NUMERICAL EVALUATION OF WAVELET BASED DAMAGE DETECTION METHODOLOGIES APPLIED TO RC MEMBERS

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Assessment of structural damage after a seismic event is required to identify unsafe structures. However, a comprehensive application of localized damage detection methods to a complex structure is a lengthy and expensive assignment that can even become dangerous in the case of post-earthquake inspections. This paper focuses on the possibility of real time (or near real-time) damage identification from the examination of the dynamic response of the structure. In this type of approaches, analysis of the structural response registered during the earthquake is performed using advanced mathematical tools (like the Wavelet or Hilbert-Huang Transforms) that allow for simultaneous time frequency examination. The occurrence of damage is then associated with any changes in the instantaneous frequency (IF) or singularities in the high frequency response. Unfortunately, many of the proposed methodologies have been validated through numerical models in which the hysteretic and non-linear behavior proper of the elements of a civil structure is misrepresented by assuming linear or multi-linear behavior at most. In this paper we evaluate the application of wavelet based methods for damage identification of reinforced concrete structures subjected to different types of dynamic loads. Evaluation of the methods is performed using state-of-the-art nonlinear-fiber-based models capable of replicating the actual nonlinear hysteretic response of reinforced concrete members. It was found that application of such techniques can be limited by the high frequency content of the excitation and the modeling assumptions.

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ABSTRACT: Assessment of structural damage after a seismic event is required to identify unsafe structures. However, a comprehensive application of localized damage detection methods to a complex structure is a lengthy and expensive assignment that can even become dangerous in the case of post-earthquake inspections. This paper focuses on the possibility of real time (or near real-time) damage identification from the examination of the dynamic response of the structure (Hou et al. 2000, Pan and Lee 2002, Montejo 2011). In this type of approaches, analysis of the structural response registered during the earthquake is performed using advanced mathematical tools (like the Wavelet or Hilbert-Huang Transforms) that allow for simultaneous time frequency examination. The occurrence of damage is then associated with any changes in the instantaneous frequency (IF) or singularities in the high frequency response. Unfortunately, many of the proposed methodologies have been validated through numerical models in which the hysteretic and non-linear behavior proper of the elements of a civil structure is misrepresented by assuming linear or multi-linear behavior at most. In this paper we evaluate the application of wavelet based methods for damage identification of reinforced concrete structures subjected to different types of dynamic loads. Evaluation of the methods is performed using state-of-the-art nonlinear-fiber-based models capable of replicating the actual nonlinear hysteretic response of reinforced concrete members. It was found that application of such techniques can be limited by the high frequency content of the excitation and the modeling assumptions.

1 INTRODUCTION

This paper focuses on the detection of structural damage based on the occurrence (or not) of singularities (spikes or novelties) in the high frequency response of the structure. The high frequency response of the structure is analyzed using the Continuous Wavelet Transform (CWT) and the Discrete Wavelet Transform (DWT). The dynamic response of the structure is simulated using the OpenSees program (McKenna et al. 2000). The performance of the methodologies is evaluated using three different non-linear hysteretic models (bilinear, multilinear and fiber-based) and three different dynamic excitations of different nature and frequency content.

2 WAVELET ANALYIS

A Wavelet is a function with oscillatory wave-like characteristics which has its energy concentrated in time. In conjunction with the Wavelet Transforms, it has the ability to allow for simultaneous time-frequency analysis. Wavelets Transforms have been proved to be a powerful analytical tool for signal processing (detection, filtering and compression), numerical analysis, and mathematical modeling. Indeed, it was called "mathematical microscope" (Burke-Hubbard 1998) because of its ability to catch hidden trends, breakdowns, and pulse-like events. The Continuous Wavelet Transform (CWT) takes a function of time *a(t)* and transforms it to a function of two variables *C(s,p)*.

