5th International Conference on
Structural Health Monitoring of Intelligent Infrastructure (SHMII-5) 2011
11-15 December 2011, Cancún, México

GENERATING ALTERNATIVES FROM MULTIPLE MODELS: HOW TO INCREASE ROBUSTNESS IN PARAMETRIC SYSTEM IDENTIFICATION

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An invaluable tool for structural health monitoring and damage detection, parametric system identification through model-updating is an inverse problem, affected by several kinds of modeling assumptions and measurement errors. By minimizing the discrepancy between the measured data and the simulated response, traditional model-updating techniques identify one single optimal model that behaves similarly to the real structure. Due to numerical and identification errors, incomplete data and modeling inaccuracies, this mathematical optimum may be far from the true solution and lead to misleading conclusions about the structural state. Instead of the mere location of the global minimum, it should be therefore preferred the generation of several alternatives, capable to express near-optimal solutions while being as different as possible from each other in physical terms.

The methodology proposed in the present paper accomplishes this goal through a two-levels approach. At the first (outer) level, multiple models are created, which may differ from each other in various respects (analytical formulation, finite element type, boundary conditions, mesh refinement, set of updating parameters, …), and then separately solved for the respective global optimum. At the second (inner) level, for each model an iterative search is run, looking for sub-optimal alternatives located increasingly far from the global optimum, and finally resulting in the "minimum envelope section", intrinsic expression of the updating robustness. Comparing "sections" corresponding to distinct models provides a much better understanding of the search space and an insight into the reliability of the calibration process, ultimately improving the analyst decision making for system identification and downstream tasks such as further measurement, preventative maintenance and structural replacement.

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GENERATING ALTERNATIVES FROM MULTIPLE MODELS: HOW TO INCREASE ROBUSTNESS IN PARAMETRIC SYSTEM IDENTIFICATION

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ABSTRACT: An invaluable tool for structural health monitoring and damage detection, parametric system identification is an inverse problem. By minimizing the discrepancy between measured data and simulated response, classical methods focus on identifying a single, globally optimum model. However, since the number of unknown parameters is usually larger than the number of equations available, and inaccuracies in measurement, processing and modeling are inevitable, more than one good candidate may exist. In this paper, a two-levels methodology is presented to identify a set of models which hold similar fitting capability but are as physically different as possible. At the outer level, multiple models are created and separately solved for the respective global optimum. At the inner level, for each model sub-optimal alternatives are generated as far as possible from the global optimum. Comparison of the overall set of solutions finally improves the understanding of the search space and the reliability of the calibration process.

1 INTRODUCTION

An invaluable tool for structural health monitoring and damage detection, parametric system identification through model-updating is an inverse problem, affected by several kinds of modeling assumptions and measurement errors (Friswell and Mottershead (1995)). By minimizing the discrepancy between the measured data and the simulated response (i.e. the objective function), traditional model-updating techniques identify one single optimal model that behaves similarly to the real structure. Due to numerical and identification errors, incomplete data and modeling inaccuracies, this mathematical optimum may be far from the true solution and lead to misleading conclusions about the structural state (Saitta *et al*. (2005)). Therefore, instead of the mere location of the global minimum, it should be preferred to generate several alternatives, capable to express near-optimal solutions while being as different as possible from each other in physical terms (Raphael and Smith (2003)).

In this paper a methodology is presented, called “Generate alternatives from multiple models” (GAMM), which accomplishes this goal through a two-levels approach. At the first (outer) level, multiple finite-element (FE) models are created, which may differ from each other in various respects (analytical formulation, FE type, boundary conditions, mesh refinement, set of updating parameters, …), and then separately solved for the respective global optimum. At the second (inner) level, for each model an iterative search is run, looking for sub-optimal alternatives located increasingly far from the global optimum, and finally resulting in the "minimum envelope section", intrinsic expression of the updating robustness.

GAMM is applied to the FE model updating of a large-scale frame prototype, built in the Structural Engineering Lab at the University of Basilicata, and recently used as a benchmark for the experimental assessment of the seismic effectiveness of different control strategies, in the framework of the inter-university Italian DPC-ReLUIS 2005-08 Project.

