# MINOR STRUCTURAL DAMAGE DETECTION USING CHAOTIC EXCITATION TECHNIQUE

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**ABSTRACT:** Vibration-based damage detection methods are popular for the structural health monitoring. However, they can only detect fairly large damages. Usually external forces such as impact pulse, ambient vibrations and sine-wave forces are applied as the excitations. In this paper, we propose the method to use the chaotic excitation to vibrate the structures. The attractors rebuild from the output responses are used for the minor damage detection. After the damage is detected, it is further quantified using the Kalman Filter technique. Simulations of a 4-story building subjected to chaotic excitation are conducted. The structural responses and related attractors are analyzed. There results show that the attractor distances increase monotonously with the increase of the damage degree. Thus, damages, including minor damages, can be effectively detected using the proposed approach. With the Kalman Filter technique, minor damages which have the stiffness decrease of about 5% or lower can be correctly quantified. The proposed approach will be helpful for detecting and evaluating minor damages at the early stage for the structural health monitoring.

**KEYWORDS:** Minor damage, Damage detection, Chaotic excitation, Kalman Filter, Structural health monitoring

## 1. INTRODUCTION

Structural health monitoring of civil engineering structures is a fundamental issue for structural safety and integrity, due to the fact that they will deteriorate just after they are built and put into services. The failure of structures will not only result in severe economic lost but may threaten the lives of people. Hence maintaining safety and reliable civil engineering structures for daily use is an extremely important issue which has received considerable attention in literature in recent years. Deterioration of the structure often refers to the structural damage and it can be reflected by the "deterioration" of the structural parameters. In practice, damage was defined as the changes introduced into a system which adversely affected its current or future performance [1]. Therefore changes in structural parameters have been extensively applied as effective tools for damage detection. Many methods are developed for this purpose. Among which, the

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vibration-based damage identification (VBDI) methods draws extensive attention and are deeply developed.

As we know, the deterioration of the civil engineering structures usually begins from the local and small damages. Small damages gradually develop and become large damages and at last cause failure of the structure. For the consideration of the structural safety and reliability, detecting small damages is essential and useful. Also, Adewuyi and Wu found that the popular modal parameter based methods could be easily degraded by noises [2]. Their research proved that even when 2% of noise was added into the signals, the damage identification results became very poor. Therefore, in order to detect miner damages to ensure the safety and reliability of the structures, development of other approaches is necessary, in which chaos attractor-based analysis seems to be a promising way. However, in many times, detection alone may not be sufficient for the purpose of damage evaluation and structural maintenance. In this case, damage requires to be further quantified. In this paper, the damage will be detected and quantified by identifying structural parameters using the Kalman Filter. Thus the structural deterioration can be detected and evaluated at the early stage and proper relevant measurement can be applied to ensure the safety of the structures.

## 2. MINOR DAMAGE DETECTION WITH CHAOTIC ATTRACTORS

Recently, some new damage detection techniques have been proposed by using chaotic excitation and attractor analysis. In the field of nonlinear dynamics, systems are often described via their state space. Given infinite time, an ensemble of trajectories evolving in the state space can trace out a dynamical attractor which may be thought of as a geometrical object in the space to which all trajectories belong. The attractor of the space actually contains useful information due to the fact that it reflects the invariant properties of the system, and therefore draws much attention to the application of system classification.

Basically, attractor-based approach requires the acknowledgement of each state variable, which will make it very inconvenient in practice. In steady, attractor reconstruction is often applied, with its advantages to allow only a small number of variables be observed in real applications. Attractor reconstruction is a technique to recreate a topologically equivalent picture to the original multi-dimensional system behavior. Considering a low-dimensional deterministic original system composed of d variables, attractor of the original system is obtained by plotting time series of d variables in d dimensional space. However, as mentioned, it is always the case that limited number of variables can only be observed. This limitation can be solved using the Takens's theorem [3]. For a time series $x_1, x_2, \ldots, x_N$ of a single variable x, the embedding vector can be defined by

$$\mathbf{X}_i = \left(x_i, x_{i+\tau}, \cdots, x_{i+(m-1)\tau}\right)^T \quad (i = 1, 2, \cdots N_m) \tag{1}$$

where m is embedding dimension, $\tau$ is delay time and $N_m = N - (m-1)\tau$. By plotting $\mathbf{X}_i$ in m dimensional space from index $i$ of 1 to $N_m$, an attractor which is not the same with the original attractor itself but is topologically equivalent to it can be reconstructed by only a single variable.

