Probabilistic Method for Estimating Remaining Fatigue Life in Steel Bridges

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Abstract. A fracture-critical highway bridge with active cracks was monitored using strain gages to determine the number and amplitude of stress cycles under service loads. The measured response was used to estimate the remaining fatigue life of fatigue-sensitive members. Previous calculations based on a deterministic approach revealed that the fatigue life had long been exceeded. Due to the scatter inherent in the fatigue data, a probabilistic approach for estimating the remaining fatigue life was developed. The probabilistic method allows a bridge owner to weigh the risk of maintaining a bridge past its design service life.

1. Introduction

Maintaining the bridge inventory with the pressure of reduced budgets and aging infrastructure is a difficult task for bridge owners. One of the more critical types of structural deterioration for steel bridges is fatigue-induced fracture. The primary method used to identify structural deterioration is hands-on visual inspections. The inspections provide bridge owners with data related to the number of cracks and rate of crack growth, but quantitative data are often needed to distinguish between bridges that can be kept in service and bridges that need to be replaced or retrofitted. Calculating the remaining fatigue life is one means of providing quantitative data to bridge owners.

The fatigue life can be estimated using either deterministic or probabilistic approaches. Deterministic approaches are frequently used because the results are easy to calculate and understand. Despite the ease of use, deterministic approaches may not be the most appropriate method because uncertainty is not included in the analyses. Metal fatigue is not a simple process; rather it involves localized damage as the metal is subjected to cyclic loading, which causes variability in the fatigue resistance. Using the AASHTO design curves for fatigue [1], the mean number of cycles to failure can be more than 2.5 times the design value. Thus, there is considerable scatter in the estimated fatigue data, and this inherent variability must be considered when evaluating an existing bridge. A probabilistic approach provides a rational method for including this uncertainty in the analysis.

Measured strain data from a fracture-critical bridge were used to estimate its remaining fatigue life based on the measured response to traffic loads. Measuring the distribution of stress ranges directly reduces the inherent uncertainty as compared with calculating the effective stress range based on design specifications. By minimizing the uncertainty, a more representative remaining fatigue life can be estimated. A deterministic

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approach was used previously to estimate the fatigue life of this bridge [2], and a probabilistic approach will be discussed in the sections below.

2. Background

The fatigue life of a particular bridge depends on the inherent variability of the fatigue response of the connections, rates of accumulation of fatigue cycles (past, current, and future), and the method of calculation (deterministic or probabilistic). The current design fatigue relationships given by AASHTO [1] are based on experimental tests of typical steel connections subjected to a given stress range (S-N curves). The majority of the tests were performed in the 1970-1980s and the results indicated that the primary variables affecting the fatigue life were the stress range and type of connection detail [3].

AASHTO uses a set of eight fatigue curves, Categories A to E′, to characterize the variety of detail types. Because they are used for design, the AASHTO S-N curves represent the 5th percentile value (95% confidence interval that 95% of the details will have a fatigue life greater than the design number of cycles). The number of cycles to failure at a specific stress range can be determined through Eq. 1.

$$N_f = \frac{A}{S_r^3} \qquad \textbf{Eq. 1}$$

where

N_f = number of cycles corresponding to failure at stress range S_r

A = fatigue constant for detail category, defined by [1]

$S_{r,i}$ = constant-amplitude stress range

Because A can be considered to be the fatigue resistance, the fatigue damage induced in a c connection by a given number of stress cycles can be defined as the number of cycles times the stress range cubed (rearranging Eq. 1).

AASHTO provides guidance on estimating past, current, and future fatigue damage. If measured data are not available, an effective stress range for the average number of trucks per day can be calculated through structural analysis. AASHTO recommends values for the analysis, such as a representative fatigue truck (HS-20 truck with a fixed axle spacing), dynamic impact factor (10% to 30%), and distribution factor. However, those values may not be appropriate for each individual bridge and could lead to significant error in the calculation of the fatigue life. The use of stresses induced from measure strains provides the actual number and amplitude of fatigue cycles during the measurement period, thereby eliminating some of the necessary assumptions and improving the estimate of fatigue life.

In real structures, the stress ranges induced by traffic vary with the weight and length of the vehicles crossing the bridge. To use measured strain data to enhance the fatigue life estimation, the local maxima and minim in the dynamic strain history, and the corresponding cycle amplitudes, must be determined. For the analysis described in this paper, a simplified rainflow counting algorithm was used to calculate the amplitude of each cycle in a strain history and accumulate the cycles to form a histogram (Figure 1(a)) [4].

