EXPERIMENTAL VERIFICATION OF A NOVEL LOAD DEPENDENT SENSOR PLACEMENT METHOD

Dongsheng LI
Dalian University of Technology
State Key Laboratory of Coastal & Offshore Engineering
Dalian,116024, China

Claus-Peter Fritzen
University of Siegen
Institute of Mechanics and Automatic Control - Mechatronics, Germany.

Hongnan LI
Dalian University of Technology
State Key Laboratory of Coastal & Offshore Engineering
Dalian,116024, China
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In the current work, experiments are conducted on a six-story truss structure to verify the proposed load dependent sensor placement method. Different loading conditions are tested and identified mode shapes are compared in terms of identification accuracy. It is found that better mode shape identification can be achieved with the proposed novel load dependent sensor placement method. The experiments have verified that loading conditions under various working environment have to be accounted for when the issue of sensor placement is arisen.

Corresponding author’s email: dsli@dlut.edu.cn

Experimental verification of a novel load dependent sensor placement method

Dongsheng LI[1], Claus-Peter Fritzen[2], and Hongnan LI[3]

[1] Dalian University. of Technology, State Key Laboratory of Coastal & Offshore Engineering, Dalian,116024, China
[2] University of Siegen, Institute of Mechanics and Automatic Control - Mechatronics, Paul-Bonatz Str. 9-11, D-57076 Siegen, Germany.
[3] Dalian University. of Technology, State Key Laboratory of Coastal & Offshore Engineering, Dalian,116024, China

ABSTRACT: A novel sensor placement method is proposed for structural health monitoring in our previous investigations. The proposed method depends on both the characteristics and the actual loading situations of a structure. The aim of the method is to select sensor locations from a set of possible candidate positions for achieving the best identification of modal frequencies and mode shapes, and therefore for best damage identification and subsequent structural health monitoring. In the current work, experiments are conducted on a six-story truss structure to verify the proposed load dependent sensor placement method. Different loading conditions are tested and identified mode shapes are compared in terms of identification accuracy. It is found that better mode shape identification can be achieved with the proposed novel load dependent sensor placement method. The experiments have verified that loading conditions under various working environment have to be accounted for when the issue of sensor placement is arisen.

1 INTRODUCTION

Structural health monitoring provides vital information for the safe operation of key civil structures and enables operational cost reduction by performing prognostic and preventative maintenance. It shows also a great potential for disaster mitigation. In general, a typical structural health monitoring (SHM) system includes three major components: a sensor system, a data processing system (including data acquisition, transmission and storage), and a health evaluation system (including diagnostic algorithms and information management)(Li,etc. 2004). The sensors utilized in SHM are required to monitor not only the structural status including stresses, displacements, and accelerations of critical structural elements, but also influential environmental parameters, such as wind speed, temperature and the quality of its foundation. On the other hand, the high costs of data acquisition systems (sensors and their supporting instruments) and accessibility limitations constrain, in many cases, the wide distribution of a large number of sensors on a structure. In particular, many structures have to be tested under operational conditions in which the sensors are not easily amenable to be displaced. It is, therefore, of increasing interest to seek as small as possible a number of sensors that contain as much information as possible about the health state of a structure.

The issue of sensor placement can be investigated from various aspects and many methodologies have been developed. Visual inspection, which later evolved into modal kinetic energy (MKE) method, examines the mode shapes of interest and selects locations with high amplitude of responses (Papadopoulos and Garcia 1996). A relatively promising method is the static flexibility approach (Flanigan and Botos 1992). This method optimizes the static transformation matrix with the assumption that the best master DOFs are those for which the FEM mode shapes can be represented as a linear combination of static flexibility shapes. It begins with Guyan reduction and ends by the minimization of the fit errors of the least squares, and a reduced number of DOFs is obtained. However, a major disadvantage of the reduction methods is that they strongly depend on the meshing size of the FEM. Udwadia(1994) examined the problem by estimation theory using sensitivity analysis. Kammer (1991) simplified and extended this analysis, and proposed the effective independence (EI) method, which tends to maximize the trace and determinant and minimize the condition number of the Fisher information matrix corresponding to the target modal partitions. Recently, Li et al. (2007) found the connection between EI and modal kinetic energy (MKE) methods, i.e., EI is an iterated version of MKE with reorthonormalized mode shapes and Li et al. (2009) further develop a fast computation scheme for the EI with QR downdationg. Sensor placement can also be investigated from other perspectives. A thorough review of sensor placement methodologies can be found in Li (Li et al.,2007a; Li et al.,2008).