Even though white noise is often used for VBDI analysis, it cannot be used for attractor-based analysis, because it is not deterministic and cannot yield deterministic responses. For applying the attractor-based analysis, the chaotic signal as the input is usually applied to be the excitation. Chaotic signals possess broad band frequency domain like noise, so that they can excite desirable number of modes. However, unlike the noise which is a random signal, chaotic signal is a low dimensional deterministic signal, so that it can provide deterministic and low dimensional responses. Also, with the deterministic chaotic excitations, noises can be significantly reduced, simply by stacking and averaging.

In literature, there are several chaotic attractors subtracted from different chaotic signals, such as Lorenz signal, Chen signal, Rössler signal, etc. In practice, Lorenz signal is the most popular and the derived Lorenz attractor is applied widely in engineering field. The Lorenz signal is the produced by three ordinary differential equations now known as the Lorenz equations. The governing equations of the typical Lorenz system can be expressed as

$$\begin{aligned} \dot{x} &= a(y-x) \\ \dot{y} &= cx - xz - y \\ \dot{z} &= xy - bz \end{aligned} \tag{2}$$

which is chaotic when a=10, b=8/3, c=28.

The three dimensional Lorenz chaotic signals can be seen in Figure 1, and corresponding constructed attractor can be found in Figure 2.

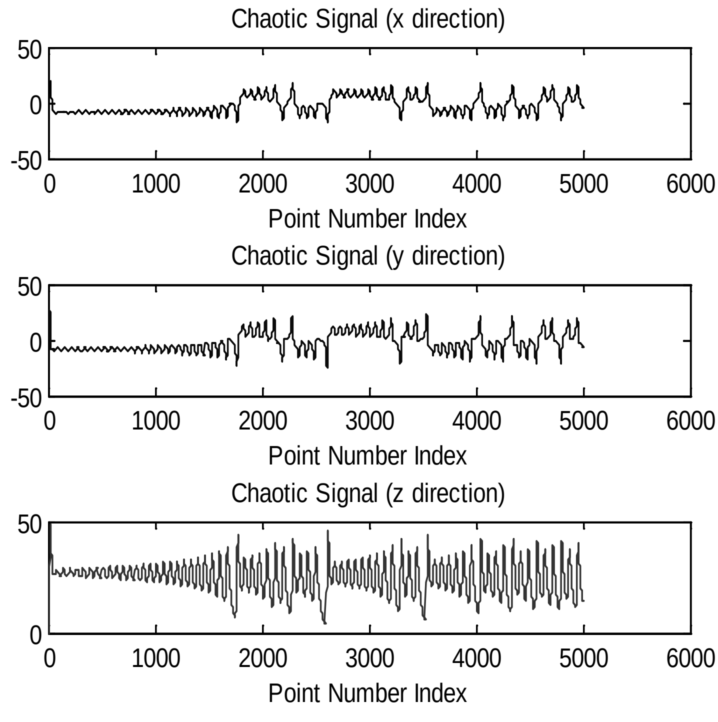



**Figure 1. Three dimensional Lorenz signals**

As mentioned before, attractors can be reconstructed by the signal in one direction, Figure 3 shows a reconstructed Lorenz attractor by only using x direction signal, with the delay $\tau = 10$.