2 GENERATING ALTERNATIVES FROM MULTIPLE MODELS: THE METHODOLOGY

2.1 Outer level: generating multiple models

Modeling errors play a major role in FE model updating. Without properly acknowledging modeling errors, an incorrect or shifted baseline model would be reconciled to the measured data, leading to an updated model which would replicate the measurements while lacking physical meaning (Saitta *et al.* (2005)). Modeling errors may be mainly distinguished into errors in the *model structure* (the form of the equations) and errors in the *model parameters* (the coefficients of the equations). Errors in the model structure depend on wrong assumptions mainly concerning mathematical modeling (differential equations, boundary conditions) and discretisation (meshing, FE type). Every effort should be made to eliminate these errors from the initial FE model, prior to updating. This can be rarely achieved without the application of considerable physical insight. On the other hand, errors in the model parameters depend on wrong values assigned to the geometrical and mechanical properties of the model. Usually only few parameters can be simultaneously corrected, due to the limited amount of available data and to the unacceptable computational cost of optimizing too many variables. Any choice of the updating set will implicitly introduce some modeling errors in the regions of the model which are excluded from updating (Sanayei *et al.* (2001)). Various parameterization techniques exist to make the overall number of *model parameters* depend on a restricted set of *updating parameters*, the latter being the variables of the optimization process. The constraint implicit in such dependence is the error in model parameters which can not be amended during updating.

The search for a proper baseline model, i.e. a model structure and an updating set characterized by minimal modeling errors, is basically a trial and error process guided by the analyst's experience. Generating multiple models and then updating each of them separately highlights the influence of the various modeling assumptions on the capability of the model to explain measured data (Robert-Nicoud *et al.* (2005)). The dispersion of the multiple solutions gives an indication about the credibility of each model and the conditioning of the inverse problem.

In the general case GAMM can handle multiple models characterized by different model structures but in the present paper the methodology will be specified by assuming models differing from each other in terms of the updating set only. In its outer level, GAMM develops through the following steps:

- Step 1: a meaningful objective (or error) function f_{ob} is selected, quantifying the discrepancy between the experimental and the analytical models;

- Step 2: a robust optimization algorithm is chosen, capable to reliably minimize f_{ob};

- Step 3: the total set $\mathbf{p} \in \Re^{N_p}$ of the *potential updating parameters* (normalized to their respective nominal value) is identified, based on the evaluation of the sensitivity of f_{ob} to changes in the model parameters (sensitivity analysis) and/or on an *a priori* estimation of their respective uncertainties;

- Step 4: $\mathbf{p}$ is partitioned into two subsets: the set of the *updating parameters*, denoted as $\mathbf{p}_u \in \Re^{N_u}$, and its complementary set of the *constrained parameters*, denoted as $\mathbf{p}_c \in \Re^{Np-N_u}$; while $\mathbf{p}_u$ contains the independent variables of the optimization process, $\mathbf{p}_c = \mathbf{F}(\mathbf{p}_u)$ contains the parameters which are excluded from updating and equaled to their respective nominal value or to any possible function $\mathbf{F}$ of $\mathbf{p}_c$; such partition will be called a model and denoted as $\mathcal{M}_{i\,(N_u)}$;

- Step 5: the model defined at Step 4 is optimally updated, i.e. $\mathbf{p}_u$ is found minimizing f_{ob};

- Step 6: Steps 4 and 5 are repeated, each time selecting a different updating set (i.e. a different model), e.g. enlarging previous sets by progressively including new parameters (plausibly the most uncertain ones); a family of N_s multiple solutions is finally obtained.

2.2 Inner level: generating alternatives for each of the multiple models

“Modeling to generate alternatives” (MGA) is a general-purpose decision-support methodology developed in the early eighty’s to provide solutions to complex, incomplete problems by coupling the computational power of computers and human intelligence, and recently applied to the parametric identification of Civil structures (Zarate and Caicedo (2008)). MGA identifies several possible good solutions for an assigned problem, designed to be as physically different as possible but providing a similar outcome to the problem. This is usually achieved, once the global optimum is found, by subsequently searching, among the solutions whose objective function is below a certain threshold, the one which is the most distant from the global optimum. If further alternatives are desired, these will be found by maximizing the weighted distance from the global optimum and all previously found alternatives.