Since sensor placement is essentially a discrete optimization problem with respect to certain objective function, each method then strives to achieve a practical suboptimal sensor combination other than an optimal one, which is, in most cases, unpractical in current engineering practice due to its computation burden if covering the entire feasible domain. It is true that each sensor placement method as discussed above has a concrete background and aims to achieve a designated objective. The application of certain sensor placement method is, of course, determined by the evaluation criteria behind the optimization objectives.

The paper is organized as follows. Theory of the load dependent sensor placement method is first introduced in Section 2. In Section 3, a truss structure is employed to validate the proposed method through modal tests, which is the major contribution of the paper. Finally, conclusions are drawn.

2 THEORY OF THE LOAD DEPENDENT SENSOR PLACEMENT METHOD

2.1 Criterion of the Fisher Information Matrix

The criterion originates from estimation theory by sensitivity analysis of the parameters to be estimated. A Fisher Information Matrix (FIM) results from minimizing the covariance matrix of the estimate error for an efficient unbiased estimator from the perspective of statistics as discussed in (Kammer 1991). From the measurement equation,

$$\mathbf{y} = \mathbf{\Phi q} + \varepsilon \tag{1}$$

where $\mathbf{q}$ is a modal coordinate vector, $\mathbf{\Phi}$ is the mode shape matrix, $\mathbf{y}$ is a measurement column vector indicating which positions of the structure are possibly to be measured, and $\boldsymbol{\varepsilon}$ is a stationary Gaussian white noise with zero mean and a variance of $\Psi_0{}^2$. The EI method as discussed in the introduction takes the covariance matrix of the estimate error for an efficient unbiased estimator as follows,

$$\mathbf{E}\left[(\mathbf{q}-\hat{\mathbf{q}})(\mathbf{q}-\hat{\mathbf{q}})^{\mathbf{T}}\right] = \left[\left(\frac{\partial \mathbf{y}}{\partial \mathbf{q}}\right)^{\mathbf{T}}\left[\mathbf{\Psi}_0^2\right]^{-1}\left(\frac{\partial \mathbf{y}}{\partial \mathbf{q}}\right)\right]^{-1} = \left[\frac{1}{\mathbf{\Psi}_0^2}\mathbf{\Phi}^{\mathbf{T}}\mathbf{\Phi}\right]^{-1} = \mathbf{A}^{-1} \tag{2}$$

in which $\mathbf{A}$ is the FIM neglecting a constant coefficient, E denotes the expected value, and $\hat{\mathbf{q}}$ is an unbiased efficient estimator of $\mathbf{q}$. Maximizing $\mathbf{A}$ will result in the best state estimate of $\mathbf{q}$.The FIM relates also to the information contained in the measured responses from the viewpoint of information community.

2.2 THEORY OF THE LOAD DEPENDENT SENSOR PLACEMENT METHOD

As shown in Section 2.1, the rationale behind the FIM criterion is that the estimator with the smallest variance gives the best solution among all the unbiased estimators, as stated by Kammer (Kammer 1991) using the well-known Cramer-Rao lower bound variance theorem in Eq.(2). An underlying implicit premise for this approach is that all the to-be-compared least squares estimators for different sensor combinations are equally unbiased. However, this premise can not generally be considered as valid without examination, especially for the sensor placement issue under discussion. The obscurely shadowed problem will be clarified if we examine the measurement Eq.(1) in further details. For simplicity, we assume hereinafter that 10 response measurements in total are available to be collected, $\mathbf{y} = [\mathbf{y}_{1,}\mathbf{y}_{2,}\ldots,\mathbf{y}_{10}]^{\mathbf{T}}$, which maybe obtained by a pretest, and that six among of the ten will be finally selected to deploy sensors for future online health monitoring. Two different groups of sensor positions maybe selected according to certain criteria and ensuing methods in Section 2, for instance, group G1 may include six candidate sensor positions as, $\mathbf{y}_{\mathbf{G}1} = [\mathbf{y}_{1,}\mathbf{y}_{3,}\mathbf{y}_{5,}\mathbf{y}_{7,}\mathbf{y}_{9,}\mathbf{y}_{10}]^{\mathbf{T}}$, and the group G2 is, $\mathbf{y}_{\mathbf{G}2} = [\mathbf{y}_{2,}\mathbf{y}_{3,}\mathbf{y}_{4,}\mathbf{y}_{6,}\mathbf{y}_{8,}\mathbf{y}_{10}]^{\mathbf{T}}$. With these two groups of measurements, the modal coordinates can be estimated using traditional ordinary least squares (OLS) method for both groups as,

$$\mathbf{q}_{\mathbf{G}1} = \mathbf{\Phi}_{\mathbf{G}1}{}^{+}\mathbf{y}_{\mathbf{G}1},\ \mathbf{q}_{\mathbf{G}2} = \mathbf{\Phi}_{\mathbf{G}2}{}^{+}\mathbf{y}_{\mathbf{G}2} \tag{3}$$