$$C(s,p) = \int_{-\infty}^{\infty} a(t)\psi_{s,p}(t)dt \tag{1}$$

The parameter *s* and *p* are called the "scale" and the "position", and can be related to frequency and time, respectively. The wavelet coefficients *C(s,p)* contain information about the function *a(t)* at the scale *s* around the time (or position) *p*. The functions $\psi_{s,p}(t)$ are defined by translating along the time axis and stretching or compressing a “mother wavelet” $\Psi(t)$:

$$\psi_{s,p}(t) = \frac{1}{\sqrt{s}}\Psi\left(\frac{t-p}{s}\right) \tag{2}$$

In order to recover the original signal *a(t)* it is necessary to use the “reconstruction formula”:

$$a(t) = \frac{1}{K_\psi}\int_{s=0}^{\infty}\left(\int_{p=-\infty}^{\infty} C(s,p)\psi_{s,p}(t)\frac{dp}{s^2}\right)ds = \frac{1}{K_\psi}\int_{0}^{\infty} D(s,t)ds \tag{3}$$

where $K\psi$ is a constant that depends on the mother wavelet. The functions within parenthesis in Eq. (3) are referred to as the “detail functions” *D(s,t)* and have a dominant frequency that depends on *s* and the wavelet used for the analysis. In the CWT the wavelet coefficients are calculated at scale values that vary continuously, the result is a highly redundant and time demanding representation of the signal. In the Discrete Wavelet Transform (DWT) the values of scale and position are choose based on a dyadic scale, for some special mother wavelets $\psi(t)$ the corresponding discretized wavelets $\psi_{s,p}(t)$ constitute an orthonormal basis. Mallat (1989) developed a fast wavelet decomposition and reconstruction algorithm for the DWT using a two-channel subband coder. In the DWT, a signal can be represented (Eq. 4) by its approximations (*A*) and details (*D*) at different levels of decomposition (*j*). The approximations are the high-scale, low-frequency components of the signal. The details are the low-scale, high-frequency components.

$$a(t) = A_j + \sum_{i \le j} D_i \tag{4}$$

To demonstrate the application of the Wavelet Transforms, the signal displayed in Fig. 1a was artificially generated. This signal is composed of a sinusoidal with frequency 1 Hz from 0 to 5 seconds, then a sinusoidal with frequency of 2Hz goes from 5 to 10 seconds (so that a discontinuity is caused at time 5 seconds). The sampling time (*dt*) used to generate the signal was 0.005 seconds. The Fourier spectrum of the signal is presented in Fig. 1b. It is seen that the Fourier spectrum captured the correct frequencies of the signal (1Hz and 2Hz), but the time where the frequencies occur is missed. The result from a CWT analysis at low frequencies (0 to 5 Hz) is displayed in Fig. 1c, this figure is called a Wavelet Map and it shows the absolute values of the CWT coefficients obtained using Eq. 1. From this figure not only the frequency content of the signal can be identified but also the instants at which each frequency is occurring. Fig. 1d is similar to Fig. 1c but this time the analysis is performed at high frequencies, from 70Hz up to the Nyquist frequency (*1/2dt=100Hz*). It is seen that a CWT analysis at high frequencies allow us to identify the discontinuity in the signal at time 5 seconds. Figs. 1e and 1f show the results of the analysis using the DWT at the first level of decomposition. The first level approximation (A_1), i.e. the low-frequency component of the signal, is displayed in Fig. 1e and the first level detail (D_1), i.e. the high frequency component of the signal, is displayed in Fig. 1f. It is seen that the discontinuity in the signal can be easily pointed out from the detail function.

Based on the properties of the Wavelet Transforms, two different damage detection approaches have been explored. One approach relies on the simultaneous time-frequency analysis capability of the Wavelet Transform. In this approach the changes in time of the dominant frequency of the structure are closely examined, the changes in frequency are then associated with changes in the stiffness of the structure. The second approach takes advantage of the ability of the Wavelet Transform to detect breakdowns and pulse-like events. Since damage is usually reflected in the high frequency response of the structure, this approach concentrates on the high frequency wavelets coefficients time history to detect any singularities in the response. The singularities detected are then related with the occurrence of damage. This paper deals only with this second approach.