In the inner level of GAMM, MGA is instead used as follows:

- Step 7: for each of the N_s global optima obtained at Step 6 a new optimization is run, searching, among the solutions whose (Euclidean) distance d from the global optimum is larger than a certain threshold distance, the one which minimizes f_{ob};

- Step 8: Step 7 is repeated for increasing values of the threshold distance;

- Step 9: results from Step 7 are plotted in a diagram reporting d in the abscissas and f_{ob} in the ordinates, which represents the “minimum envelope section” (MES) of the objective function, i.e. the minimum envelope among the ∞^{Np-1} radial sections of f_{ob} centred in its global minimum;

- Step 10: the various MES are finally collected in a unique diagram, which describes the performance and robustness of the N_s solutions.

Contrary to the traditional use of MGA, focused on identifying only few alternatives far from the global minimum and from each other, here MGA is aimed at finding several sub-optimal solutions at progressively increasing distance from the global minimum in order to construct the MES. The latter eventually expresses the robustness of the global solution (the flatter the section, the less robust the solution), in a certain way representing the global counterpart of the local concept of second order approximation of the objective function at its global minimum.

3 AN EXPERIMENTAL CASE STUDY

3.1 The building structure

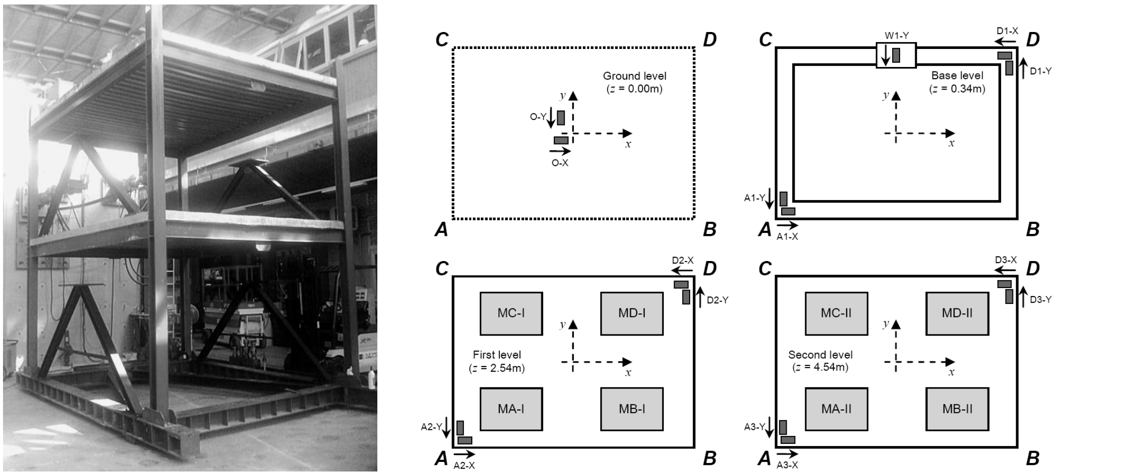
The test structure is a large-scale (2:3) model of a two-storey steel frame building with composite steel-concrete floors (figure 1(a)). The steel structure, consisting of columns and beams orthogonally interconnected into a regular (doubly-symmetrical) three-dimensional frame with one bay in both directions and two rectangular floors (level 1 and 2), is mounted on a rigid horizontal base (level 0), resting on two sliding guides and connected to a dynamic hydraulic actuator which can impart the desired mono-dimensional excitation to the structure. Four HE140B equal columns, fixed to the base, extend continuously to the top floor. Eight IPE180 lateral beams, welded to the columns, support the two composite floors, made up of concrete slabs cast on coffer profiled steel sheeting. The columns free length is 4.00m, divided into two 2.00m inter-storey heights. The beams length is 4.00m in the along-excitation (longitudinal) direction and 3.00m in the across-excitation (transverse) direction. The floors thickness is larger than expected because of the sagging effect occurred during concrete casting. In order to house the dissipating devices during tests on the controlled structure, four V-inverted braces, crowned with gusset plates, are bolted at both storeys, parallel to the longitudinal direction.