Where, $\mathbf{\Phi}_{\mathbf{G}1}$ and $\mathbf{\Phi}_{\mathbf{G}2}$ are the reduced mode shapes consisting of the rows of the full ones specified by $\mathbf{y}_{\mathbf{G}1}$ and $\mathbf{y}_{\mathbf{G}2}$, respectively. In addition, the estimate of the modal coordinates with the initial total ten measurements is,

$$\mathbf{q}_0 = \mathbf{\Phi}^{+}\mathbf{y}. \tag{4}$$

It is apparent that the three estimates of modal coordinates, $\mathbf{q}_{\mathbf{G}1}$, $\mathbf{q}_{\mathbf{G}2}$ and $\mathbf{q}_0$, are unbiased them-selves in terms of the group data points they are computed, i.e., $\mathbf{q}_{\mathbf{G}1}$ is unbiased for the data group G1, $\mathbf{q}_{\mathbf{G}2}$ for G2, and $\mathbf{q}_0$ for all ten response measurements

(G1∪G2). However, the unbiasedness of the three estimators, $\mathbf{q}_{\mathbf{G}1}$, $\mathbf{q}_{\mathbf{G}2}$ and $\mathbf{q}_0$ with respect to their groups does not necessarily guarantee that they equal to each other. In fact, they may differ. Consequently, it is questionable to compare the variances of different least squares estimates without first examination of their unbiasedness, which is a prerequisite. What we are discussing here different from conventional statistics, lies in the fact that different groups of data are used in sensor placement problem, whereas different estimators with various hypothesis (for instance, maximum likelihood) for the same data group are compared in statistics.

In fact, unbiased estimators are always sought as a priority in statistics or linear regression than biased ones. The estimation accuracy of $\mathbf{q}$ determines the quality of the linear fitting equation (1) and thus the accuracy of the mode shapes to be identified. However, an unbiased estimator of $\mathbf{q}$, the true value of the parameter being estimated, is not attainable in many cases for sensor placement issues. Consequently, compromises have to be made. Among all biased estimators of $\mathbf{q}$, the one which is much closer to the unbiased one turns out to be more desirable. As a result, the objective is then to minimize the Euclidean distance between the ideal unbiased estimator and an almost unbiased one as follows,

$$\mathbf{J}_{\mathbf{gu}} = (\hat{\mathbf{q}}_{\mathbf{s}} - \hat{\mathbf{q}}_{\mathbf{OLS}})^{\mathbf{T}}(\hat{\mathbf{q}}_{\mathbf{s}} - \hat{\mathbf{q}}_{\mathbf{OLS}}). \tag{5}$$

where $\mathbf{J}_{\mathbf{gu}}$ is the objective function to achieve almost global unbiasedness of an estimator with regard to $\hat{\mathbf{q}}_{\mathbf{OLS}}$ which equals to the best attainable unbiased estimator, $\hat{\mathbf{q}}_0$ with all n cases; $\hat{\mathbf{q}}_{\mathbf{s}}$ is the ordinary least squares estimator with a partial subset of s cases (required s sensor positions).

The new objective, as define in Eq.(5), is a novel criterion for sensor placement issue to achieve almost global unbiasedness. Under the term ‘almost global unbiasedness’, we mean that unbiasedness is of great priority than variance (dispersion) when sensor placement is considered and that the objective is to choose a given number of sensor positions which obtain an almost unbiased coefficient estimator nearest to the unbiased estimator achieved by all candidate sensor positions as follows,

$$\hat{\mathbf{q}}_{\mathbf{S}} = (\mathbf{\Phi}_{\mathbf{s}}^{\mathbf{T}}\mathbf{\Phi}_{\mathbf{s}})^{-1}\mathbf{\Phi}_{\mathbf{s}}^{\mathbf{T}}\mathbf{y}_{\mathbf{s}},\ \hat{\mathbf{q}}_{\mathbf{OLS}} = (\mathbf{\Phi}^{\mathbf{T}}\mathbf{\Phi})^{-1}\mathbf{\Phi}^{\mathbf{T}}\mathbf{y} \tag{6}$$

where the subscript s denotes a set of s cases including possible combinations of selecting s out of n rows.

