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Health monitoring of short- and medium-span bridges using an improved modal strain energy method

By:

Parviz Moradipour
Tommy H.T Chan
And Chaminda Gallage

School of Civil Engineering & Built Environment, Queensland University of Technology, Brisbane, Qld, Australia

Oct. 2013

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Introduction

- Infrastructure (mostly bridges) are becoming more complex
- ↑ Importance & Complexity of structures → ↑ Safety assessment
- During the service life → Structure maybe damage

What is damage?

- Decreasing the structural bearing capacity →
  - exposing to collapse

Bridges

I-35W Bridge, US, 2007

Damage detection techniques

- Non-destructive Testing (NDT Or NDE)
  - Visual Inspection
  - Ultrasonic Testing (UT)
  - Laser testing
  - Electromagnetic testing (ET)
  - Radiographic testing (RT)
  - GPS
  - Vibration Analysis → Vibration-Based-Damage–Detection (VBDD) techniques

Some of the SHM techniques

Vibration-Based-Damage–Detection (VBDD) methods

- Natural Frequency
- Mode Shape
- Modal Strain Energy (MSE)
- Frequency Response Function (FRF)
- Genetic Algorithm (GA)
- Artificial Neural Network (ANN)

Literature review

Researcher/S	Type	Verified For
Yuen (1985)	eigenparameters	cantilever beam
Kaouk and Zimmerman (1994)	developed an eigenstructure technique	a space truss structure model
Lim and Kashangaki (1994)	concept of eigenvectors damage indicator	
Pandey and Biswas (1995)	flexibility matrix	
Doebling et al. (1997)	modal strain energy finite-element model (FEM) correlation	
Shi et al. (2002)	change of modal strain energy (MSE) or MSEC	
Wang et al. (2012)	Correlation MSE	for steel truss bridges
Wahalathantri et al. 2012	damage index based MSE	simply supported and two-span beam

Research gaps

1. Many numerical VBDD studies have been validated for elementary structures such as beam and plate like structures.
2. It is desired to propose a more sensitive and reliable damage indicator capable of capturing
   - The small values of damage at the initial formation stage.
   - Also it should be compatible with incomplete data in presence of noise.
3. The VBDD methods theoretically are often based on FEM analysis. Therefore, the more exact analytical solution and proper optimization of high-DOF structures (bridges), the more accuracy of the damage detection.

Objectives

1. Improve an MSE method to precisely locate and quantify the damage in short- and medium-span bridges.

2. Validate the method for 2D short- and medium-span bridges.

Methodology

1. Mathematically improvement of an available MSE equation.
2. Using MSE equation improved to;
    - Locate the damage
    - Derive a sensitivity matrix
    - Quantify the damage
    - Apply the noise effect
3. Verification for 2D short- and medium span bridges.

Improved MSE Method

Theoretical Background

Mode shape: $\{\phi_i^d\} = \{\phi_i\} + \{\Delta\phi_i\}$

Natural Frequency: $\lambda_i^d = \lambda_i + \Delta\lambda_i$

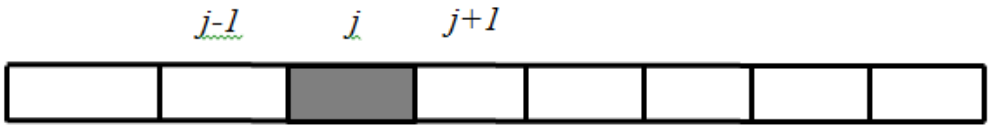
Global stiffness matrix: $[K^d] = K + \sum_{m=1}^{L} \alpha_m [K_m \qquad -1 < \alpha_m \leq 0$

α_m: Fractional reduction coefficient of m^{th}elemental stiffness matrix

L：Total no. of elements

Concept- Elemental Modal Strain Energy (MSE)