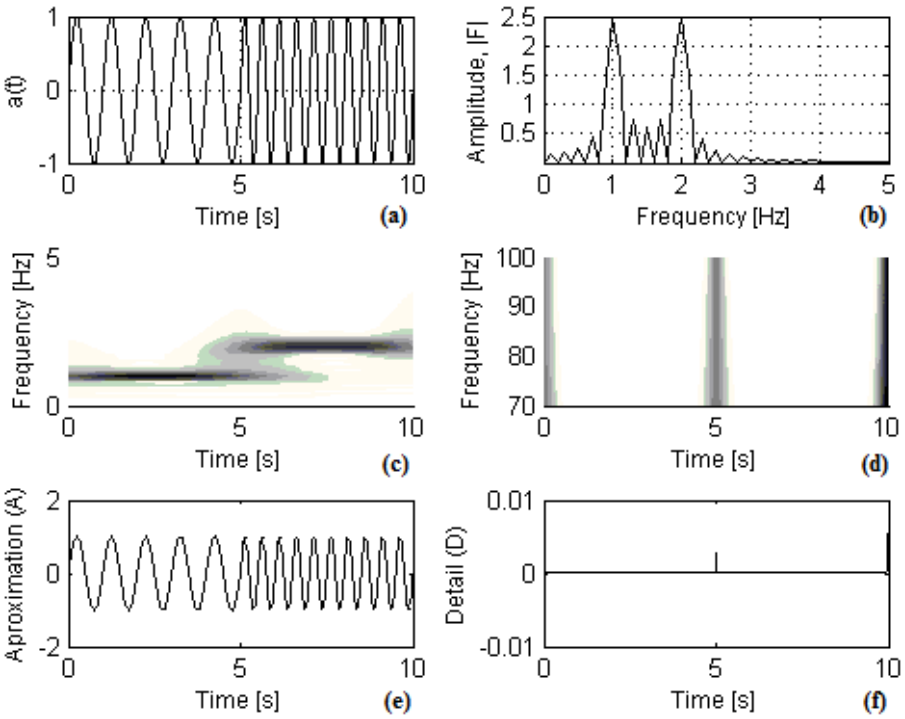

Figure 1. (a) Signal in time domain, (b) Fourier spectrum, (c) Low frequencies Wavelet Map via CWT, (d) High frequencies Wavelet Map via CWT, (e) Approximation via DWT and (f) Detail via DWT

3 NONLINEAR MODELING

Fig. 2a shows a typical force-displacement hysteretic response for a reinforced concrete column subjected to cyclic reversals of load (Montejo et al., 2009). Notice that the response of the column is highly non-linear. Non-linearity is observed even in the frequently called "linear range", i.e. the displacement range where the reinforcing steel strains are below the yield strain. Therefore, linear-elastic models used to validate damage detection methodologies may not adequate in the case of civil structures. Some methodologies have been validated using bi-linear or more advanced hysteresis rules like the Takeda model (Takeda et al. 1970) to represent the behavior of reinforced concrete members. While this type of models have been successful used for years by the structural engineering community to analyze the non-linear behavior of civil structures, their use for validation of damage detection techniques may not adequate due to the unrealistic abrupt change in stiffness proper of any multi-linear hysteretic model. In order to evaluate the effect of different modeling assumptions on the accuracy of wavelet-based damage detection approaches, in this paper we use three different hysteretic models to simulate the non-linear dynamic response of the structure: bilinear model (Fig. 2b), hysteretic "Takeda like" model (Fig. 2c) and fiber-based model (Fig. 2d). All three models where implemented using material and element types available in OpenSees. The *Steel01* material was used for the bilinear model. The "Takeda like" model was implemented using the *Hysteresis* material with strength and stiffness degradation parameters adjusted to emulate the Takeda model. In the fiber-based model, the member is represented by unidirectional fibers and constitutive-material relationships are specified to each type of fiber. In RC members, for example, fibers representing the reinforcing steel, cover concrete (unconfined) and core concrete (confined) can be employed (Fig. 3). The fiber-based model was implemented using the

BeamWithHinges element (Scott and Fenves 2006, Fig. 3). For comparison purposes the experimental response is also plotted in Figs. 2b, c and d (gray line). It is seen that the methodology that most closely resembles the behavior of the column is the fiber-based model.