At the initial stage of the Project, the need of an accurate numerical model of the benchmark building motivated a preliminary campaign of dynamic tests on the uncontrolled structure (i.e. with no dissipative device installed). In order to increase the amount of data available, classical “perturbed boundary condition testing” was pursued, consisting in perturbing both the structure and the analytical model by adding the same amount of mass at given positions. A total of eight concrete blocks (about 340kg each) were fixed onto the floors during testing according to three different configurations. Starting from the basic configuration (BC), characterized by no additional mass, a second doubly-symmetric configuration (SC) was obtained by the addition of four blocks on each storey, then a non-symmetric configuration (NC) was obtained removing two blocks from the SC configuration at each storey.

For each of the three mass configurations, ambient vibration tests were conducted on the uncontrolled structure, keeping the sliding guides locked. The dynamic response was measured by 15 accelerometers deployed in the most significant observation points. The location of the accelerometers and of the concrete blocks is described in figure 1(b).

3.2 The nominal finite element model

First, the nominal FE numerical model $\mathscr{M}_0$ is formulated which will undergo experimental calibration next. Stiffness and mass matrices are statically condensed to the six translational and rotational displacement components of the first and second floors’ geometrical centres, included in the vector $\eta = \{d_{x1}\ \ d_{y1}\ \ d_{\theta 1}\ \ d_{x2}\ \ d_{x2}\ \ d_{\theta 2}\}^{\mathrm{T}}$, under the hypotheses of axial rigidity of Euler-Bernoulli type columns and beams, in-plane rigidity of floor slabs, and lumped mass formulation for columns and beams. Columns are clamped at their base (i.e.: no dynamics attributed to the base level), the V-inverted braces deprived of any dynamics of their own, and no connection is explicitly recognized between the floor slab and the steel beams. Each of the additional concrete blocks is accounted for as a translational inertia, and enters the system mass matrix with its own mass, static moment and polar inertia. With these assumptions, for each mass configuration the nominal (reference) FE model is obtained through equalling each geometrical and mechanical parameter to its expected (nominal) value.



(a) (b)

Figure 1. The structural prototype: (a) overall view; (b) location of accelerometers and additional blocks.

3.3 The experimental modal analysis

The experimental modal model is identified on the basis of ambient vibration tests. Three classical output-only methods are used to extract frequencies and modeshapes, respectively working in the time-domain, in the frequency-domain and in the time-frequency domain: ERA (Juang and Pappa (1984)), FDD (Brincker *et al.* (2001)) and TFIE (De Stefano *et al.* (2009)).

The details of the experimental modal analysis are not reported here for brevity's sake. The interested reader is referred to Antonacci *et al.* (2011). Two results are, however, worth mentioning. The first, well expected, is the progressive reduction of all the natural frequencies with increasing additional masses on the structure, and a slight modification of the modeshapes too (with the loss of their double-symmetry in the NS configuration). The second, totally unexpected, is the dynamic interaction (due to frequency closeness) between the out-of-plane mode of the V-inverted braces attached to the first storey and the fourth mode (i.e. the second flexural mode in the transverse direction) of the main frame (Matta *et al.* (2009)). Such interaction explains the existence of seven structural modes instead of the expected six ones, the modes 4a and 4b having distinct frequencies but nearly undistinguishable modeshape components at both stories. The 6-DOF analytical model is obviously inadequate to simulate such modal coupling effect. Although including braces dynamics in the model would easily remove such inadequacy (a clear proof of an error in the model structure), this is left for future work, and the present paper applies instead an "experimental model reduction", consisting in the mere removal of the resonant modes from the experimental data. In this way, the FE model is calibrated relying exclusively on fitting the five modes which do not feel the coupling effect.

3.4 The FE multi-model updating

The nominal model $\mathscr{M}_0$ derived in Subsection 3.2 and the experimental modal model identified in Subsection 3.3 are here reconciled by means of GAMM, through the following steps:

- Step 1: f_{ob} is introduced as the normalized discrepancy, averaged over the three mass configurations, between numerical and experimental eigenfrequencies and modeshapes (with the exclusion of modes 4a and 4b), so conceived as to represent the weighted error of the numerical model with respect to the experimental modal model;

- Step 2: a hybrid direct-search algorithm is chosen as the optimization algorithm, consisting of a genetic algorithm followed by a nonlinear least-square solver;

