HEALTH MONITORING OF SHORT- AND MEDIUM-SPAN BRIDGES USING AN IMPROVED MODAL STRAIN ENERGY METHOD

Parviz Moradipour [1], Tommy H.T Chan [2] and Chaminda Gallage [3]

ABSTRACT: Increasing the importance and use of infrastructures such as bridges, demands more effective structural health monitoring (SHM) systems. SHM has well addressed the damage detection issues through several methods such as modal strain energy (MSE). Many of the available MSE methods either have been validated for limited type of structures such as beams or their performance is not satisfactory. Therefore, it requires a further improvement and validation of them for different types of structures. In this study, an MSE method was mathematically improved to precisely quantify the structural damage at an early stage of formation. Initially, the MSE equation was accurately formulated considering the damaged stiffness and then it was used for derivation of a more accurate sensitivity matrix. Verification of the improved method was done through two plane structures: a steel truss bridge and a concrete frame bridge models that demonstrate the framework of a short- and medium-span of bridge samples. Two damage scenarios including single- and multiple-damage were considered to occur in each structure. Then, for each structure, both intact and damaged, modal analysis was performed using STRAND7. Effects of up to 5 per cent noise were also comprised. The simulated mode shapes and natural frequencies derived were then imported to a MATLAB code. The results indicate that the improved method converges fast and performs well in agreement with numerical assumptions with few computational cycles. In presence of some noise level, it performs quite well too. The findings of this study can be numerically extended to 2D infrastructures particularly short- and medium-span bridges to detect the damage and quantify it more accurately. The method is capable of providing a proper SHM that facilitates timely maintenance of bridges to minimise the possible loss of lives and properties.

KEYWORDS: Structural health monitoring, modal strain energy, finite element method (FEM), modal analysis, damage detection, short- and medium-span bridges

1. INTRODUCTION

Occurrence of damage in a structure, regardless of its appearance source/mode, will alter the global dynamic characteristics of the structure such as natural frequencies and mode shapes. Theoretically, as these parameters are measureable, evaluating the changes of these parameters over the time may be beneficial in determining the location and severity of the possibly damage/s in the structure. However, applying this perception on real structures demands more deliberations and considerations [1]. Short- and medium-span (length smaller than 200 ft) bridges [2] are usually prone to possible damage because of the environmental and service loads, which requires an effective and inexpensive surveillance technique rather than visual inspections [3].

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A proper structural monitoring and safety evaluation could ensure the structural health [4] and in case of (existing) the damage, it will facilitate timely maintenance of bridges to provide a safe transportation by minimizing the possible loss of lives and properties. This study deals with improving an MSE method for detecting the damage of civil infrastructures especially short- and medium-span bridges. Some previous studies on MSE method are briefly summarized in the following paragraphs;

Carrasco and others [5] proposed an MSE method using pre- and post-damage MSE measured via modal analysis. It was implemented for an 3D space truss subjected to 18 different damage scenarios using experimental data. The results showed that the method was able to detect most of the damage types and cases that were tested. However, the method is unique for truss structure and it is also capable of capturing only some damage types properly. Doebling and others [6] developed an MSE based method for selecting a subset of identified structural vibration modes to be used in FEM correlation and structural damage detection. The MSE modes measured were then ranked and used in descending order. It was observed that a mode selection strategy based on MSE provided more accurate results than a strategy using modal frequency.
Shi and others [7, 8] established an MSE based method for detecting damage using the change of elemental MSE. This approach is simple and capable of detecting single or multiple damages in structures. The sensitivity of the MSE with respect to damage was also derived as a function of the analytical mode shape changes and the stiffness matrix. Since in this approach only the incomplete measured mode shapes and analytical system matrices are used for damage location and quantification, there is a need to more accurately quantify the damage severity. The results show that though the proposed approach is noise sensitive, it can locate single and multiple damages.
Shih and others [9] blended a multi-criteria procedure incorporating modal flexibility and MSE methods applying to a plate structure and a beam structure. The purpose was to identify single and multiple damages via a computer simulation. Nine damage scenarios were considered in each element and it was found that for single damage, modal flexibility changes (MFC) and MSE changes provide similar results with no locating error. Although for multiple damage scenarios MSE changes increased the accuracy of the damage locating in the plate, the simulation of multiple-damage needs further study.

Wang and others [10] theoretically improved a modal strain energy correlation (MSEC) method using an MSE-to-damage sensitivity variable. The method was then further developed to overcome the effect of noise contamination. Subsequently, it was validated for complicated steel truss bridges using multi-layer genetic algorithm which became more efficient and feasible even in presence of noise [11, 12]. However, this method has not been verified for other type of bridge structures or buildings. Wahalathantri and others [13, 14] validated a damage index based MSE method on a simply supported and two-span beam. It was observed that it was capable of capturing the damage at any one of the measured modes inexpensively and with less time-consumption. However, this method has been applied to simple beams only.

From the literature reviewed, it was observed that MSE has been widely used for structural damage detection. However, the MSE methods either have often been validated for only specific type of structures such as beam like structures and steel truss bridges or have no sufficient accuracy. It is desired to identify the location and severity of structural damage more accurately in a practical manner. Therefore it is essential to enhance/improve the available MSE methods in order to provide a more applicable and reliable approach for damage detection and quantification of any structure.