Changes (↑) in MSE ≈ existence a damage



$$MSE_{i,j} > MSE_{i,j+1}$$

$$MSE_{i,j} > MSE_{i,j-1}$$

Mathematically Improved MSE Method

a) Locate the damage

$$MSE_{i,j} = \frac{1}{2}\{\phi_i\}^T[K_j]\{\phi_i\} \qquad MSE^d_{i,j} = \frac{1}{2}\{\phi^d_i\}^T[K^d_j]\{\phi^d_i\}$$

$$MSE^d_{i,j} > MSE_{i,j}$$

MSE Change (MSEC) = $\Delta MSE = MSE^d - MSE$

MSEC Ratio (MSECR) $MSECR = \frac{|MSE^d - MSE|}{MSE}$

b) Quantify the damage - Changes in str. parameters

Mode shape: ↑ $\{\phi_i^d\} = \{\phi_i\} + \{\Delta\phi_i\}$

Natural Frequency: ↑ $\lambda_i^d = \lambda_i + \Delta\lambda_i$

Local stiffness matrix: ↓ $[K_m^d] = [K_m] + \alpha_m[K_m]$ $\quad -1 < \alpha_m \leq 0$

Global stiffness matrix: ↓ $[K^d] = K + \sum_{m=1}^{L} \alpha_m[K_m$

α_m: Fractional reduction coefficient of m^{th}elemental stiffness matrix

L : Total no. of elements

Contribution 1: Improve MSE Equation

$$\Delta MSE_{i,j} = MSE_{i,j}^{d} - MSE_{i,j} = \frac{1}{2}\{\phi_i^d\}^T [K_j^d]\{\phi_i^d\} - \frac{1}{2}\{\phi_i\}^T [K_j]\{\phi_i\}$$

Previous study:

$$\Delta MSE_{i,j} = \frac{1}{2}\sum_{i=1}^{L} 2\alpha_i \{\phi_i\}^T [K_j] \sum_{r=1}^{md} \frac{\{\phi_r\}^T [K_i]\,\{\phi_i\}}{\lambda_i - \lambda_r}\{\phi_r\}$$

Current study:

$$\Delta MSE_{i,j} = \frac{1}{2}\alpha_j \{\phi_i\}^T [K_j]\{\phi_i\} + \frac{1}{2}\left[\{\phi_i\}^T [K_j] \sum_{i=1}^{L} \alpha_i \sum_{r=1}^{md} \frac{\{\phi_r\}^T [K_i]\,\{\phi_i\}}{\lambda_i - \lambda_r}\{\phi_r\} + \sum_{i=1}^{L} \alpha_i \sum_{r=1}^{md} \frac{\{\phi_r\}^T [K_i]\,\{\phi_i\}}{\lambda_i - \lambda_r}\{\phi_r\}^T [K_j]\{\phi_i\}\right]$$

Contribution 2: Improve Sensitivity Matrix

$$\beta = S^{MSE} = \frac{\partial MSE}{\partial \alpha}$$

Previous study:

$$2\sum_{r=1}^{n}\{\phi_i\}^T[K_s]\frac{\{\phi_r\}^T[K_t]\{\phi_i\}}{\lambda_i-\lambda_r}\{\phi_r\}$$

Current study:

$$\{\phi_i\}^T[K_j]\{\phi_i\}-\{\phi_i^d\}^T[K_j]\{\phi_i^d\}+\sum_{r=1}^{n}\{\phi_i\}^T[K_s]\frac{\{\phi_r\}^T[K_t]\{\phi_i\}}{\lambda_i-\lambda_r}\{\phi_r\}+\sum_{r=1}^{n}\frac{\{\phi_r\}^T[K_t]\{\phi_i\}}{\lambda_i-\lambda_r}\{\phi_r\}^T[K_s]\{\phi_i\}$$

