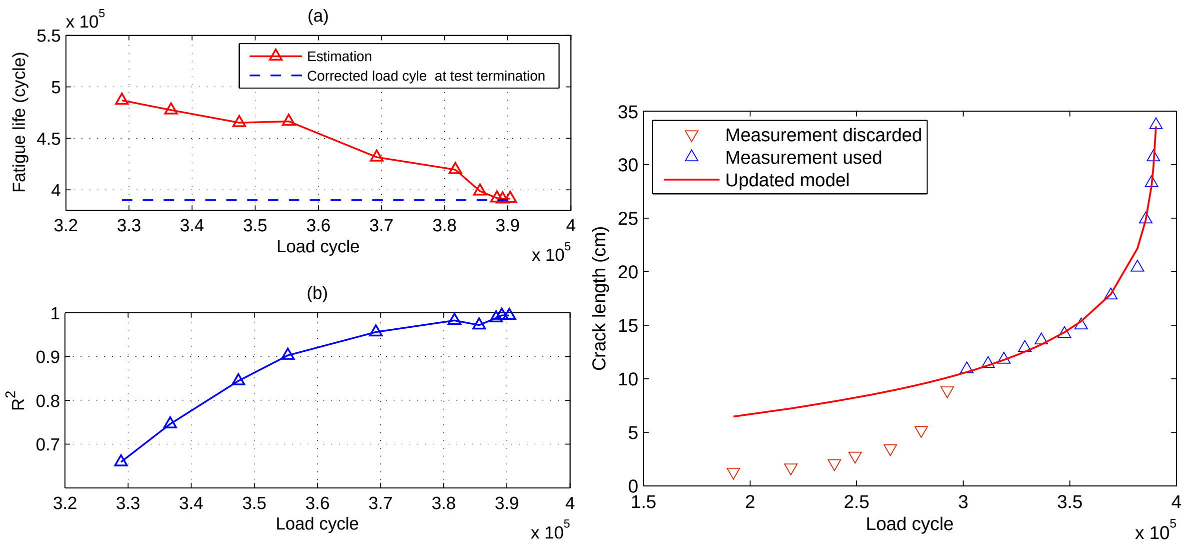

Figure 5. Results of fatigue failure time prediction using empirical failure model with data selection

PREDICTION OF LANDSLIDE FAILURE TIME

Landslides constitute a major geologic hazard because they are widespread, occur in all 50 states and U.S. territories, and cause $1-2 billion in damages and more than 25 fatalities on average each year (see http://landslides.usgs.gov/). Expansion of urban and recreational developments into hillside areas leads to more people that are threatened by landslides each year. The empirical failure model can also be used to model the final stage of slope failure process. At the early stage of landslide, the displacement of the slope is usually so small that the measured data are dominated by noise. However, when the landslide reaches its final stage, the slope becomes unstable and displacement at the monitoring location gradually increases at an accelerating rate. Therefore, in landslide failure time prediction, only the data set within the final stage of the landslides should be used for model updating. It thus raises an interesting question as to how to define the starting point at which the landslide process is considered as approaching its final stage. The coefficient of variation for the predicted failure time is chosen here as this criteria for judging if the landslide has reached its final stage.
The critical value for coefficient of variation is denoted as CV_{cr}. At the early stage of the landslide when

coefficient of variation value is larger than CV_{cr}, only a small portion of the lately measured displacement data is used for updating Eq. 6. The displacement data at earlier stage are thrown away. When the coefficient of variation value goes below the critical value CV_{cr}, all incoming displacement data are kept and used for updating. By doing so, more accurate landslide failure time can be estimated.

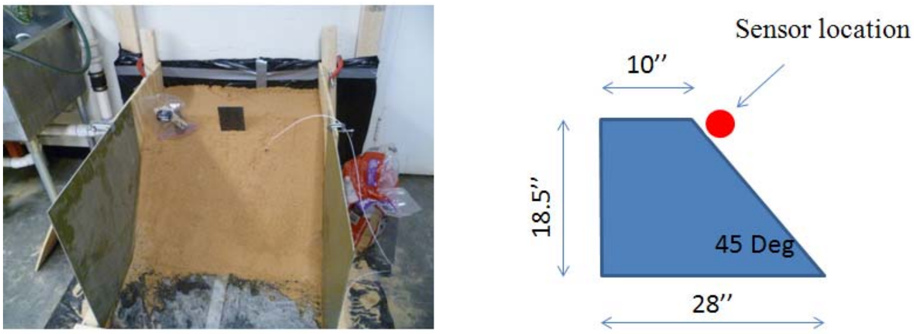

Figure 6. Experimental set up for landslide simulation in lab

In order to evaluate the empirical failure model for landslide failure time prediction, a lab-scale landslide test was carried out at the University of Maryland. Sand slope was built in the lab and its configuration is shown in Figure 6. The sand slope has a slope angle of 45 degree and height of 18.5”. Water from a garden sprinkler was used to simulate the rain by spreading onto the sand slope. Three types of sensors were installed on the top of the sand slope. The first one is a laser displacement sensor, which measures the displacement of the sand slope. The second one is a high-sensitivity wireless accelerometer used to measure the rotation of the sand slope. Tiltmeter was also used to measure the rotation variation of the slope. For illustration purpose, only the data from the laser displacement sensor is used here for landslide failure time estimation. The time history of the laser sensor data from one test is shown in Figure 7, which shows that the complete failure occurred at time t = 837 seconds. However, the final stage of the slope failure started until after t= 500 seconds.