This study aims to develop an available MSE scheme to be more accurate and feasible for damage detection of infrastructures. An MSE method is mathematically developed and then numerically applied to two plane structures including a steel truss bridge and a concrete frame bridge structures which demonstrate framework of a short- and medium-span bridge samples respectively. The research contribution can be numerically extended to 2D infrastructures particularly short- and medium-span

bridges to detect the damage and quantify it more accurately. The method is capable of providing a proper SHM that facilitates the maintenance by preventing unexpected structural damages and consequently decreasing the possible loss of lives and properties.

2. THEORETICAL BACKGROUND

In structural dynamics, for a free vibration case in the absence of damping, the changes in global dynamic characteristics of a structure including mode shapes and natural frequencies and the change in stiffness are given by equations 1, 2 and 3 respectively [7];

$$\{\phi_i^d\} = \{\phi_i\} + \{\Delta\phi_i\} = \{\phi_i\} + \sum_{r=1}^{md} c_{ir}\{\phi_r\} \quad (1)$$

where $c_{ir} = \frac{\{\phi_r\}^T[\Delta K]\{\phi_i\}}{\lambda_i - \lambda_r} \quad (i \neq r)$,
md= number of analytical modes and
$\{\phi_i^d\}$ and $\{\phi_i\}$ are damaged and undamaged mode shapes at mode i respectively.

$$\lambda_i^d = \lambda_i + \Delta\lambda_i \quad (2)$$

where λ_i^d and λ_i are the damaged and undamaged eigenvalues at mode i.

$$[K^d] = [K] + [\Delta K] = K + \sum_{m=1}^{L} \alpha_m [K_m] \qquad (-1 < \alpha_m \leq 0) \quad (3)$$

where α_m is the fractional reduction coefficient of m^{th} elemental stiffness matrix and
K^d and K are the global damaged and undamaged stiffness of the structure respectively.

3. MATHEMATICALLY IMPROVED MSE METHOD

An attempt was made to improve the previous study proposed by Shi and others [7, 15] in order to quantify the damage more accurately. Firstly, the damaged structural stiffness matrix (instead of undamaged structural stiffness matrix) was used to obtain the more accurate MSE equation. By this improvement, it is expected to get the appropriate damage severity with less computational cycles and iterations. The procedure is stated as follows;

The change in MSE at mode *i* and element *j* is:

$$\Delta MSE_{i,j} = MSE_{i,j}^d - MSE_{i,j} = \frac{1}{2}\{\phi_i^d\}^T[K_j^d]\{\phi_i^d\} - \frac{1}{2}\{\phi_i\}^T[K_j]\{\phi_i\} \quad (4)$$

Substituting for $\{\phi_i^d\}$ and $[K_j^d]$ in equations 4 from equations 1 and 3 respectively:

$$\Delta MSE_{i,j} = \frac{1}{2}\{\phi_i + \Delta\phi_i\}^T([K_j] + \alpha_j[K_j])\{\phi_i + \Delta\phi_i\} - \frac{1}{2}\{\phi_i\}^T[K_j]\{\phi_i\} \quad (5)$$

Simplifying and neglecting the higher order term leads to;

$$\Delta MSE_{i,j} = \frac{1}{2}\alpha_j\{\phi_i\}^T[K_j]\{\phi_i\} + \frac{1}{2}(1+\alpha_j)\left[\{\phi_i\}^T[K_j]\sum_{r=1}^{md}\frac{\{\phi_r\}^T[\Delta K]\{\phi_i\}}{\lambda_i-\lambda_r}\{\phi_r\} + \sum_{r=1}^{md}\frac{\{\phi_r\}^T[\Delta K]\{\phi_i\}}{\lambda_i-\lambda_r}\{\phi_r\}^T[K_j]\{\phi_i\}\right] \quad (i \neq r) \tag{6}$$

where *i* is normally in the range of 1 to 5
and *r* is the number of analytical modes under consideration ($r \leq$ no. of DOFs)

Substituting for $[\Delta K]$ ($[\Delta K] = \sum_{i=1}^{L}\alpha_i[K_i]$) from equations 3 into equations 6, simplifying and ignoring the higher order terms

$$\Delta MSE_{i,j} = \frac{1}{2}\alpha_j\{\phi_i\}^T[K_j]\{\phi_i\} + \frac{1}{2}\left[\{\phi_i\}^T[K_j]\sum_{i=1}^{L}\alpha_i\sum_{r=1}^{md}\frac{\{\phi_r\}^T[K_i]\{\phi_i\}}{\lambda_i-\lambda_r}\{\phi_r\} + \sum_{i=1}^{L}\alpha_i\sum_{r=1}^{md}\frac{\{\phi_r\}^T[K_i]\{\phi_i\}}{\lambda_i-\lambda_r}\{\phi_r\}^T[K_j]\{\phi_i\}\right] \quad (i \neq r) \tag{7}$$

Even though the change in MSE primarily in equation 4 was expressed in terms of damaged stiffness matrix (K_j^d), in equation 7, it has been transformed in terms of undamaged stiffness matrix (K_j) which is a known variable.