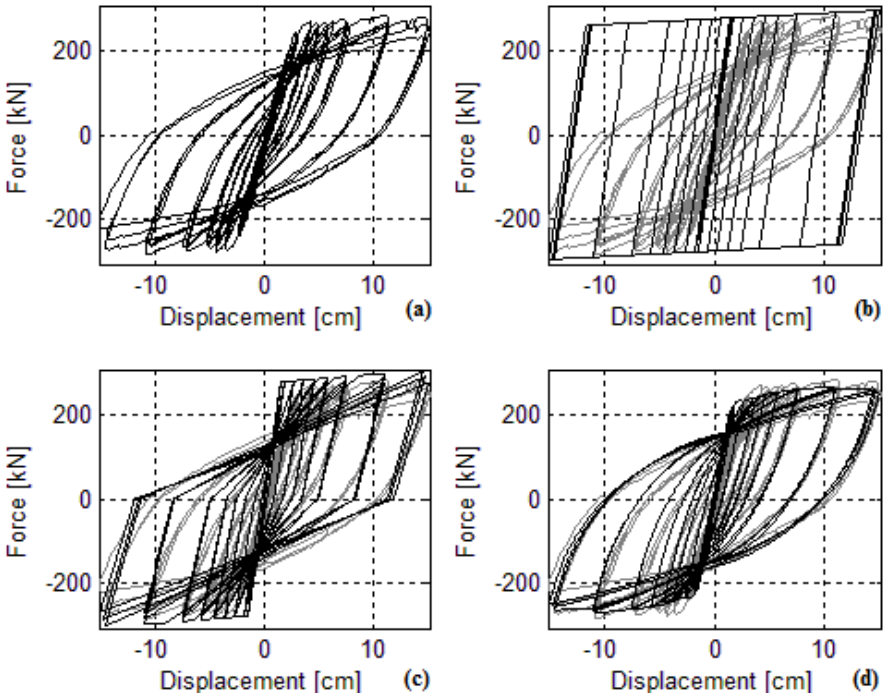

Figure 2. Hysteretic force-displacement response: (a) experimental test, (b) bilinear model, (c) multilinear "Takeda like" model and (d) fiber-based model

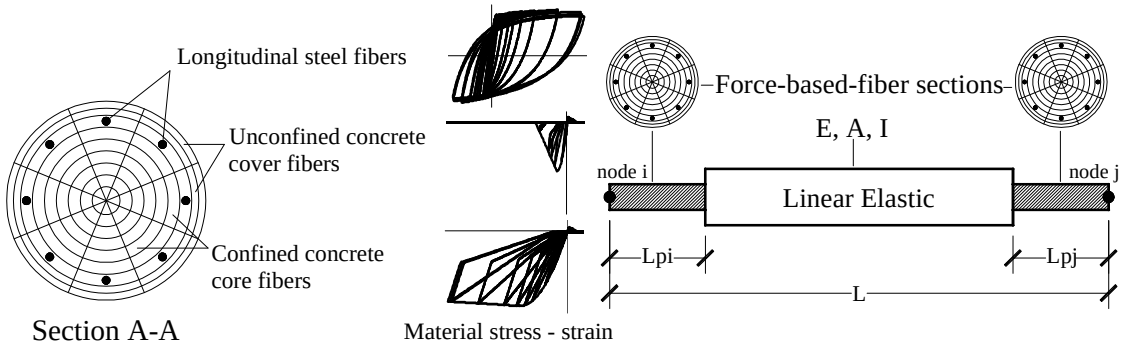

Figure 3. Fiber-based model using the BeamWithHinges (Scott and Fenves 2006) element in OpenSees

4 ANALYSIS OF RESULTS

In order to evaluate the damage detection capabilities of the previously discussed methodologies when the structure is subjected to different type of excitations, the structure is subjected to the three acceleration histories displayed in Fig. 4. The input signals were selected to study the influence of different frequency contents and non-stationary characteristics in the results of the detection methodologies. The first signal is a 5Hz sinusoidal tapered twice by a Hanning window. The other two signals are artificially generated white noises that have been passed through a Hanning window and low pass filters at 20 Hz (wn20) and 40 Hz (wn40). All four acceleration histories have a sampling frequency of 200 Hz. The acceleration response of the structure was

examined using the Complex Morlet wavelet for the CWT analysis and the Bior6.8 for the DWT analysis.