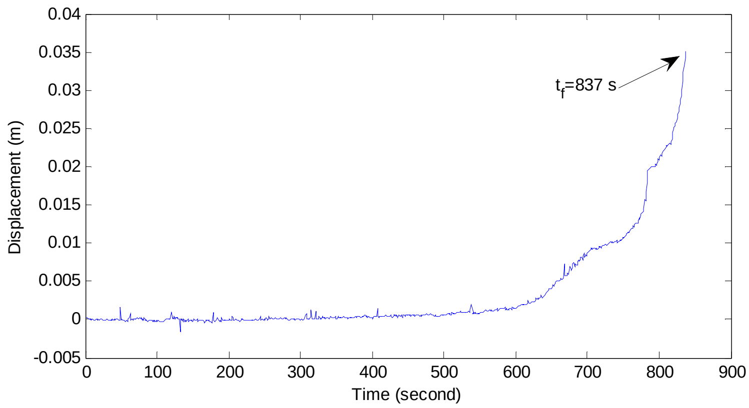

Figure 7. Displacement data of the laser diplacement sensor int he landslide test

The distributions of γ_1 and t_f of the landslide test displacement data are listed in Table 1. For γ_2, it is assumed to be within the range of [0.01 0.30]. Optimization was implemented to automatically select the value. The value with the smallest mean square error for displacement differences are chosen for γ_2 in each updating. Bayesian updating of the random variables are carried out every 25 seconds. CV_{cr} is set to be 0.10. The estimated landslide failure time t_f is shown in Figure 8a. Also, the CV values of t_f are plotted in Figure 8b. Figure 8b shows that the *CV* value become smaller than 0.1 at around 590 seconds. Starting from the time point on, all sensor data are kept for model updating. The prognosis result converges to the true values shortly after *CV* value get smaller than CV_{cr}.

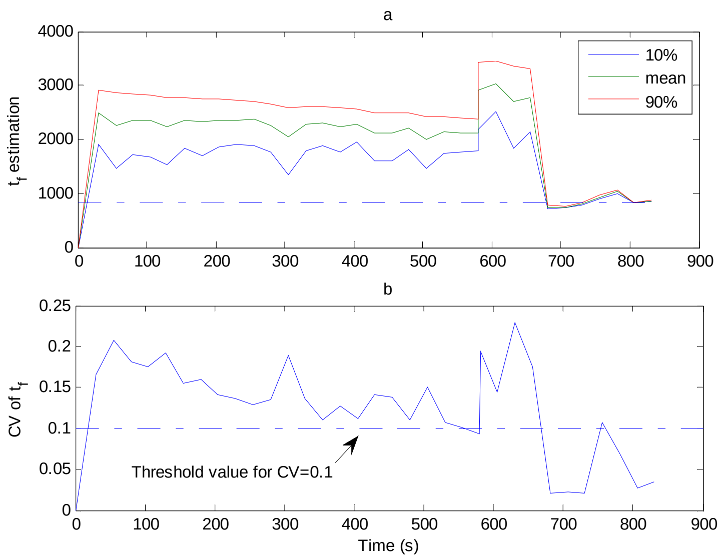

Figure 8. Updated t_f estimation and CV values during the landslide test

Table 1. Random parameters of empirical failure model for the landslide test

	Mean	Standard Deviation	Distribution Type
γ_1	0.01	0.2	Lognormal
t_f	1500	NA	Uniform

CONCLUSION

Prognosis using the Empirical Failure Model is able to provide a good estimation for system's failure time using monitoring data for model updating. As an explicit parameter, the system failure time is directly obtained from updating of the Empirical Failure Model using measured data. No explicit definition of the limit state is thus necessary in comparison with the prognosis procedure based on other explicit model such as the linear elastic fracture model for fatigue modelling. One limitation of the Empirical Failure Model is that it can only fit the deterioration process which has monotonically increasing growth rate. In practical applications, the growth rate might not always be monotonic. In order to make the Empirical Failure Model viable for such cases, a data selection method is proposed to determine which portion of the measure data is suitable for model updating and failure time prediction use. The data set of one welded tubular joint specimen subjected to fatigue test is used as an example to demonstrate the use of data selection method. With the data selection method, the empirical failure model based prognosis method is shown to yield a good estimate of the fatigue failure time. The Empirical Failure Model is also demonstrated for use in landslide failure time prediction, which shows reasonably well results after comparing with experimental data.

ACKNOWLEDGEMENTS

The work described in this paper was partially supported through research grants from the National Science Foundation under Award No. CMMI-1031304 and CMMI-1031414. The authors are also grateful to the University of Maryland for providing additional financial support through a GRB award for this research project. However, the opinions and conclusions expressed in this article are solely those of the writers and do not necessarily reflect the views of the sponsors.

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