3.1 LOCATE THE DAMAGE

The procedure proposed by Shi and others [7, 16] is directly used to locate the damage. A damage location indicator which is called the modal strain energy change ratio (MSECR) obtained from equation 8 is used. MSECR can be derived for mode *i* and element *j* as given in equation 8.a or normalized for the first five mode shapes of element *j* as given in equation 8.b. As the improved ΔMSE obtained from equation 7 is used in equation 8 to calculate the MSECR parameter, the recent MSECR is expected to be more accurate in locating the damage/s. In the plot of MSECR versus element numbers, the elements with higher amounts of MSECR are the probably damaged elements.

$$MSECR_{ij} = \frac{\left|MSE_{i,j}^d - MSE_{i,j}\right|}{MSE_{i,j}} \tag{8.a}$$

$$MSECR_j = \frac{1}{m}\sum_{i=1}^{5}\frac{MSECR_{ij}}{MSECR_{i,max}} \tag{8.b}$$

where $MSECR_j$ is the average of summation of $MSECR_j^i$ for the first five modes normalized with respect to the largest value $MSECR_{max}^i$ of each mode.

3.2 QUANTIFY THE DAMAGE

The second improvement of the present study is derivation of a sensitivity matrix using the previously improved equation of MSE. After locating the damage/s by selecting the most probably suspected elements from the previous section, quantifying the damage is performed within those elements. It is trying to find the amount of α's as the fractional reduction coefficient of elemental stiffness. In each

iteration, the value of α for true damaged elements will converge to non-zero values while for other suspected elements converge to zero values. The exact value of each α may be obtained after a number of iterations. The improved procedure is as follows;

In previous study [7], MSEC has been defined as following terms:

$$MSEC_{ij} = \{\phi_i^d\}^T[K_j]\{\phi_i^d\} - \{\phi_i\}^T[K_j]\{\phi_i\} \quad (9)$$

However theoretically the value of $MSE_{i,j}^d$ is a function of $[K_j^d]$. Definitely it is expected by using K_j^d instead of K_j will give more exact value for $MSEC_{ij}$, therefore

$$MSEC'_{ij} = \{\phi_i^d\}^T[K_j^d]\{\phi_i^d\} - \{\phi_i\}^T[K_j]\{\phi_i\} \quad (10)$$

Substituting for K_j^d from equation 3 into equation 10 and then simplifying and arranging:

$$MSEC'_{ij} = \alpha_j\{\phi_i^d\}^T[K_j]\{\phi_i^d\} + \{\phi_i^d\}^T[K_j]\{\phi_i^d\} - \{\phi_i\}^T[K_j]\{\phi_i\} \quad (11)$$

Substituting equation 10 into equation 11 gives

$$MSEC'_{ij} = \alpha_j\{\phi_i^d\}^T[K_j]\{\phi_i^d\} + MSEC_{ij} \quad (12)$$

Partial differentiating of the equation 7 with respect to α_i leads to sensitivity matrix as follows;

$$\frac{\partial MSE_{i,j}}{\partial \alpha_i} = S_{i,j}^{MSE} =$$
$$\frac{1}{2}\{\phi_i\}^T[K_j]\{\phi_i\} + \frac{1}{2}\left[\{\phi_i\}^T[K_j]\sum_{i=1}^{L}\sum_{r=1}^{md}\frac{\{\phi_r\}^T[K_i]\{\phi_i\}}{\lambda_i-\lambda_r}\{\phi_r\} + \sum_{i=1}^{L}\sum_{r=1}^{md}\frac{\{\phi_r\}^T[K_i]\{\phi_i\}}{\lambda_i-\lambda_r}\{\phi_r\}^T[K_j]\{\phi_i\}\right]$$
$$(i \neq r) \quad (13)$$

From equation 13 ignoring the coefficient of $\frac{1}{2}$, it can be expressed as:

$$[\beta]\{\alpha\} = \{MSEC'\} \quad (14)$$

where $MSEC'$ is the difference between the damage and undamaged cases as equation 10 and β equals to

$$\beta_{s,t} = \frac{\partial MSE}{\partial \alpha} =$$
$$\{\phi_i\}^T[K_j]\{\phi_i\} + \sum_{r=1}^{n}\{\phi_i\}^T[K_s]\frac{\{\phi_r\}^T[K_t]\{\phi_i\}}{\lambda_i-\lambda_r}\{\phi_r\} + \sum_{r=1}^{n}\frac{\{\phi_r\}^T[K_t]\{\phi_i\}}{\lambda_i-\lambda_r}\{\phi_r\}^T[K_s]\{\phi_i\}$$
$$(i \neq r) \quad (15)$$

where *s* is a selected element for computation of the MSEC and
t is a suspected damaged element.