Sensitivity matrix in the three different methods

No		Sensitivity equation	Sensitivity matrix
1	Shi et al. (2002).	$2\sum_{r=1}^{n}\{\phi_i\}^T[K_s]\frac{\{\phi_r\}^T[K_t]\{\phi_i\}}{\lambda_i-\lambda_r}\{\phi_r\}$	$\begin{bmatrix}\beta1_{11} & \beta1_{12} & \dots & \beta1_{1q}\\ \beta1_{21} & \beta1_{22} & \dots & \beta1_{2q}\\ \vdots & \vdots & \ddots & \vdots\\ \beta1_{J1} & \beta1_{J1} & \dots & \beta1_{Jq}\end{bmatrix}$
2	Wang et al. (2010).	$-\sum_{r=1}^{n}\{\phi_i\}^T[K_s]\frac{\{\phi_r\}^T[K_t]\{\phi_i\}}{\lambda_i-\lambda_r}\{\phi_r\}-\sum_{r=1}^{n}\frac{\{\phi_r\}^T[K_t]\{\phi_i\}}{\lambda_i-\lambda_r}\{\phi_r\}^T[K_s]\{\phi_i\}$	$\begin{bmatrix}\beta2_{11} & \beta2_{12} & \dots & \beta2_{1q}\\ \beta2_{21} & \beta2_{22} & \dots & \beta2_{2q}\\ \vdots & \vdots & \ddots & \vdots\\ \beta2_{J1} & \beta2_{J1} & \dots & \beta2_{Jq}\end{bmatrix}$
3	Current study	$\{\phi_i\}^T[K_j]\{\phi_i\}-\{\phi_i^d\}^T[K_j]\{\phi_i^d\}+\sum_{r=1}^{n}\{\phi_i\}^T[K_s]\frac{\{\phi_r\}^T[K_t]\{\phi_i\}}{\lambda_i-\lambda_r}\{\phi_r\}+\sum_{r=1}^{n}\frac{\{\phi_r\}^T[K_t]\{\phi_i\}}{\lambda_i-\lambda_r}\{\phi_r\}^T[K_s]\{\phi_i\}$	$\begin{bmatrix}\beta^*_{11} & 0 & \dots & 0\\ 0 & \beta^*_{22} & \dots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \dots & \beta^*_{qq}\end{bmatrix}+\begin{bmatrix}\beta'_{11} & \beta'_{12} & \dots & \beta'_{1q}\\ \beta'_{21} & \beta'_{22} & \dots & \beta'_{2q}\\ \vdots & \vdots & \ddots & \vdots\\ \beta'_{q1} & \beta'_{q2} & \dots & \beta'_{qq}\end{bmatrix}$

Sensitivity matrix

$$S^{MSE} = \frac{\partial MSE}{\partial \alpha} = \begin{vmatrix} \frac{\partial MSE_1}{\partial \alpha_1} & \frac{\partial MSE_1}{\partial \alpha_2} & \cdots & \frac{\partial MSE_1}{\partial \alpha_N} \\ \frac{\partial MSE_2}{\partial \alpha_1} & \frac{\partial MSE_2}{\partial \alpha_2} & \cdots & \frac{\partial MSE_2}{\partial \alpha_N} \\ \vdots & \vdots & & \vdots \\ \frac{\partial MSE_{md}}{\partial \alpha_1} & \frac{\partial MSE_{md}}{\partial \alpha_2} & \cdots & \frac{\partial MSE_{md}}{\partial \alpha_N} \end{vmatrix}$$

α_N : No. of suspected elements

No. of selected elements for computation

md: No. of (analytical) modes in consideration

$md \leq no.of\ DOFs$

Contribution 3: Calculate the more exact α's

$$\frac{\partial MSE}{\partial \alpha} \times \{\alpha\} = \{MSE\}$$

$$\beta \times \{\alpha\} = \{MSE\}$$

$$\left(\begin{bmatrix} \beta^*_{11} & 0 & \dots & 0 \\ 0 & \beta^*_{22} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \beta^*_{qq} \end{bmatrix} + \begin{bmatrix} \beta'_{11} & \beta'_{12} & \dots & \beta'_{1q} \\ \beta'_{21} & \beta'_{22} & \dots & \beta'_{2q} \\ \vdots & \vdots & \ddots & \vdots \\ \beta'_{q1} & \beta'_{q2} & \dots & \beta'_{qq} \end{bmatrix} \right) \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_q \end{bmatrix} = \begin{bmatrix} MSEC_{i1} \\ MSEC_{i2} \\ \vdots \\ MSEC_{iJ} \end{bmatrix}$$

$$\begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_q \end{bmatrix} = \left(\begin{bmatrix} \beta^*_{11} & 0 & \dots & 0 \\ 0 & \beta^*_{22} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \beta^*_{qq} \end{bmatrix} + \begin{bmatrix} \beta'_{11} & \beta'_{12} & \dots & \beta'_{1q} \\ \beta'_{21} & \beta'_{22} & \dots & \beta'_{2q} \\ \vdots & \vdots & \ddots & \vdots \\ \beta'_{q1} & \beta'_{q2} & \dots & \beta'_{qq} \end{bmatrix} \right)^{-1} \begin{bmatrix} MSEC_{i1} \\ MSEC_{i2} \\ \vdots \\ MSEC_{iJ} \end{bmatrix}$$

Effect of noise

$$\bar{\varphi}_{ij} = \varphi_{ij}(1 + \gamma_i^{\varphi}\rho^{\varphi}|\varphi_{max,j}|)$$