The physical significance of the almost global unbiasedness is clarified at this stage. The objective function $\mathbf{J}_{\mathbf{gu}}$ measures the distance between an OLS estimator $\hat{\mathbf{q}}_{\mathbf{OLS}}$ with all n cases and a possibly biased estimator $\hat{\mathbf{q}}_{\mathbf{s}}$ with only partially s cases. In essence, the criterion of almost global unbiasedness shares many common aspects as that of the mean-squared error in statistics. The estimation method with the objective defined in Eq.(5) is named representative least squares (RLS) for naming convenience. By RLS, the selected data subset is required to be representative and sufficiently approaching to portrait the scenario defined by the original full data set, i.e., a partial estimator $\hat{\mathbf{q}}_{\mathbf{s}}$ with only s cases approximates the original least squares estimator with the whole data set (Li, 2009).

Generally, the RLS estimator can be found through genetic algorithm, through subspace approximation, and through backward and forward combinational approach. The last two methods are suboptimal in the sense that they are approximate and only optimal in the single step in which the computation is processed. The first computation approach is exact, however, it has the disadvantage of complexity and suffers from huge computation burden. For detailed discussion for the RLS and its comparison with traditional linear regression method, the readers are invited to consult the reference (Li, 2007).

Figure 1. Truss structure in experiment.

3 EXPERIMENTAL VARIFICATON

3.1 Model structure

The experimental structure is a space frame truss with 6 bays as shown in Fig.1. The dimension of each bay is 500mm in x direction, 300mm in y direction, and 500mm in z direction. The total height of the truss is 3 meters and the 6 bays are equally divided. The truss is pinned firmly at one end on the floor with x direction upward pointed through a steel base. The bays of the truss are numbered 1 to 6 from the bottom up-wards. Fourteen modes fall below 100 Hz, among which five are bending modes in the weak y direction.

3.2 Experiment setup

In our laboratory, 12 accelerometers are available. Two electromagnetic shakers are fixed horizontally on the wall of our laboratory in parallel to excite the truss synchronously. To minimize the torsional responses of the truss, the excitation signals input to the shakers share the same signal generator. Moreover, a small crane, which was used to move the two shakers with steel ropes, was fixed just under the room ceiling as indicated on the top of Fig.1. The two shakers can, therefore, be conveniently lifted up or lowered down by the crane to any of the six bays in order to excite the truss at different horizontal levels. This experimental arrangement is deliberately designed to test the influence of different excitation positions on the placement of sensors, which is the central concept in Section 2, i.e. sensors placement depends not only on the structure itself, but also on its actual loading conditions in order to achieve better modal identification accuracy.

Six setups with different shaker positions are tested. In each setup, both shakers are bolted to two horizontal nodes of a bay in both sides of the truss to excite it horizontally. To take the Setup 2 for example, both shakers are connected to the Bay 2 at side nodes as shown in Fig.1. Similarly, Setup 1 to Setup 6 are tested when the shakers are located at the Bay 1 to Bay 6 to excite the truss sequentially.

3.3 Sensor placement for the truss with traditional method - EI

As discussed in Section 2, the existing sensor placement methods share a common feature, which is that the sensor positions are solely determined by the structure. Once a structure model is given, the sensor positions are thus determined as well. According to the EI, the candidate sensor positions are ranked in sequence as 6, 1, 5, 2, 4 3, with the top position is the most important.

3.4 Experimental verification of the load dependent sensor placement method

In this section, the difference between the mode shapes identified with all the six candidate sensor positions and the mode shapes identified with only certain five candidate sensor positions is compared, which is used to verify the theory of the load dependent sensor placement method proposed in Section 2.

3.4.1 Experiment results in Setup 2

In this subsection, the measured responses of Setup 2 will be used to identify the mode shapes of the truss. The identified mode shapes with five components are then compared with the theoretical ones that are reduced accordingly. First, Sensor 1 at Bay 1 will be excluded and then Sensor 2 at Bay 2 until Sensor 6 at Bay 6. These cases are dubbed as Case 1 to Case 6 and listed in Table 1. All the cases are conducted under Setup 2.

Table 1. Mode shape identification error in Setup 2.