Substituting the equations 15, 12 and 9 respectively into equation 14 and simplifying, it gives:

$$\left[-[\mathrm{MSEC}] + \sum_{r=1}^{n}\{\phi_i\}^T[K_s]\frac{\{\phi_r\}^T[K_t]\{\phi_i\}}{\lambda_i-\lambda_r}\{\phi_r\} + \sum_{r=1}^{n}\frac{\{\phi_r\}^T[K_t]\{\phi_i\}}{\lambda_i-\lambda_r}\{\phi_r\}^T[K_s]\{\phi_i\}\right]\{\alpha\} = \{\mathrm{MSEC}\} \quad (i \neq r) \qquad (16)$$

Denoting $\beta^*_{s,t} = -\mathrm{MSEC}_{ij}$ and

$$\beta'_{s,t} = \sum_{r=1}^{n}\{\phi_i\}^T[K_s]\frac{\{\phi_r\}^T[K_t]\{\phi_i\}}{\lambda_i-\lambda_r}\{\phi_r\} + \sum_{r=1}^{n}\frac{\{\phi_r\}^T[K_t]\{\phi_i\}}{\lambda_i-\lambda_r}\{\phi_r\}^T[K_s]\{\phi_i\} \quad (i \neq r),$$

Then $\beta_{s,t}$ can be written in the following form;

$$\beta_{s,t} = \beta^*_{s,t} + \beta'_{s,t} \qquad (17)$$

Reconstructing the equation 14 in matrix notation,

$$([\beta^*] + [\beta'])\{\alpha\} = \{\mathrm{MSEC}\} \qquad (18)$$

From equation 18, $\{\alpha\}$ can be obtained in an expanding form as;

$$\begin{bmatrix}\alpha_1\\ \alpha_2\\ \vdots\\ \alpha_q\end{bmatrix} = \left(\begin{bmatrix}\beta^*_{11} & 0 & \cdots & 0\\ 0 & \beta^*_{22} & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & \beta^*_{qq}\end{bmatrix} + \begin{bmatrix}\beta'_{11} & \beta'_{12} & \cdots & \beta'_{1q}\\ \beta'_{21} & \beta'_{22} & \cdots & \beta'_{2q}\\ \vdots & \vdots & \ddots & \vdots\\ \beta'_{q1} & \beta'_{q2} & \cdots & \beta'_{qq}\end{bmatrix}\right)^{-1}\begin{bmatrix}\mathrm{MSEC}_{i1}\\ \mathrm{MSEC}_{i2}\\ \vdots\\ \mathrm{MSEC}_{ij}\end{bmatrix} \qquad (19)$$

3.3 EFFECT OF NOISE

The effect of noise is considered as equation 20 [7];

$$\bar{\varphi}_{ij} = \varphi_{ij}\left(1 + \gamma_i^{\varphi}\rho^{\varphi}\left|\varphi_{max,j}\right|\right) \qquad (20)$$

where $\bar{\varphi}_{ij}$ and φ_{ij} are the mode shape components of the j^{th} mode at i^{th} DOF
γ_i^{φ} are the random numbers with the mean of zero and a variance of 1
ρ^{φ} is the noise level (per cent)
$\varphi_{max,j}$ is the largest component of the j^{th} mode shape

4. VERIFICATION

Verification of the improved method is performed by applying to two 2D structures namely a steel truss bridge and a concrete frame bridge structure which demonstrate the framework of a short- and medium-span of bridge samples respectively. For each structure, modal analysis is performed for both the intact and damaged cases (both single- and multiple-damage scenarios) using STRAND7 [17]. For intact structures, all mode shapes and natural frequencies are derived while for damaged cases only the first five mode shapes and natural frequencies are derived. The improved method then is applied to structures using numerical data derived from the modal analysis through a MATLAB code [18, 19]. The details of calculation for each sample are presented in the following sections.

4.1 NUMERICAL EXAMPLE 1

The first numerical example is an 2D steel truss structure with truss element of two degrees-of-freedom (DOFs) at each end. As shown in Figure 1, the truss consists of 12 nodes and 21 elements with 20 DOFs. The material properties and geometric data are as follow;
Length =L= 5 m
Modulus of elasticity = E=207×10^9 N/m^2
Cross-sectional area = A=0.04 m^2
Mass density = 7870 kg/m^3

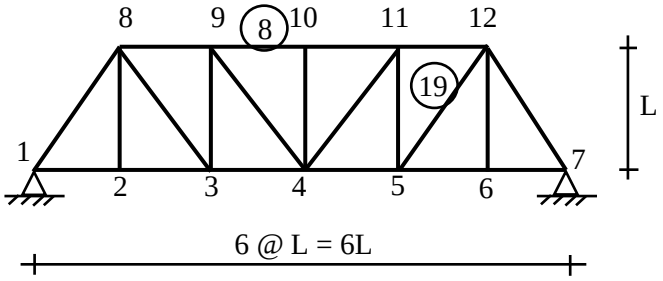

Figure 1. (Pratt) Truss bridge with 21 elements
(Note: Circled numbers are the damaged elements.)

Two damage scenarios are assumed to occur in the structure. Scenario1 is a single-damage that occurs in element 8 with a stiffness loss of 15% and scenario 2 is a multiple-damage with damages in elements 8 and 19 with stiffness loss of 10% in both of them. To apply the improved method and determine the suspected elements, MSECR parameter is calculated using equation 8. The search for finding the true damaged elements is then performed through the suspected elements using the improved equation 19. The effect of noise is also included using equation 20 for two different assumed percentages of 3 and 5.
The single- and multiple-damage locations are shown in Figures 2 and 3 respectively. The alpha coefficients of single- and multiple-damage quantified with first mode using the improved method in this study are shown in Figures 4 and 5 respectively. The values of alpha of suspected elements for single and multiple-damage scenarios at different cases are shown in Tables 1 and 2 respectively as well.