Where $\bar{\varphi}_{ij}$ and φ_{ij} are the mode shape components of the jth mode at ith DOF

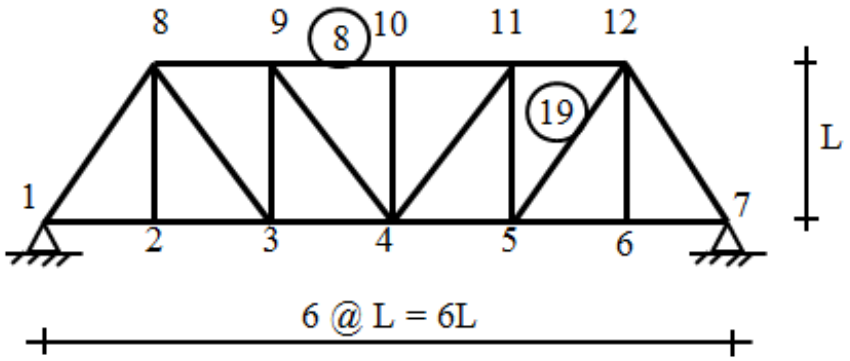
γ_i^{φ} are the random numbers with the mean equal to zero & variance equal to 1

ρ^{φ} is the random noise level (per cent)

$\varphi_{max,j}$ is the largest component in the jth mode shape

Verification

Numerical example 1:

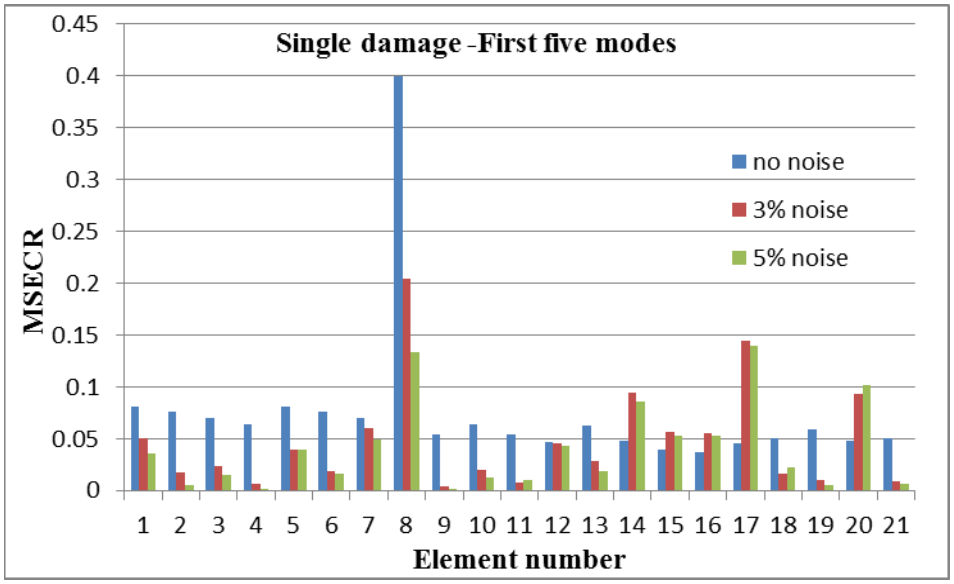


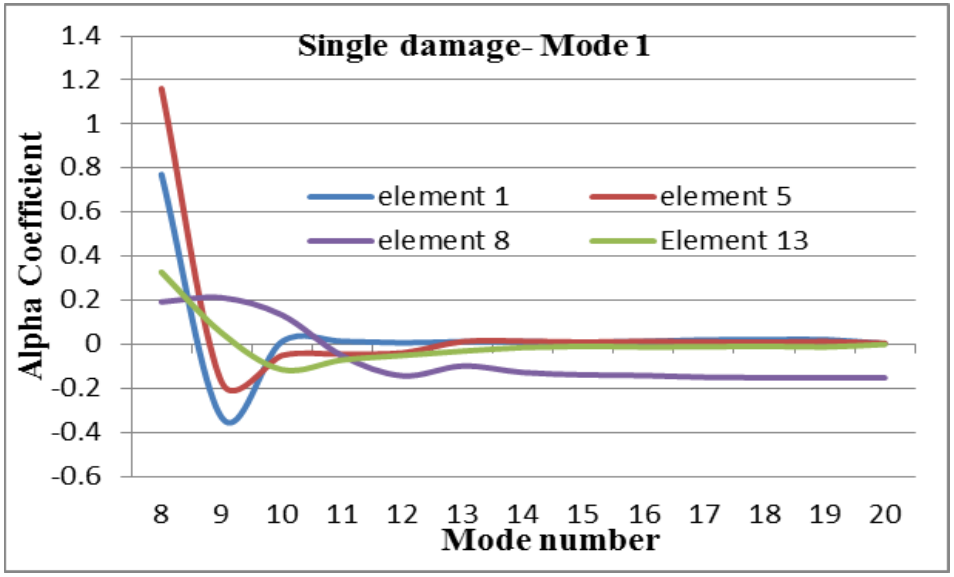
- **An 2D (Pratt) Truss bridge**
- 12 nodes and 21elements with 20 DOFs.