Figs. 5 to 6 show the results obtained for the bilinear, Takeda and fiber-based models for each of the three load excitations. The two horizontal lines in each of the force histories indicate the column yield force. For the bilinear model (Fig. 5) it is seen that both approaches (CWT and DWT) where able to detect the yield episodes for the sinusoidal and wn20 excitations. However, in the case of the wn40 excitation the yield events are only detected by the CWT while the DWT results exhibit a number of spikes that are not related to yield in the structure. For the multilinear hysteretic model (Fig. 6) it is noticed that several anomalies that are no related to yield episodes are exposed by both approaches. A closer look to the results shows that some of these additional spikes occur at the time instants when the force goes through cero, they are a reflection of the abrupt stiffness change at this point proper of the Takeda model (Fig. 2c). Finally, Fig. 7 shows the results obtained for the fiber-based model, in this case the yield occurrences are more difficult to detect. Satisfactory results are obtained only for the sinusoidal excitation. For the white noise excitations a number of anomalies appear that are no related with any damage/yield in the structure.

5 CONCLUSIONS

Results from the numerical evaluation performed suggest that damage detection methodologies based on the direct examination of the structural response via Wavelet Transforms can be limited by the frequency content of the excitation and the hysteretic characteristics of the response (or the modeling assumptions). Good results are obtained when the frequency content of the excitation is limited. However, as the high frequency content of the excitation increases, the results obtained after a high frequency analysis may include not only spikes due to structural damage but also anomalies proper of the excitation. Regarding the characteristics of the hysteretic behavior, better results are obtained when the hysteretic behavior exhibits drastic/abrupt changes in the stiffness, as in any bilinear or multilinear model. When the system exhibits smooth stiffness degradation in its hysteretic response, as in reinforced concrete structures, both methodologies displayed a number of anomalies that were not related to any damage /yield event.

Although more computational expensive, the results obtained using a redundant decomposition of the structural response at high frequencies via CWT seem more robust than the results obtained from the detail functions from the DWT.

While wavelet based damage detection methodologies have been validated in the past using data from actual structures subjected to earthquake loads, the structures analyzed even suffer no damage (Rodgers and Çelebi 2006) or the damage was severe and obvious (Todorovska and Trifunac 2009). Since fiber-based modeling can be used to generate models that closely resemble the non-linear hysteretic response of the structure, such approaches present a unique opportunity to validate and calibrate damage detection methodologies aimed for civil structures.

6 ACKNOWLEDGEMENTS

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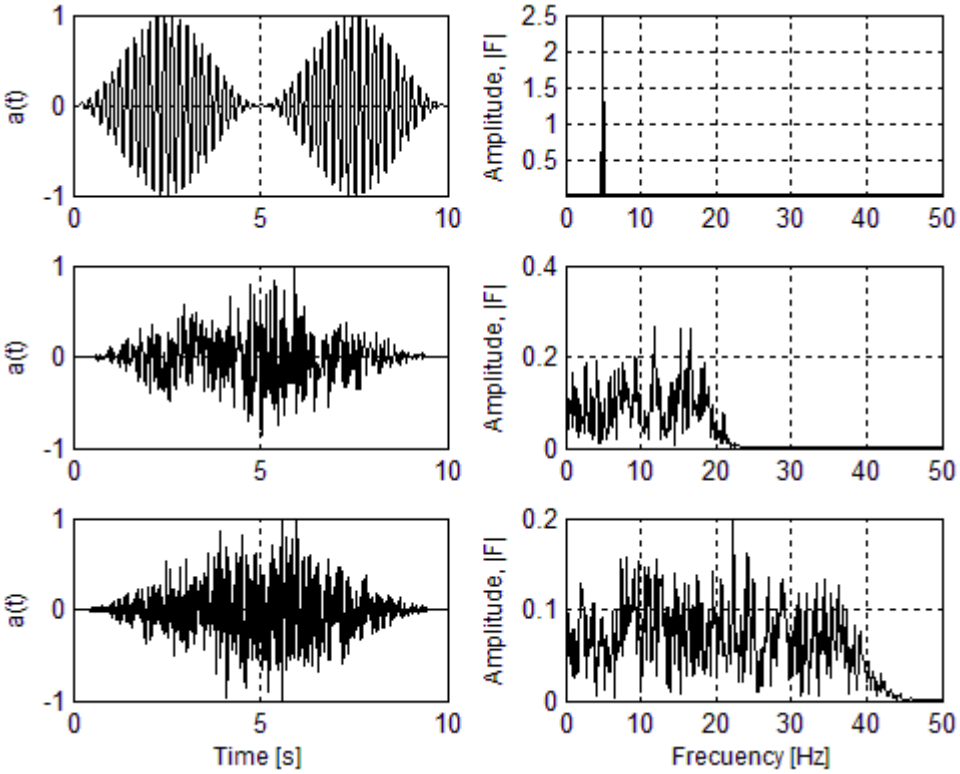