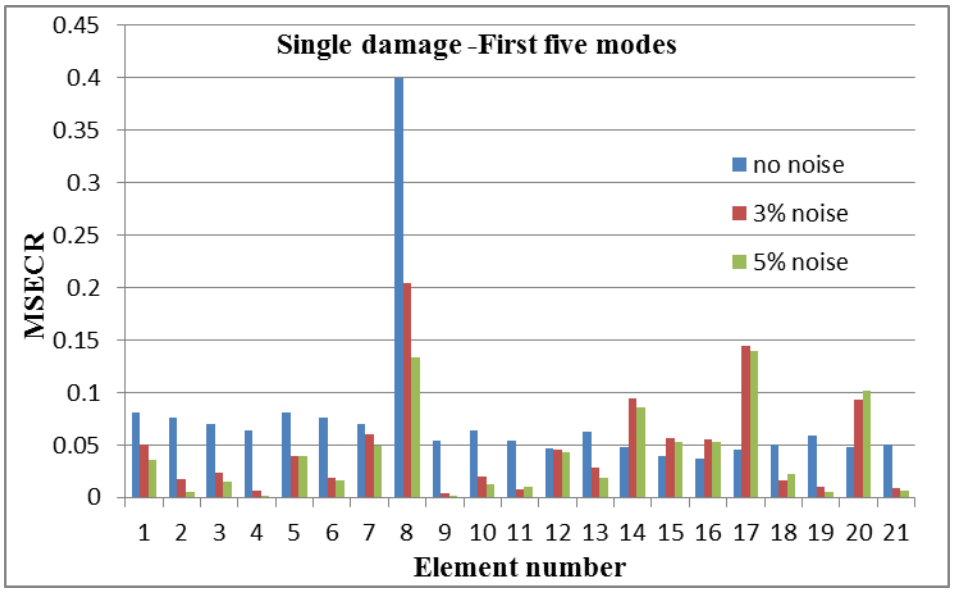

Figure 2. Elemental damage of single-damage scenario located with the first five modes- element 8 (using equation 8)

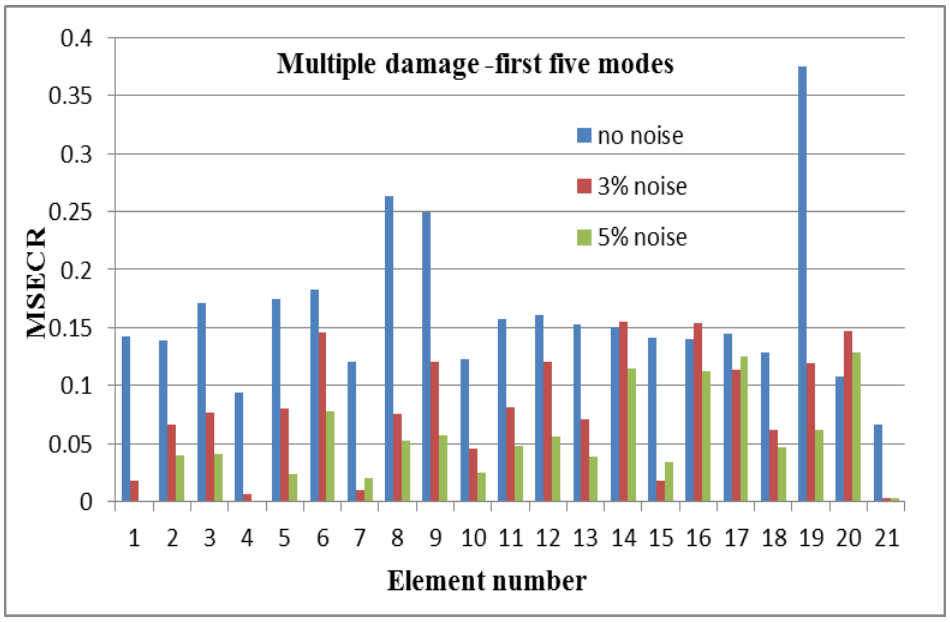

Figure 3. Elemental damage of multiple-damage scenario located with the first five modes- elements 8 and 19 (using equation 8)

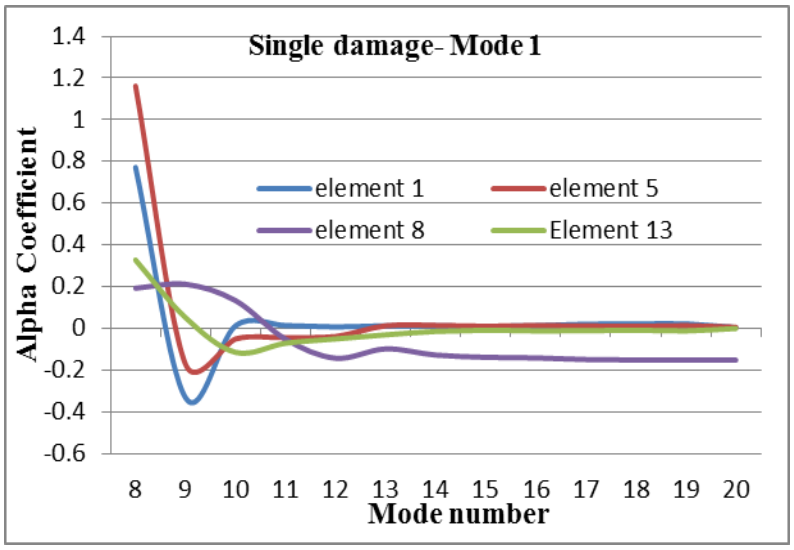

Figure 4. Alpha coefficients of single-damage scenario (no noise) quantified with the first mode for suspected elements