Material properties and geometric data;

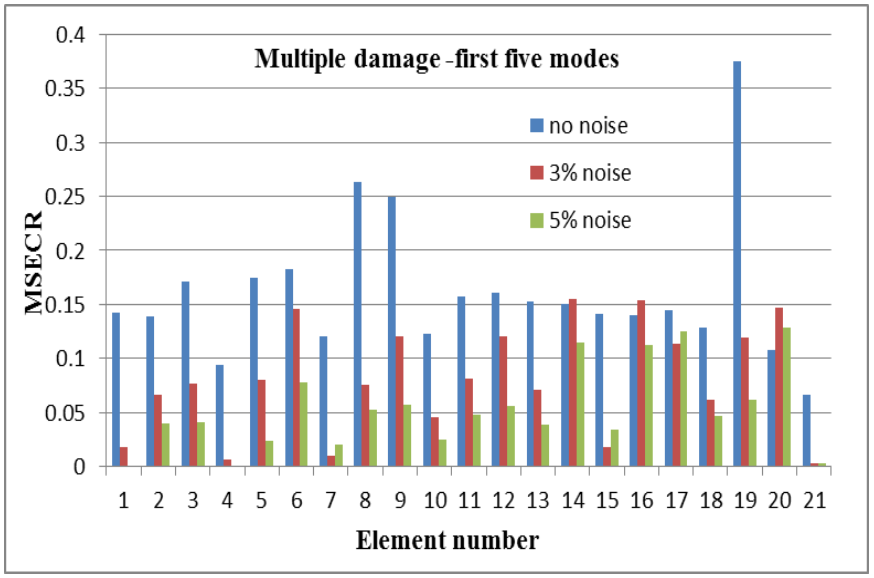
- Length of each element =L= 5.0 m
- Modulus of elasticity = $E = 207 \times 10^9$ N/m^2
- Cross-sectional area = A=0. 04 m^2
- Mass density = 7870 kg/m^3