Case No.	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Absolute Error	Relative Error
1	0.0652	0.1631	0.0369	0.1377	0.1351	0.5380	0.2637
2	0.0611	0.1875	0.0594	0.1520	0.1759	0.6358	0.3067
3	0.1029	0.1767	0.0488	0.2406	0.1269	0.6959	0.3422
4	0.1093	0.1515	0.0642	0.2125	0.1475	0.6849	0.3338
5	0.1076	0.4094	0.0507	0.2371	0.1977	1.0025	0.4912
6	0.1452	0.1067	0.0676	0.2438	0.1476	0.7110	0.3519

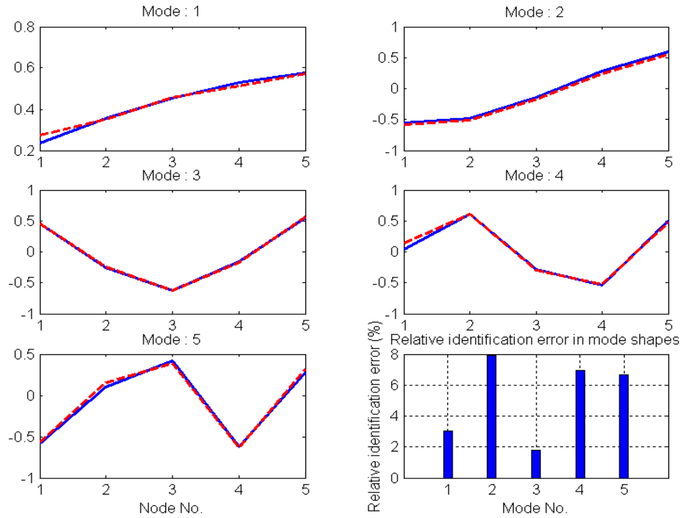

Figure 2. Mode shape comparison for Case 1, blue solid line – theoretical; red dashed line – identified.

The lower-right plot in Fig.2 shows relative identification errors (%) of the identified mode shapes compared with theoretical ones. The absolute identification error for each mode and each case are numerically listed in Table 1. If one sensor position is absent, for instance, Node 1 absent in Case1 and the identification error is large, then that sensor position would be more important. Consequently, the relative importance of all the six candidate sensor positions is ranked decreasingly as: 5, 6, 3, 4, 2, and 1 according to Table 1.This sensor sequence differs much from those ranking sequences by the traditional methods as in Section 3.3, where EI indicate that Node 1 is very important. However, Node 1 ranks the least importance in Setup 2.

3.4.2 Experiment results in Setup 5

Similar to Section 3.4.1, we will sequentially exclude one measurement at a certain sensor position and identify the five mode shapes with the remaining five sensor positions and compare it with the reduced theoretical mode shape in Setup 5 First, Sensor 1 at Bay 1 will be excluded and then Sensor 2 at Bay 2 until Sensor 6 at Bay 6, and these cases are dubbed as Case 1 to Case 6 and listed in Table 2.

Table 2. Mode shape identification error in Setup 5.

Node No.	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Absolute Error	Relative Error
1	0.0973	0.2293	0.0529	0.1238	0.0570	0.5604	0.2728
2	0.0954	0.2749	0.0336	0.1146	0.1234	0.6417	0.3098
3	0.0881	0.2827	0.0468	0.2011	0.1203	0.7390	0.3624
4	0.0976	0.1693	0.0545	0.1850	0.1282	0.6346	0.3084
5	0.2093	0.1719	0.0442	0.1709	0.1203	0.7166	0.3527
6	0.1231	0.2017	0.0412	0.1757	0.1079	0.6496	0.3199

The candidate sensor sequence, 3, 5, 6, 2, 4 and 1 in Setup 5, is much different from that determined in Setup 2, in which the sequence is: 5, 6, 3, 4, 2, and 1. Especially, the identification error in Setup 2 as in the last section is in reverse order for Case 3 and Case 5 compared to the error in Setup 5. Therefore, we can easily observe that the relative importance of the six candidate sensor positions are changing with the loading conditions of the truss structure. The experiments have, therefore, powerfully verified that loading conditions under various working environment have to be accounted for when the issue of sensor placement is arisen.

4 DISCUSSION OF EXPERIMENTAL RESULTS

Since the load dependent sensor placement method deploys sensors taking actual loading conditions of a structure into consideration, the sensor positions are not solely determined by the structure itself. The load dependent sensor placement method indicates two different sensor topologies for the same structure with two loading conditions. On the other hand, the traditional methods give only one sensor positions' ranking sequence no matter what the actual loading conditions of the structure change. In the comparison of theoretical and experimentally identified mode shapes in the investigation, only a reduction of 6 to 5 sensors is compared due to available experiment facilities. The validation experiment is, thus, rather limited. If more candidate sensor positions are involved, the comparison will be much apparent.

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