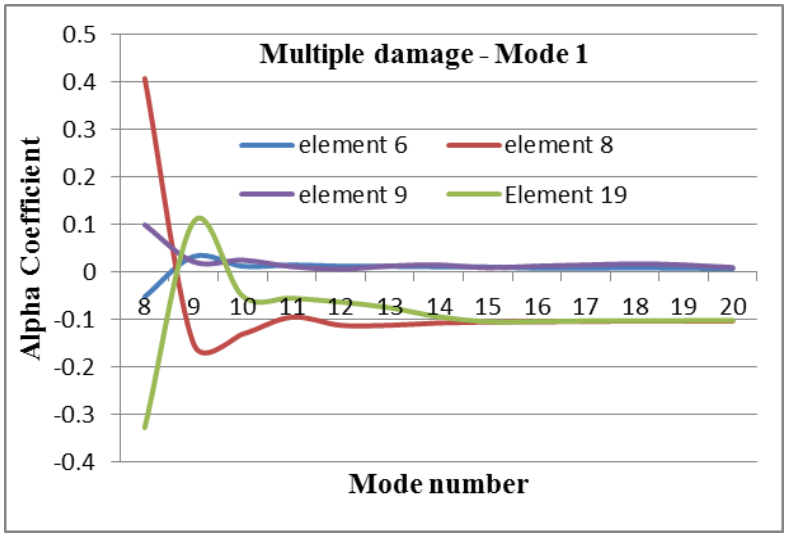

Figure 5. Alpha coefficients of multiple-damage scenario (no noise) quantified with the first mode for suspected elements

Table 1. The amount of alphas of suspected elements for single-damage scenario at different cases – example 1

Case	α_1	α_5	α_8	α_{13}
No noise	0.0033	0.0047	-0.1527	-0.001
3% noise	-0.0073	-0.0092	-0.1471	-0.0056
5% noise	-0.0202	-0.0175	-0.1537	-0.0129

Table 2. The amount of alphas of suspected elements for multiple-damage scenario at different cases – example 1

Case	α_6	α_8	α_9	α_{19}
No noise	0.0071	-0.1033	0.0097	-0.1018
3% noise	-0.0114	-0.0974	-0.0129	-0.0983
5% noise	-0.0161	-0.1057	-0.0178	-0.1041

4.2 NUMERICAL EXAMPLE 2

The second example is a concrete frame bridge structure with frame elements of three degrees-of-freedom (DOFs) at each end. It consists of 8 nodes and 7 elements with 12 DOFs as shown in Figure 6. The material properties and geometric data are as follows;

Length =L= 6.0 m

Modulus of elasticity = E=30×10^9 N/m^2

Cross-sectional area = A=0.75 m^2

Second moment of area = I= 0.140625 m^4

Mass density = 2500 kg/m^3

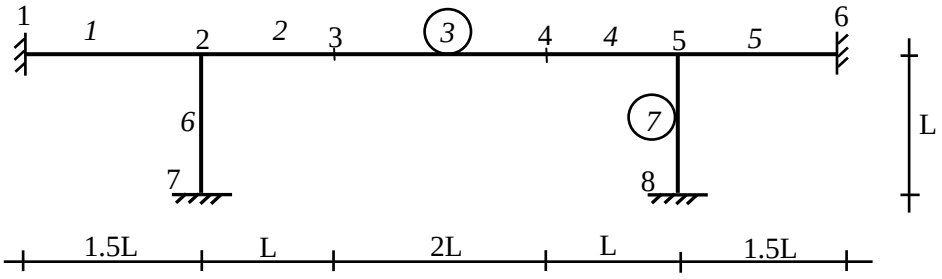

Figure 6. Frame structure with seven elements
(Note: Circled numbers are the damaged elements.)

Similarly, two damage scenarios are assumed to happen in the frame structure. Scenario 1 is a single-damage that occurs in a beam with element number of 3 with a stiffness loss of 15% and scenario 2 is a multiple-damage with damages in elements 3 and 7 with stiffness loss of 10% in both of them. Similarly, equations 8, 19 and 20 were used to calculate MSECR parameter, amount of alphas and effects of 3 and 5 percent noises respectively.

The single- and multiple-damage locations are shown in Figures 7 and 8 respectively. The alpha coefficients of single- and multiple-damage quantified with first mode using the proposed method in this study are also shown in Figures 9 and 10 respectively. The amount of alphas of suspected elements for single and multiple-damage scenarios at different cases of this example are shown in Tables 3 and 4 respectively too.