- Step 3: given the simplified structure of the FE model, 8 stiffness parameters and 4 mass parameters completely characterize the model; these N_p = 12 parameters, each normalized to its nominal value, are described in figure 2 and include: 4 parameters, p_1 to p_4, representing columns bending inertias (I_{cx1}, I_{cy1}, I_{cx2}, I_{cy2}), where the subscripts denote the in-plan direction and the storey level; 4 parameters, p_5 to p_8, representing beams bending inertias (I_{bx1}, I_{by1}, I_{bx2}, I_{by2}); and 4 parameters, p_9 to p_{12}, representing slabs masses and polar inertias (m_1, m_2, J_{o1}, J_{o2});

- Steps 4 to 6: eight different subsets of the 12 parameters, i.e. eight multiple models, are generated and then separately solved, with N_u varying from a minimum of 2 to a maximum of 10; the case N_u = 0 is included for comparison; solutions are summarized in table 1; their objective functions and their distance from the best model, $\mathscr{M}_{8(10)}$, are reported in figure 3;

- Steps 7 to 10: MGA is applied and the MES is created for each model; all the MES are finally collected in figure 4.

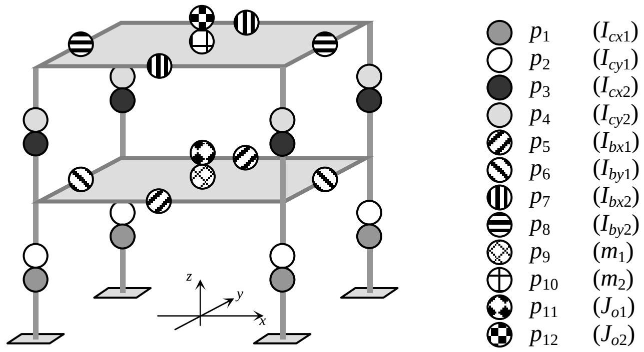
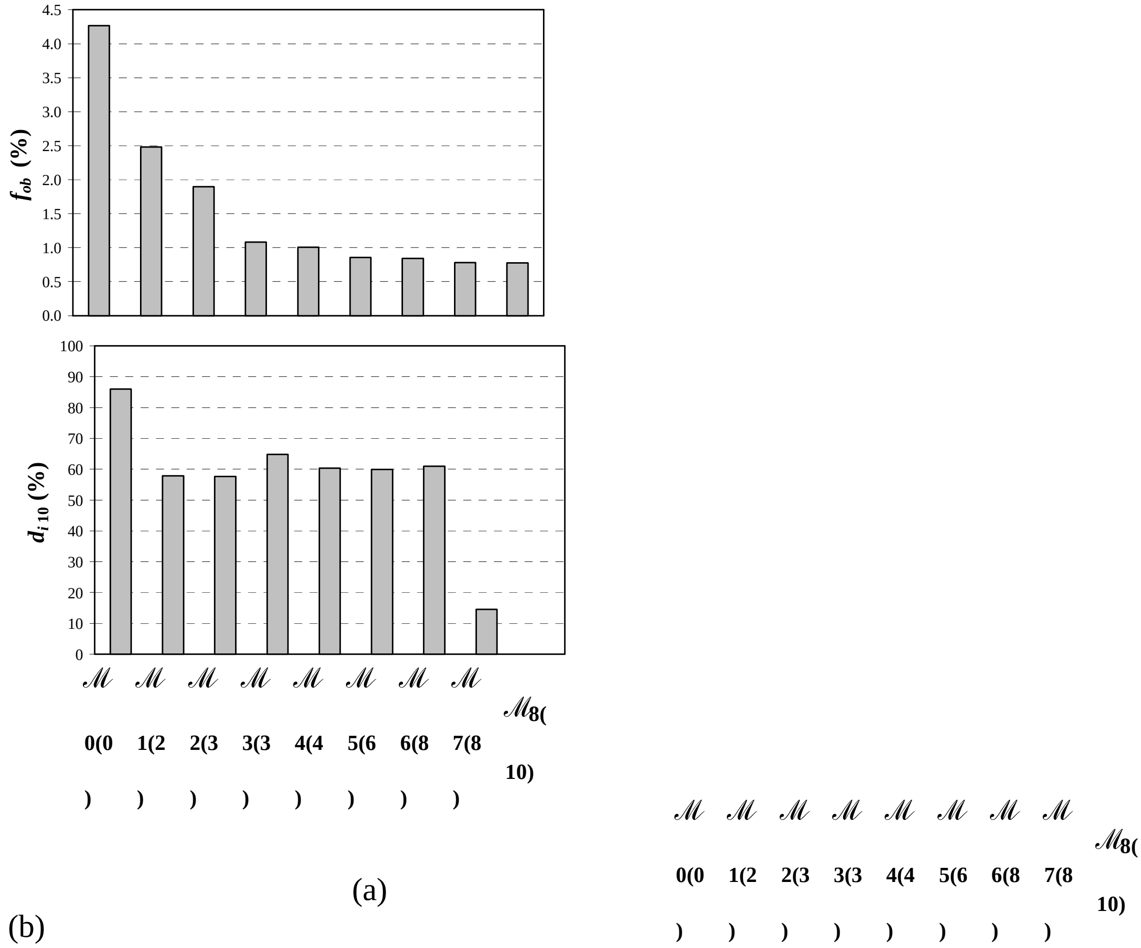