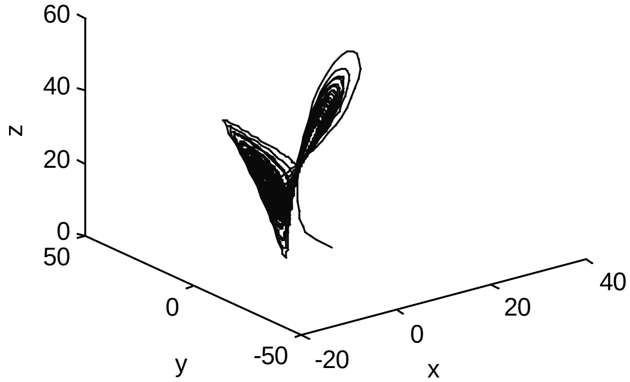



**Figure 2. Constructed Lorenz attractor**

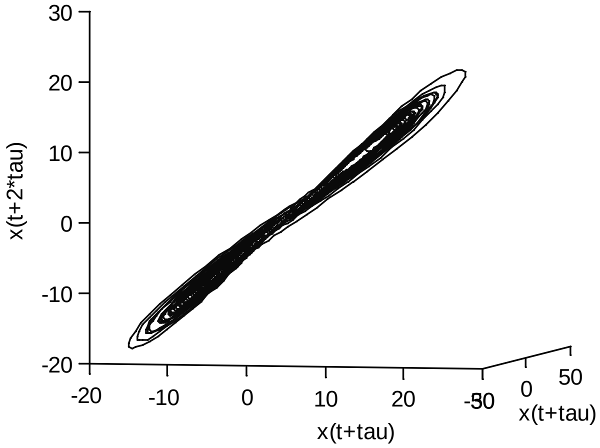



**Figure 3. Reconstructed Lorenz attractor by only using x direction signal**

In the structural health monitoring field, attractors, as a special feature, were also used for the damage detection. Nichols et al. detected the damage by comparing the defined “features” based on attractors reconstructed from healthy and damaged structural responses using chaotic excitation [4]. Sato et al. also proposed an attractor based damage detection method using chaotic excitation and recurrence analysis [5]. Since damages, even small ones, can change the state of a system which can be amplified in the attractor space, damages can be detected by studying the amplified change of the attractor trajectories. In this paper, distances of attractors between a health system and a damaged one are used to describe the system change, and hence to detect damages. Distance of two attractors between a health system and a damaged one can be expressed as

$$Dis = \sum_i \left\| \mathbf{H}_A^i - \mathbf{D}_A^i \right\| \qquad (i = 1, 2, \cdots n) \tag{3}$$

Where $\mathbf{H}_A^i$ and $\mathbf{D}_A^i$ are two discrete points in attractor space for a healthy structure system and a damaged structure system respectively. Superscript i is the serial number, means the ith point, n is the point number, while subscript A stands for the attractor space. $\mathbf{H}_A^i$ and $\mathbf{D}_A^i$ can be further be expressed as $\mathbf{H}_A^i = (h_1^i, h_2^i, \cdots, h_m^i)$ and $\mathbf{D}_A^i = (d_1^i, d_2^i, \cdots, d_m^i)$, where m is the dimension of the attractor space.

## 3. MINOR DAMAGE QUANTIFICATION WITH KALMAN FILTER

Kalman filtering is a power tool in the state estimating problem. It provides an efficient computational method to estimate the state of a process. After it was first proposed by R.E. Kalman in 1960, it was extensively studied and applied in many areas. In civil engineering filed, it is often used for the damage identification for structural health monitoring [6].
For a system, it has a state transfer function as

$$x_t = \Phi_{t-1} x_{t-1} + \Gamma_t w_t \tag{4}$$

where $x_t$ is the state vector, $\Phi_t$ is the state transfer matrix, $\Gamma_t$ is the noise effect matrix and $w_t$ is the system noise vector.

For Structural monitoring, we will have the observation equation as

$$\mathrm{z}_t = \mathbf{H}_t x_t + v_t \tag{5}$$

where $\mathrm{z}_t$ is the observed vector, $\mathbf{H}_t$ is the observation matrix, and $v_t$ the observation noise vector.