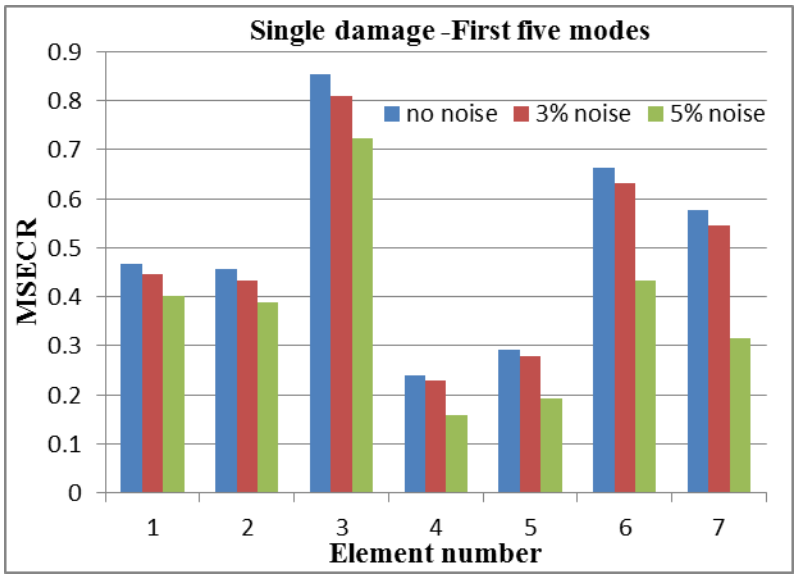

Figure 7. Elemental damage of single-damage scenario located with the first five modes- element 3 (using equation 8)

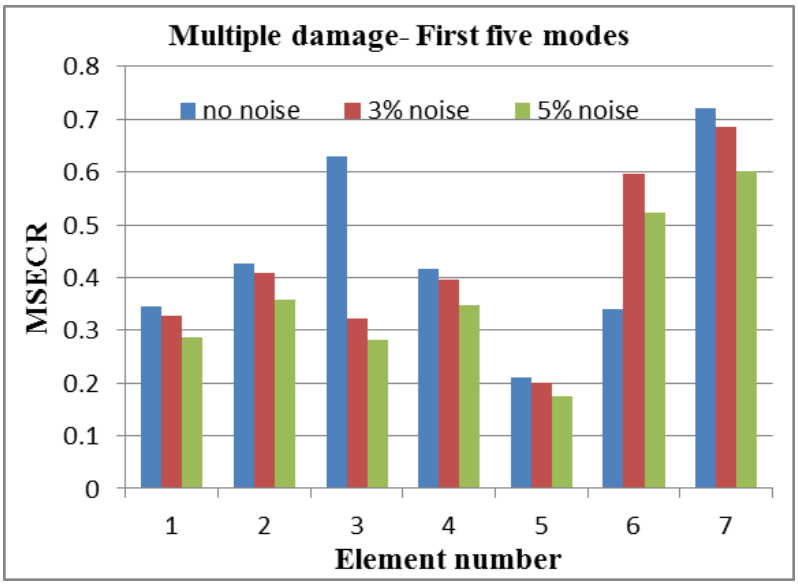

Figure 8. Elemental damage of multiple-damage scenario located with the first five modes –elements 3 and 7 (using equation 8)

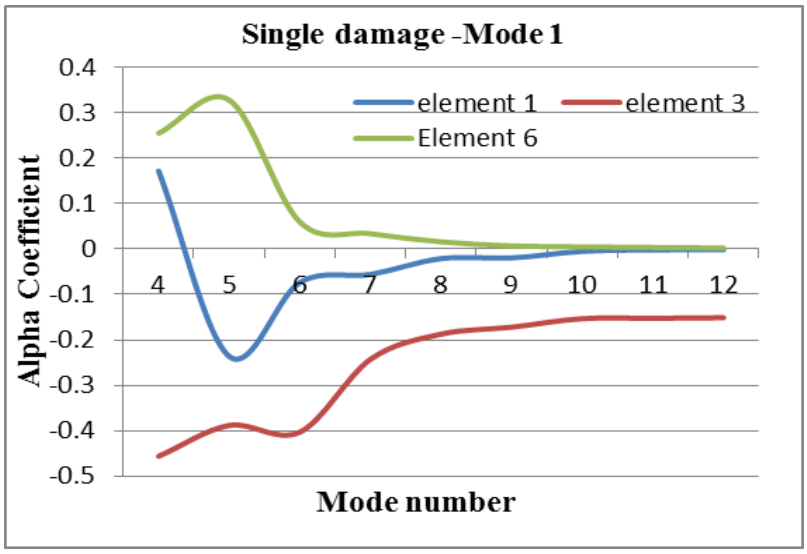

Figure 9. Alpha coefficients of single-damage scenario (no noise) quantified with the first mode for suspected elements

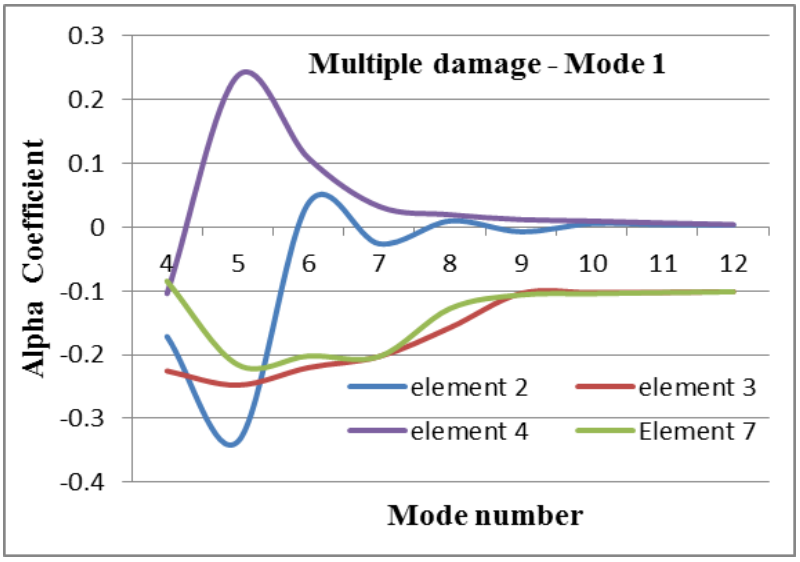

Figure 10. Alpha coefficients of multiple-damage scenario (no noise) quantified with the first modes for suspected elements