Figure 2. The FE model with the 12 potential updating parameters.

Table 1. The global optima obtained with the multiple models.

Model	p_1 (I_{cx1})	p_2 (I_{cy1})	p_3 (I_{cx2})	p_4 (I_{cy2})	p_5 (I_{bx1})	p_6 (I_{by1})	p_7 (I_{bx2})	p_8 (I_{by2})	p_9 (m_1)	p_{10} (m_2)	p_{11} (J_{o1})	p_{12} (J_{o2})	f_{ob} (%)

$\mathscr{M}_{0(0)}$	1	1	1	1	1	1	1	1	1	1	1	1	4.26
$\mathscr{M}_{1(2)}$	.751	.751	1	1	1	1	1	1	.770	.770	.770	.770	2.48
$\mathscr{M}_{2(3)}$	.623	.731	1	1	1	1	1	1	.710	.710	.710	.710	1.90
$\mathscr{M}_{3(3)}$	.830	.830	1	1	1	1	1	1	.932	.742	.932	.742	1.08
$\mathscr{M}_{4(4)}$	.755	.795	1	1	1	1	1	1	.878	.716	.878	.716	1.01
$\mathscr{M}_{5(6)}$	.756	.792	1	1	1	1	1	1	.887	.716	.830	.694	.856
$\mathscr{M}_{6(8)}$	.782	.778	1.07	.915	1	1	1	1	.862	.726	.834	.707	.843
$\mathscr{M}_{7(8)}$	.629	.755	1	1	1.25	1.19	1.25	1.19	.816	.777	.790	.740	.780
$\mathscr{M}_{8(10)}$	.620	.739	.956	.964	1.28	1.28	1.28	1.28	.794	.783	.757	.749	.775



(a)

(b)

Figure 3. Comparison among the multiple global optima: (a) objective function; (b) distance from $\mathscr{M}_{8(10)}$.

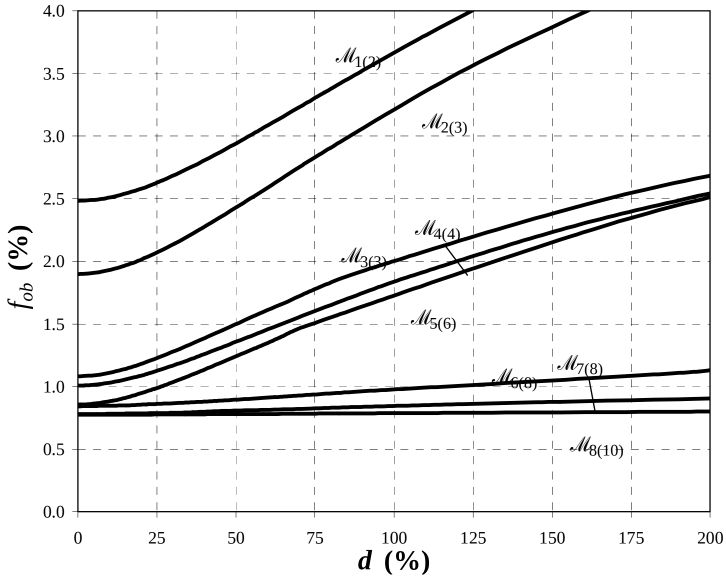


Figure 4. Comparison among the multiple models: minimum envelope sections.