In the application to a system identification problem, state transfer matrix can be the unit matrix, i.e., $\Phi_t = \mathrm{I}$, while $x_t$ is still the unknown parameter vector. If we further assume that the system noise is also zero, which means we only consider the noise in the observation vector, then the simplified Kalman filter can be obtained as

$$\overline{x}_t = \hat{x}_{t-1} \tag{6}$$

$$\mathbf{G}_t = \mathbf{P}_{t-1} \tag{7}$$

$$\hat{x}_t = \overline{x}_t + \mathbf{B}_t (z_t - \mathbf{H}_t \overline{x}_t) \tag{8}$$

$$\mathbf{P}_t = \mathbf{G}_t - \mathbf{K}_t \mathbf{H}_t \mathbf{G}_t \tag{9}$$

$$\mathbf{B}_t = \mathbf{G}_t \mathbf{H}_t^T (\mathbf{H}_t \mathbf{G}_t \mathbf{H}_t^T + \mathbf{R}_t)^{-1} \tag{10}$$

$$\mathbf{B}_t = \mathbf{G}_t \mathbf{H}_t^T (\mathbf{H}_t \mathbf{G}_t \mathbf{H}_t^T + \mathbf{R}_t)^{-1} \tag{11}$$

where $\mathbf{P}_t$ is the state vector covariance matrix, while $\mathbf{R}_t$ is the covariance matrix of the observation noise vector.

$$\mathbf{P}_t = \mathrm{E}[(x_t - \hat{x}_t)(x_t - \hat{x}_t)^T] \tag{12}$$

$$\mathbf{G}_t = \mathrm{E}[(x_t - \overline{x}_t)(x_t - \overline{x}_t)^T] \tag{13}$$

$$\mathbf{R}_t = \mathrm{E}[v_t v_t^T] \tag{14}$$

For a structure, its motion equation is

$$\mathbf{M}\ddot{y} + \mathbf{C}\dot{y} + \mathbf{K}y = F \tag{15}$$

Where M is the mass matrix, C is the damping matrix, K is the stiffness matrix, and F is the out excitation force vector.

In our previous study, the case when only structural stiffness is unknown and need to be identified while mass and damping of the structure are known was identified using the Kalman filter (wan et al., 2013). Satisfied results were achieved. In this paper, however, the case when mass, stiffness and damping of the structure are all unknown is further studied. In this case, the structure with all parameters unknown actually becomes a black box. M, C, and K are all need to be identified. For identify them using Kalman Filter method, other parameters in Equation 15 should all be known. Fortunately, $\ddot{y}$ can be easily and precisely captured by install accelerometers on the structure, $\dot{y}$ and $y$ actually are the velocities and displacements, which can be derived based on the acceleration data. While outer force $\mathbf{F}$ of the structure is the inputted Lorenz signal which is deterministic and unknown. Thus $\ddot{y}$, $\dot{y}$, $y$ and $\mathbf{F}$ are all observed values, and therefore Equation 15 can be used as the observation equation in time marching integration scheme.

Since $\mathbf{k}\{y\}$ can be transformed into $[\mathrm{O}(y)]\bar{\mathbf{K}}$, where $[\mathrm{O}(y)]$ is a matrix composed from the components of displacement vector of the damaged structure while $\bar{\mathbf{K}}$ is the vector indicating the stiffness of the structure. Similarly, $M(\ddot{y})$ can be transformed into $[\mathrm{O}(\ddot{y})]\bar{\mathbf{M}}$, $\mathrm{C}\{\dot{y}\}$ can be transformed into $[\mathrm{O}(\dot{y})]\bar{\mathbf{C}}$. Then Equation 15 can be rewritten as

$$\left[\mathbf{O}(\ddot{y}) \quad \mathbf{O}(\dot{y}) \quad \mathbf{O}(y)\right]\cdot\begin{bmatrix}\bar{\mathbf{M}}\\ \bar{\mathbf{C}}\\ \bar{\mathbf{K}}\end{bmatrix}=\mathbf{F} \tag{16}$$

Let

$$\mathbf{H}=\left[\mathbf{O}(\ddot{y}) \quad \mathbf{O}(\dot{y}) \quad \mathbf{O}(y)\right] \tag{17}$$

$$\mathbf{X}=\begin{bmatrix}\bar{\mathbf{M}}\\ \bar{\mathbf{C}}\\ \bar{\mathbf{K}}\end{bmatrix} \tag{18}$$

Then Equation 16 can be further expressed as

$$\mathbf{F}=\mathbf{H}\cdot\mathbf{X} \tag{19}$$

Which shows that outer force vector $\mathbf{F}$ can be the observed vector and $\mathbf{H}$ is the observation matrix as indicated in Equation 5. X is the parameters which can be identified by Kalman filter.