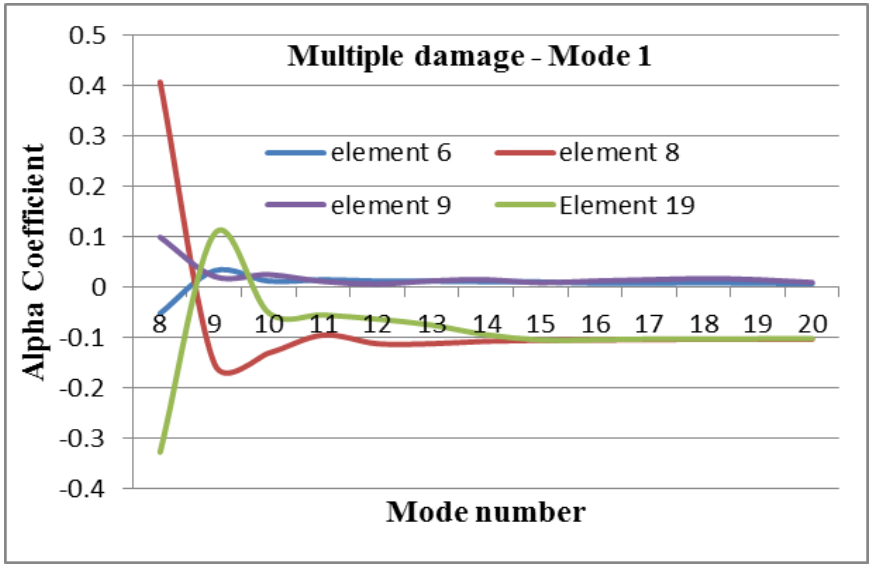
Single-damage scenario



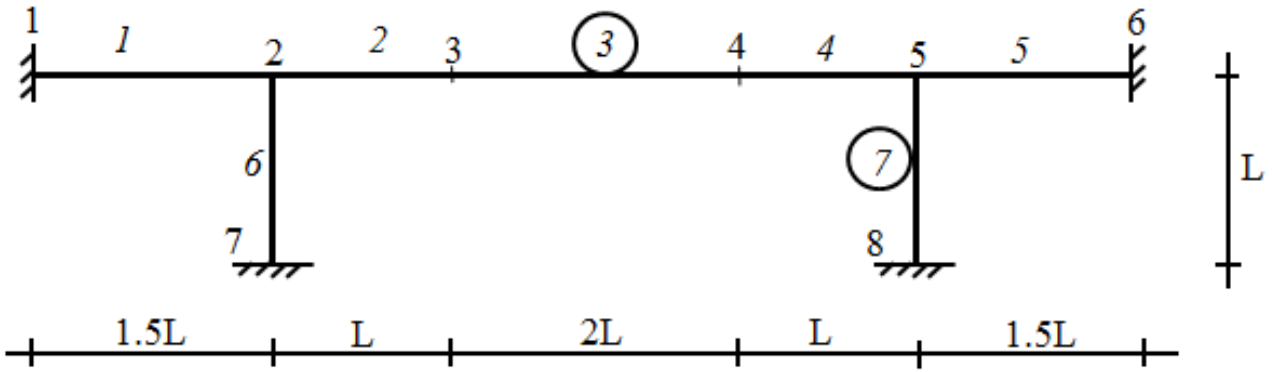


Multiple-damage scenario





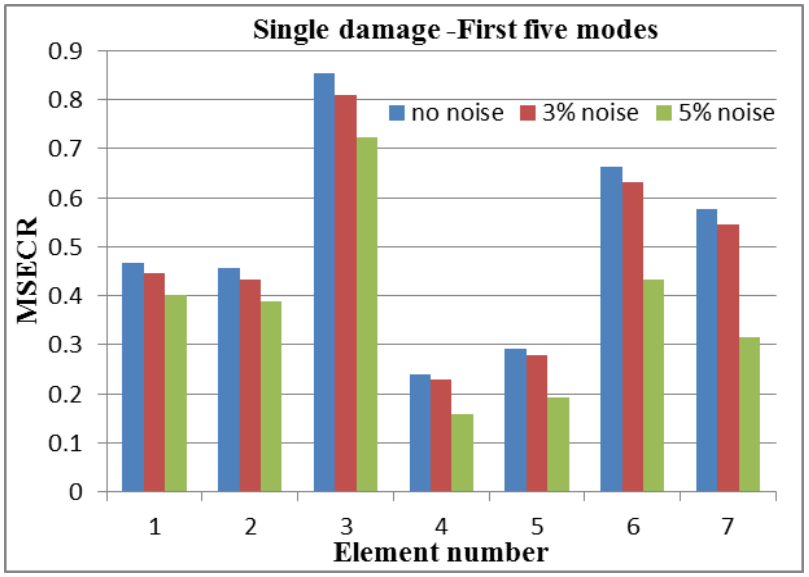
Numerical example 2:

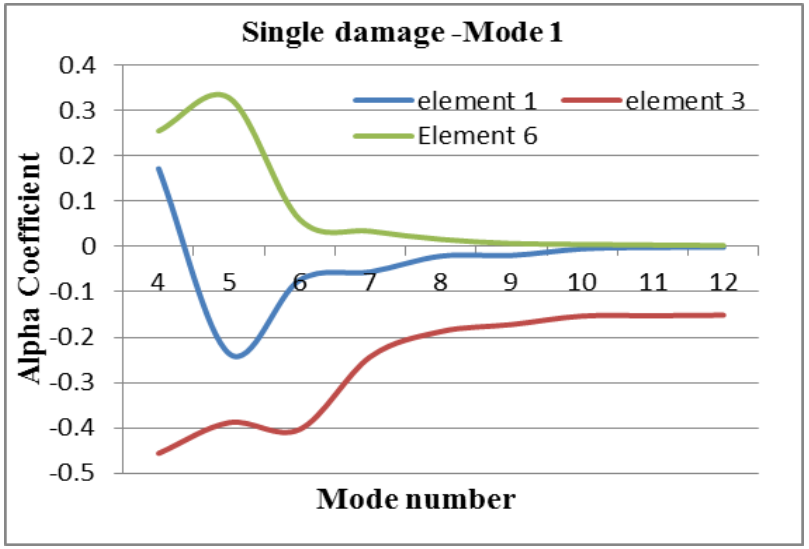


Frame structure (bridge)
8 nodes and 7 elements with 12 DOFs

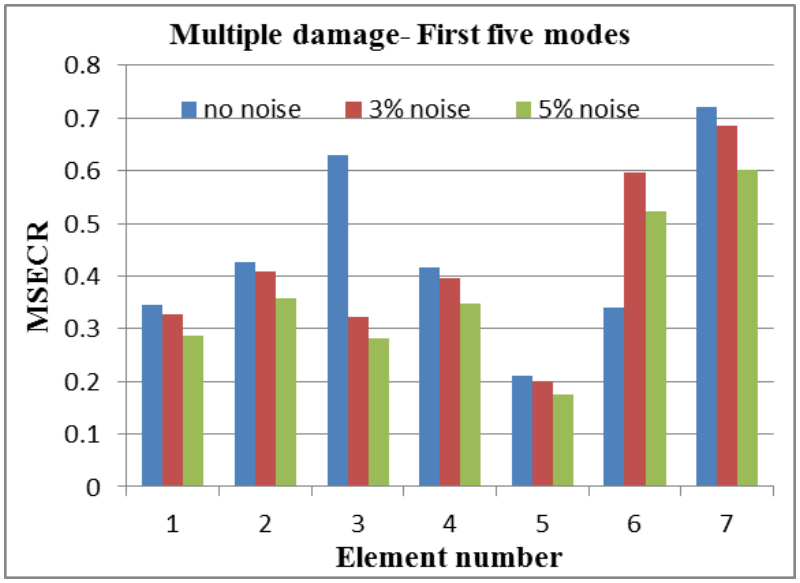
- Length of each element =L= 6.0 m
- Modulus of elasticity = E=30×10^9 N/m^2
- Cross-sectional area = A= 0.75 m^2
- 2nd moment of area = I=0.140625 m^4
- Mass density = 2500 kg/m^3

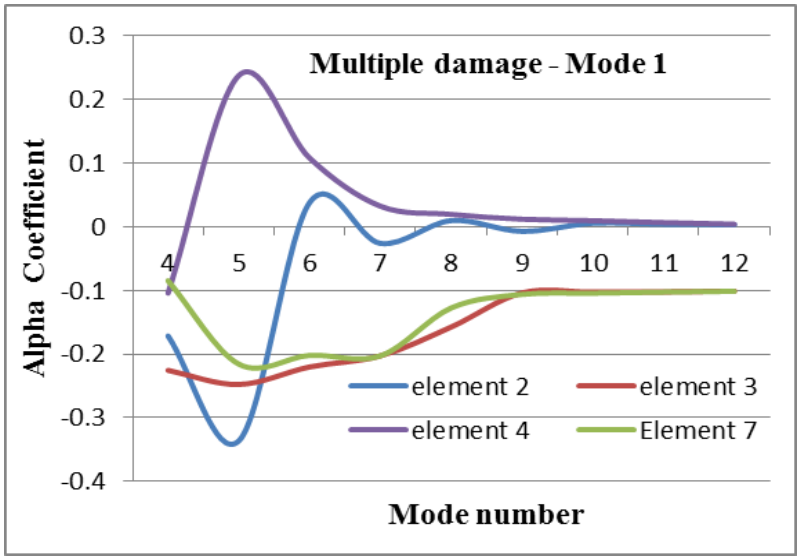
Single-damage scenario





Multiple-damage scenario





Conclusions;

The present method is;

1. More accurately capable of detecting the damage in short- and medium-span of bridges.
2. Converges faster with higher rate i.e. converges with less number of analytical modes. The number of iterations for structures with high DOFs it more inexpensive.
3. In presence of low value of the noise up to 5 percent, it performs quite well.

Significance

- Predict the possible parts exposure to damage

  Facilitates timely maintenance of bridges.

- Predict the unexpected failure and collapse in bridges → minimize the loss of lives and properties.

Thanks for your attention