Results in table 1 and in figures 3 and 4 suggest the following comments:

• Expectedly, increasing the number N_u of updating parameters generally improves fitting the experimental data. The f_{ob} decreasing trend is pronounced for small values of N_u but vanishes as N_u increases. If considered in absolute terms, the improvement of f_{ob} from $\mathscr{M}_{5(6)}$ to $\mathscr{M}_{8(10)}$ is seemingly unsubstantial.

• In order to genuinely capture improvements in the model, the multiple solutions must be compared in terms not only of their fitting capability but also of the plausibility of the underlying parametric description. A reduction of f_{ob} might in fact not correspond to approaching the “true” model in the search space. This is clearly demonstrated by figure 3: while moving from $\mathscr{M}_{0(0)}$ to $\mathscr{M}_{8(10)}$, f_{ob} monotonically decreases but the distance of each model from $\mathscr{M}_{8(10)}$ does not. Such distance is conspicuous for all the models (about 60%) except model $\mathscr{M}_{7(8)}$ (about 15%), implying that remarkably different models exist providing similar objective functions (ill-conditioning?).

• Two approaches can be followed to deal with the obtained multiple solutions. The first approach assumes that all the solutions whose objective function is below a determined threshold (e.g. $\mathscr{M}_{5(6)}$, $\mathscr{M}_{6(8)}$, $\mathscr{M}_{7(8)}$ and $\mathscr{M}_{8(10)}$) are admissible candidates to represent the physical system. The second approach relies on engineering insight to state the plausibility of each candidate and isolate the most likely one(s). So, $\mathscr{M}_{5(6)}$ (f_{ob} = 0.856%) makes the stiffness matrix depend only on the lower columns stiffness, achieving poor approximation. $\mathscr{M}_{6(8)}$ (f_{ob} = 0.843%) slightly improves fitting by adding two more variables, i.e. the upper columns stiffness, but the improvement is more seeming than real, because upper and lower columns are in fact a unique structural element. $\mathscr{M}_{7(8)}$ (f_{ob} = 0.780%) is superior to both $\mathscr{M}_{5(6)}$ and $\mathscr{M}_{6(8)}$, because in this case the two additional variables, characterizing the beams stiffness, possess a larger degree of uncertainty than that pertaining to the upper columns; in the updated $\mathscr{M}_{7(8)}$ model, the stiffness of the lower columns is less than nominal, due to the imperfect clamp at their base, and the beams stiffness is larger than nominal, due to the partial collaboration of the floor slabs. Finally, $\mathscr{M}_{8(10)}$ (f_{ob} = 0.775%), updating both the upper columns stiffness and the beams stiffness, obviously improves fitting with respect to all previous models.

From a physical viewpoint, $\mathscr{M}_{8(10)}$ is characterized by a convincing set of updated parameters, with a substantial uniformity in both the beams stiffness and the mass properties, and with the upper columns stiffness only slightly reduced with respect to their nominal value. In conclusion, both $\mathscr{M}_{7(8)}$ and $\mathscr{M}_{8(10)}$ possess enough fitting capabilities and sufficiently sound physical meaning to be accepted as reliable representations of the benchmark building structure.

- The MES in figure 4 clearly shows the trade-off between fitting capability and ill-conditioning, as a function of the number of updating parameters N_u. If N_u gets large, the MES flattens and distant alternatives are obtained with competing objective functions. Considering for example N_u = 10, because of the little sensitivity of f_{ob} to the beams stiffness, a solution is obtained at d = 200% having f_{ob} = 0.800%, i.e. performing better than the global optimum of $\mathscr{M}_{5(6)}$. In order to avoid convergence to such distant and incorrect solutions, as a general rule every effort must be done in model updating to minimize all possible sources of errors.

4 CONCLUSIONS

A methodology is presented to increase robustness in parametric system identification, based on (i) updating several distinct models of an assigned structure, and (ii) subsequently generating, for each model, distant alternatives. The methodology, exemplified on an experimental case study, proves an effective and robust decision-support tool to assist the engineer’s physical insight in structural system identification.

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