The above Equations are based on the three observation quantities of displacement, velocity and acceleration, corresponding to the three quantities, mass, damping, and stiffness, which actually are need to be identified. In this case, observation matrix $\mathbf{H}$ is non-singular. However, in some cases, we can hardly get all required observations, say , we can only get the acceleration observation data. In this case, observation matrix $\mathbf{H}$ then becomes singular.

Considering the Newmark-Beta method for the motion equation ( $\beta=1/6$ )

$$(M+\frac{\Delta t}{2}C+\frac{\Delta t^2}{6}K)\ddot{y}_{n+1}=f_{n+1}-C(\dot{y}_n+\frac{\Delta t}{2}\ddot{y}_n)-\mathrm{K}(\mathrm{y}_n+\Delta t\cdot\dot{y}_n+\frac{\Delta t^2}{3}\ddot{y}_n) \tag{20}$$

$$y_{n+1} = y_n + \Delta t \cdot \dot{y}_n + \frac{\Delta t^2}{2}(\frac{1}{3}\dot{y}_{n+1} + \frac{2}{3}\dot{y}_n) \tag{21}$$

$$\dot{y}_{n+1} = \dot{y}_n + \Delta t \cdot (\frac{1}{2}\ddot{y}_{n+1} + \frac{1}{2}\dot{y}_n) \tag{22}$$

Then the Kalman filter Equations can be written as

$$x_t = \begin{Bmatrix} z_t \\ \dot{z}_t \\ \ddot{z}_t \\ c_t \\ k_t \end{Bmatrix} = \begin{Bmatrix} a_{11} & a_{12} & a_{13} & 0 & 0 \\ a_{21} & a_{22} & a_{23} & 0 & 0 \\ a_{31} & a_{32} & a_{33} & 0 & 0 \\ 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & I \end{Bmatrix} \begin{Bmatrix} z_{t-1} \\ \dot{z}_{t-1} \\ \ddot{z}_{t-1} \\ c_{t-1} \\ k_{t-1} \end{Bmatrix} + \begin{Bmatrix} b_1 \\ b_2 \\ b_3 \\ 0 \\ 0 \end{Bmatrix} F_t + w_t \tag{23}$$

$$y_t = H_t \cdot x_t + v_t \tag{24}$$

Where

$$y_t = \begin{Bmatrix} 0 \\ 0 \\ \ddot{z}_t \\ 0 \\ 0 \end{Bmatrix} \tag{25}$$

and

$$H_t = \begin{Bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{Bmatrix} \tag{26}$$

Obviously, observation matrix $\mathbf{H}$ is singular. With the Equations (23-26), due to the special characteristics of these matrices, C and K actually can not be updated. However, $z_t$ and $\dot{z}_t$ can be updated, which means with only acceleration, velocity and displacement can be predicted with such kalman filter. Combining with the first kalman filter which is defined by Equations (16-19), C, and K can be updated, with the velocity and displacement values being updated and fed back step by step. Thus even when the observation is not enough, required quantities still are possible to be identified combine more than one kalman filters.

## 4. SIMULATIONS

In order to verify the approaches introduced above, corresponding simulations are conducted. A model of a four-story structure is used for the numerical simulations. The mass of each floor is assumed to be concentrated to a mass point as shown in Figure 4, and defined to be 100kg respectively, while the stiffness of each floor is defined to be 1000N/m. An exciter is placed on the top of the structure to produce chaotic signals, as depicted in Figure 4. The exciting force is simulated using a chaotic signal, which is shown in Figure 5.