Figure 4. The three different excitations loads in the time domain (left) and their Fourier spectra (right)

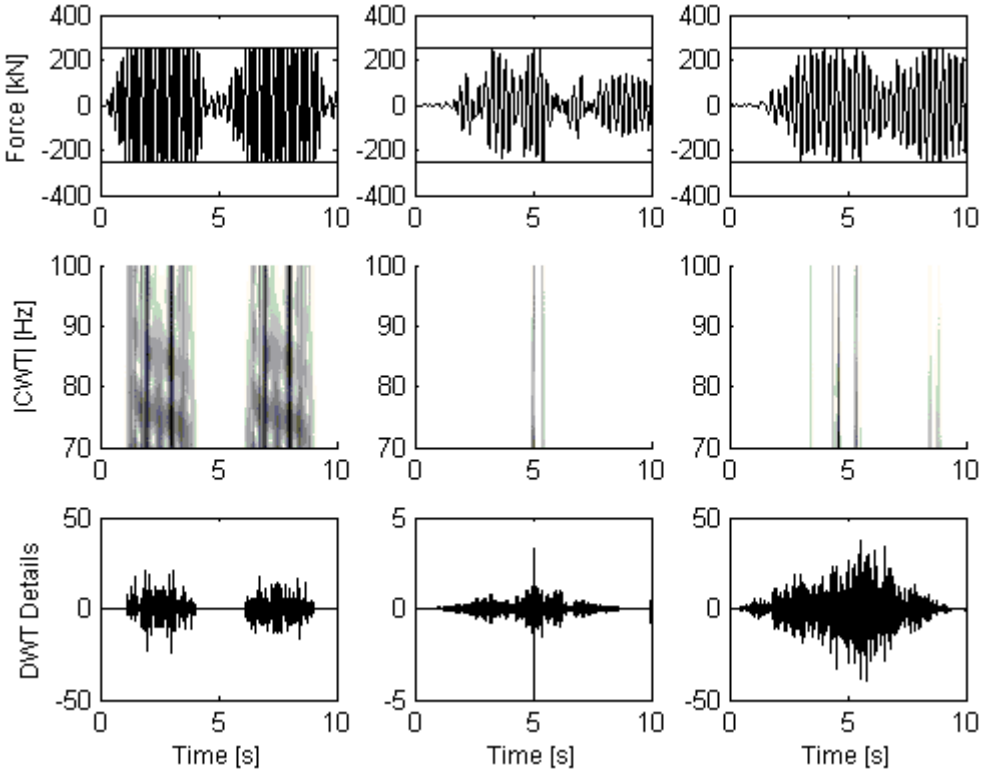

Figure 5. Bilinear model subjected to: sinusoidal (left), noise < 20 Hz (center) and noise <40 Hz (right)