Table 3. The amount of alphas of suspected elements for single-damage scenario at different cases – example 2

Case	α_1	α_3	α_6	--
No noise	-0.0019	-0.1513	0.0021	--
3% noise	-0.0106	-0.1484	-0.0096	--
5% noise	-0.0211	-0.1539	-0.0147	--

Table 4. The amount of alphas of suspected elements for multiple-damage scenario at different cases - example 2

Case	α_2	α_3	α_4	α_7
No noise	0.0032	-0.1011	0.0041	-0.1014
3% noise	-0.0081	-0.0969	-0.0094	-0.0976
5% noise	-0.0158	-0.1026	-0.0140	-0.1055

5. RESULTS AND DISCUSSION

For the truss structure, in the single-damage scenario, shown in Figure 2, it is seen that the MSECR crests at element 8 which represents it is the highly suspected element to damage. Elements 1, 5 and 13 also have probably damaged because of getting the high value of MSECR. Figure 4 shows a view of α's versus modes 8 and above that indicates the amount of all α's converge to zero except α_8 which converges to the non-zero amount around -0.1527. In other words, it means for this damage scenario, only element 8 is a true damaged element with the amount of α_8 as the reduction percentage of elemental stiffness. It is evident that since other suspected elements 1, 5 and 13 have got the α with the amount of almost zero so no damage has occurred in those elements.

Similarly, in the multiple-damage scenario, shown in Figure 3, the MSECR peaks at both elements 8 and 19 which represents their highly possibility to damage. Elements 6 and 9 are also probably damaged elements because of having the high amount of MSECR. Figure 5 shows that the amount of α_6 and α_9 converge to near zero while α_8 and α_{19} converge to the non-zero amount of -0.1033 and -0.1018 respectively. Likewise the single-damage scenario, it can be concluded that element 8 and 19 are the true damaged elements with α_8 and α_{19} amount of reduction percentage of elemental stiffness however elements 6 and 9 have not experienced any damage.

Tables 1 and 2 show the amount of α's from the last iteration of computations for the single- and multiple-damage scenarios respectively. It is seen in both scenarios, for the case of "No noise" the more exact values have achieved rather than the other cases with noise. It clearly indicates that although in presence of noise, the same elements (with the case of no noise) are recognized as the suspected elements, there is a few error in the amount of α's of the cases contaminated with noise. In other words, the method is slightly noise sensitive.

In the second structure, for single-damage scenario, elements 1, 3 and 6 are recognized as the suspected elements as shown in Figure 7. The coefficient of α_3 obtained is -0.1513 while α_1 and α_6 converge to zero as drawn in Figure 9. In other word, for this damage scenario, only element 3 is the true damaged element with reduction stiffness of α_3 percent.

In multiple-damage scenario also among the suspected elements of 2, 3, 4 and 7 as shown in Figure 8, the amount of α_3 and α_7 are calculated as -0.1011 and -0.1014 respectively whereas α_2 and α_4 converge to zero which are drawn in Figure 10. This also represents that elements 3 and 7 are the true damaged elements with reduction stiffness of α_3 and α_7 percent respectively. Tables 3 and 4 also show the amount of example 2 α's from the last iteration of computations for the single- and multiple- damage scenarios respectively. The effect of noise on accuracy of α's is similar with example 1.

6. CONCLUSIONS

In this article an MSE method was mathematically formulated to precisely quantify the structural damage and numerically validated for specifically 2D short- and medium-span bridges. Firstly, an accurate MSE equation was formulated considering damaged elemental stiffness. The MSE equation derived then was used to obtain an accurate sensitivity matrix to perfectly quantify the damage. Verification of the improved method was made by applying to two 2D structures of a steel truss bridge and a concrete frame bridge structures representing the framework of a short- and medium-span of bridge samples. Two damage scenarios including single- and multiple-damage were considered to occur in each structure. Then, for each structure, both intact and damaged (of two damage scenarios), modal analysis was performed. For undamaged cases, all mode shapes and natural frequencies were calculated whereas for damaged cases only the first five modes and natural frequencies were derived. Effects of 3 and 5 percent of noise were also included in the calculation.

The results indicate that the performance of the improved method is in a good agreement with the numerically assumed damages with few computational cycles. It is because of that the improved method converges very fast using few numbers of the analytical modes. Also in presence of low value of the noise up to 5 percent, it performs quite well with the same number of iterations that requires for the case of no noise.

The findings of this study can be numerically extended to real civil infrastructures specifically short- and medium-span bridges to detect the damage and especially quantify it more accurately. The method is capable of providing a proper SHM that in case of (existing) the damage facilitates the maintenance and transportation by preventing the possible loss of lives and properties.

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