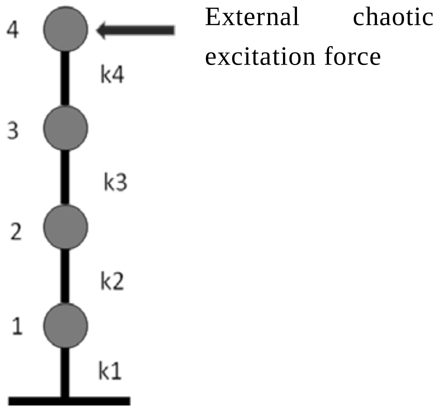



**Figure 4. Simulation model**

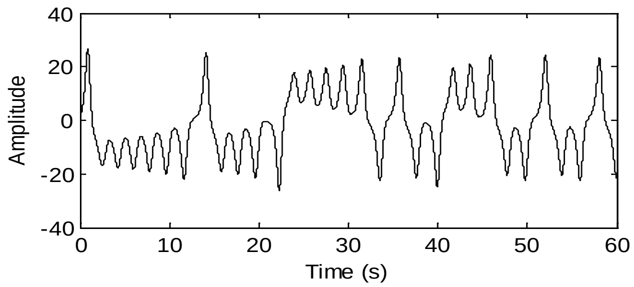
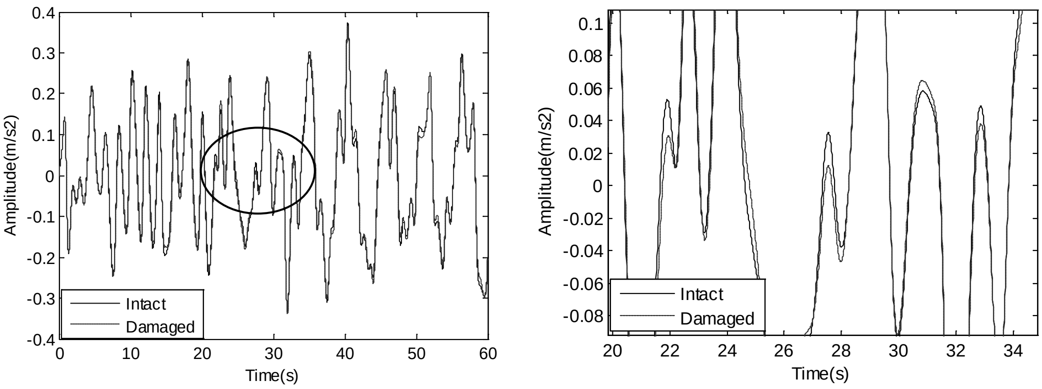



**Figure 5. Input chaotic excitation**

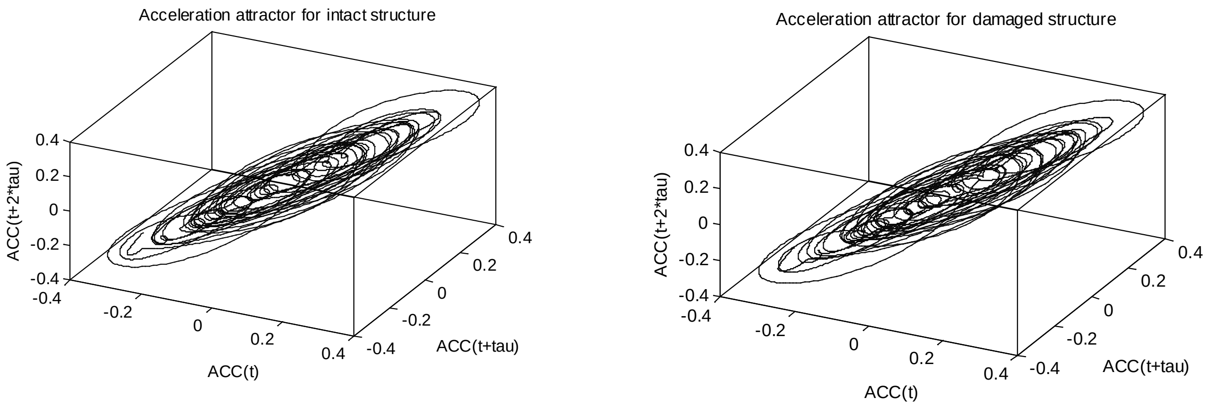



(a) Acceleration responses (b) Details of the difference

**Figure 6. Acceleration at the top floor for the intact structure and damaged structure with 5% stiffness decrease at the 3rd floor column**