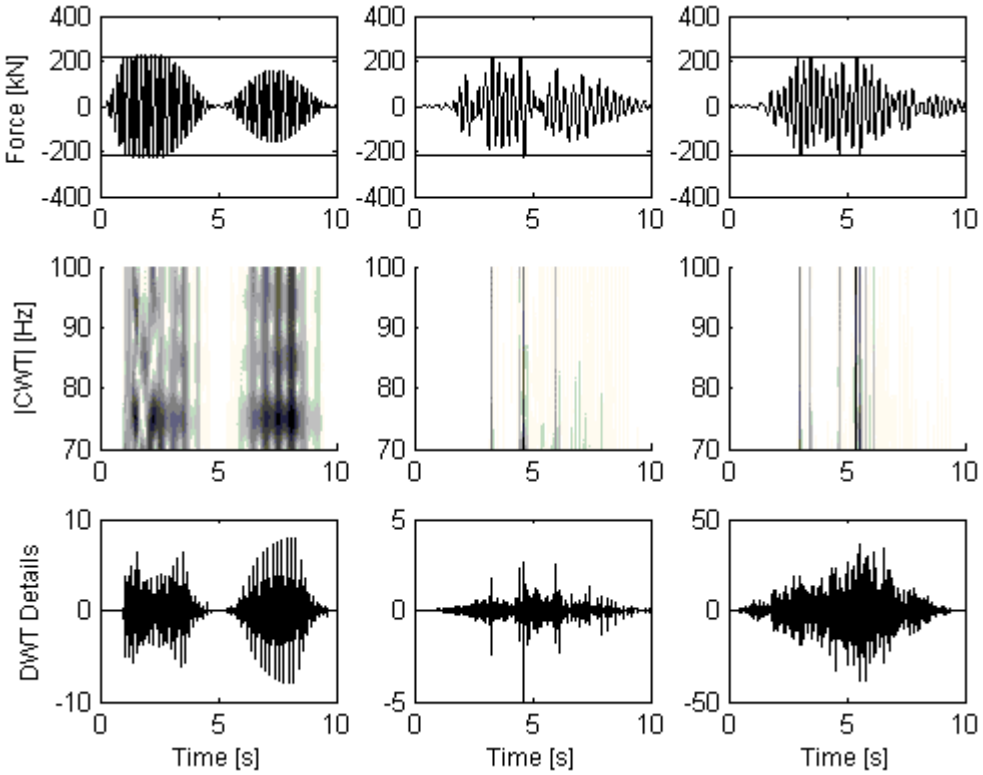

Figure 6. Takeda model subjected to: sinusoidal (left), noise < 20 Hz (center) and noise <40 Hz (right)

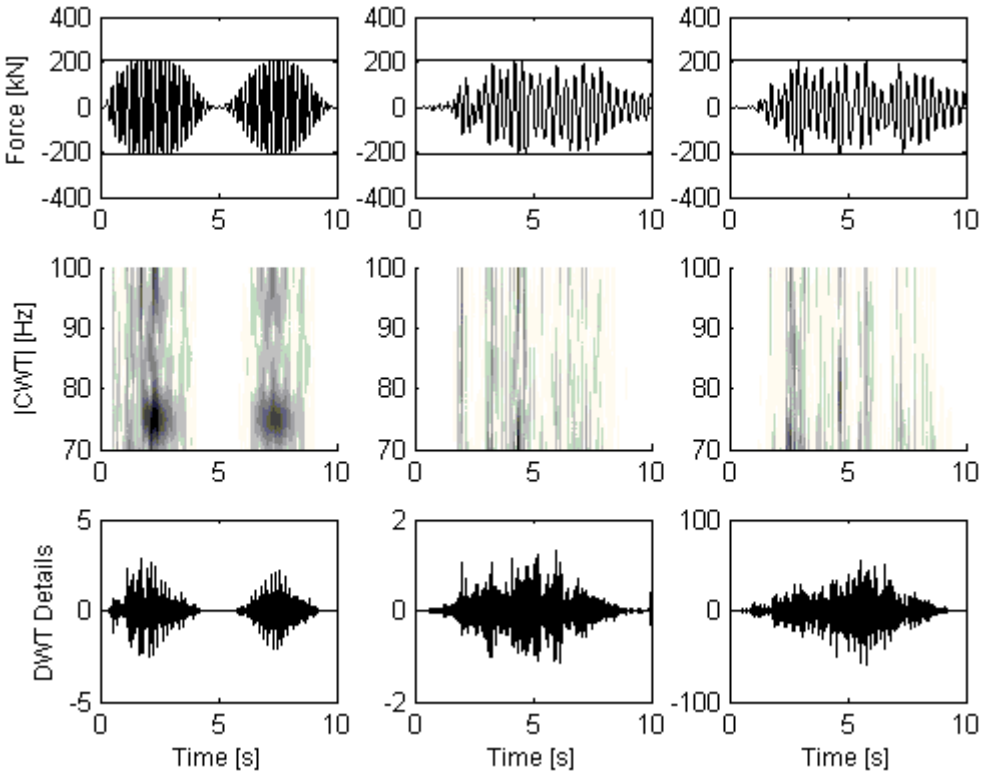

Figure 7. Fiber model subjected to: sinusoidal (left), noise < 20 Hz (center) and noise <40 Hz (right)