Structural responses subjected to the input force are calculated using newmark-beta method. Figure 6(a) shows the accelerations for the intact structure and damaged structure with 5% stiffness decrease at the 3rd floor column. With the structure being damaged, its responses also slightly changed accordingly. Figure 6(b) shows the details of some differences. Theoretically, by studying the response difference under the same excitations, the change of the structural status can be detected. However, such difference of the responses are very small when the damage degree is very small, which makes the determination of the health status extremely difficult.

In this case, attractors are therefore applied since it can amplify the difference. Figure 7 shows the constructed attractors from the time series of the acceleration responses at the top floor for the intact structure and damaged structure with 5% stiffness decrease at the 3rd floor column.




(a) Intact structure (b) Damaged structure

**Figure 7. Reconstructed attractors from accelerations at the top floor for the intact structure and damaged structure with 5% stiffness decrease at the 3rd floor column**

From the figure, even though difference can be observed, it can hardly tell us how large the difference is. Attractor distance is therefore applied to indicate the difference extent. Figure 8 shows the attractor distances to the intact structure for all floors when the 1st floor is damaged.

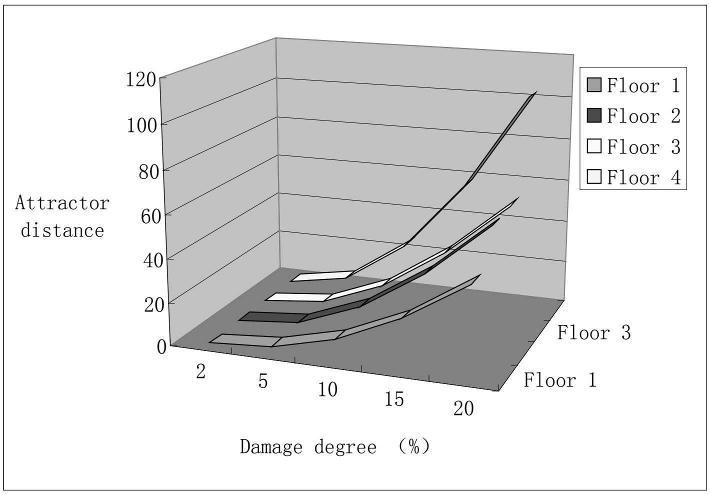



**Figure 8. Attractor distances to the intact structure when the 1st floor is damaged**

While figure 8 shows the single damage case, Figure 9 shows the multi-damage case. It shows the attractor distances to the intact structure when the 2nd and 3rd floors are damaged, with respected to different damage degree. In this paper, for simplicity, we just show the case when only two columns are damaged with the same damage rate for these two columns. For other multi-damage case, the results are actually similar.

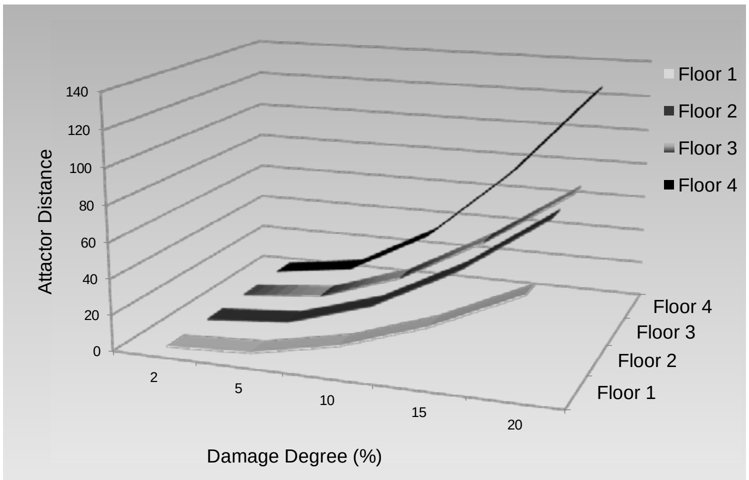



**Figure 9. Attractor distances to the intact structure when the 2nd & 3rd floors are damaged**

From Figure 8 and 9, it can be found that the attractor distance of each floor between the intact structure and damaged structure increases strictly in accord with the increase of the damage rate. Thus the degrading of the structure due to the damage can be indicated by a relative large attractor distance. Relatively larger the distance, more severe the damage.

With the study of the attractor distance, we can easily detect the structural deterioration. However, such detection is mainly the Qualitative analysis. It can hardly clearly tell us how large the deterioration is, and also can not distinguish the single damage case or multi-damage case. Therefore, identification of structural parameters and their deterioration is further implemented with the Kalman Filter. Table 1 shows the structural parameter identification results using Kalman Filter when the 3rd floor is supposed to be damaged with the damage degree increased step by step. It can be found that the identified structural parameters are very close to the real model parameters, especially for the stiffness K and mass M. Values of C in Table 1 actually are the values for the diagonal element of the damping matrix with the damping ratio 0.05. With the stiffness decreased step by step, identified stiffness is decrease accordingly at the corresponding column. It shows that with the structural parameter identification, the deterioration extent of the structural and also its location can be determined.

Table 1 Structural parameter identification using Kalman Filter when the 3rd floor is damaged.

| Structural Parameter | | Real model parameter | Identified parameter | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | intact | 3rd floor damaged (%) | | | | |
| | | | | 2 | 5 | 10 | 15 | 20 |
| K | 4 | 1000 | 1011.88 | 1011.75 | 1011.59 | 1010.87 | 1010.50 | 1010.06 |
| | 3 | **1000** | **1015.81** | **995.38** | **964.50** | **912.97** | **861.63** | **810.31** |
| | 2 | 1000 | 1016.81 | 1016.63 | 1016.26 | 1015.26 | 1014.56 | 1013.67 |
| | 1 | 1000 | 1017.49 | 1017.37 | 1016.93 | 1015.95 | 1015.19 | 1014.37 |
| C | 4 | 27.26 | 32.23 | 32.19 | 32.23 | 32.26 | 32.22 | 32.23 |
| | 3 | 39.62 | 35.60 | 35.00 | 34.08 | 32.48 | 30.91 | 29.32 |
| | 2 | 40.69 | 40.15 | 40.26 | 40.38 | 40.57 | 40.75 | 40.94 |
| | 1 | 42.91 | 40.85 | 40.94 | 41.05 | 41.18 | 41.31 | 41.45 |
| M | 4 | 100 | 100.69 | 100.68 | 100.64 | 100.58 | 100.54 | 100.49 |
| | 3 | 100 | 101.93 | 101.89 | 101.82 | 101.70 | 101.58 | 101.47 |
| | 2 | 100 | 101.97 | 101.96 | 101.93 | 101.80 | 101.73 | 101.64 |
| | 1 | 100 | 102.42 | 102.42 | 102.35 | 102.26 | 102.16 | 102.06 |

## 5. CONCLUSION

Minor damage detection and quantification is very important for the structural health monitoring. In this paper, minor damage is proposed to be detected and identified using the chaotic analysis and Kalman filter. Simulations are conducted. Chaotic excitation is used as the external force and is put on a 4-story lumped mass shear model. The attractor distances between the reconstructed attractor of the structure in intact status and in damaged status are calculated and analyzed. It is found that the attractor distance will be increased once the structure has damages, even the damages are very small. Therefore it can be used for minor damage detection. However, it is rather a qualitative detection than a quantitative detection. Also it can not distinguish the single damage case and the multi-damage case. For the quantitative analysis, Kalman filter is introduced. Analysis results show that it can identify the structural parameters and then trace the minor damages and their development in the structure. The proposed approach will be helpful for detecting and evaluating minor damages at the early stage for the structural health monitoring.

## 6. ACKNOWLEDGEMENT

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