A cumulative damage theory must be used to relate the damage from the variable-amplitude stress ranges in the bridge to the constant-amplitude fatigue data. The most popular theory is Palmgren-Miner's rule [5], which is based on a linear damage hypothesis. Palmgren-Miner's rule is simple to use and has been shown to agree well with test results [6]. Following Palmgren-Miner's rule, the fatigue damage from a particular bin in a histogram of number of cycles for a particular day can be determined (Eq. 2). The fatigue damage for that particular day ($d_{i,day}$) is then the sum of all the bins.

$$d_{i,day} = n_i * S_{i,avg}^3 \quad \textbf{Eq. 2}$$

where

$d_{i,day}$ = damage in bin i from single day

n_i = number of cycles for a single day in bin corresponding to stress range i

$S_{i,day}$ = average stress range corresponding to bin i

An example histogram of number of cycles and stress range can be seen in Figure 1(a). This histogram corresponds to a service-level traffic crossing the bridge during one 24-hr period. There are a wide variety of stress ranges captured throughout the day, with a maximum stress range of 16.6 ksi. If Eq. 2 is applied to the data in Figure 1(a), the contribution of each bin to the fatigue damage for that particular day can be evaluated. As seen in Figure 1(b), though there were a few cycles with a stress ranges above 15 ksi (very heavy trucks), the majority of damage is caused by vehicles between 5 and 8 ksi (typical truck traffic). By summing the damage from each bin, the total amount of damage for that particular day was found to be 601,500 ksi^3.

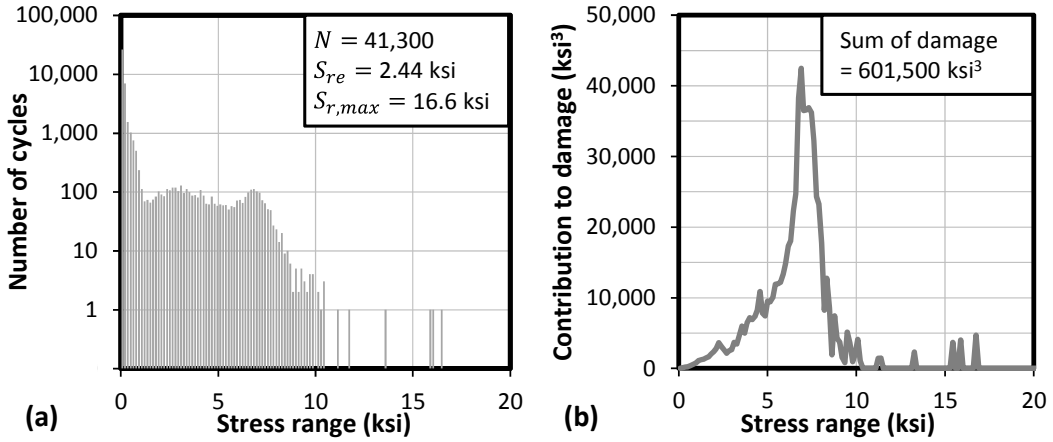

Figure 1. (a) Histogram of stress ranges captured by data acquisition system during a weekday. (b) Transformed histogram into fatigue damage using Eq. 2.

Calculating the remaining fatigue life is challenging because of the scatter in the fatigue data and the amount of fatigue damage each year will likely change due to traffic growth or decay. Typically, the annual volume of traffic increases geometrically at a rate between 2% and 6%. To minimize the uncertainty from estimating traffic growth, the growth can be compared to historic traffic counts for the site or measured data from multiple years can be used to estimate the rate of growth.

Due to the uncertainty in calculating the fatigue life, reliability equations may be the most appropriate method of calculation. Reliability concepts have mainly been used to determine the optimal schedule for inspections [7-8]. The process involves setting a target level of reliability (β_{target}) and optimizing the inspection interval in terms of the cost of the inspections, repairs, and failure. The reliability concepts have also been used to estimate the remaining fatigue life. Kwon & Frangopol [9] developed a model that accounted for measurement errors from misalignment of the strain gages and/or data acquisition error.

3. Probability Equations

A probabilistic model for determining the remaining fatigue life was evaluated by considering the fatigue limit state function (Eq. 3). The fatigue limit state function ($g(X)$) involves products and can be modeled by a lognormal distribution function. Failure is assumed to occur when $g(X) < 0$. For this analysis, failure is when the fatigue cracks at the connection detail grow exponentially, such that fracture occurs.

$$g(X) = \Delta - D(t) = \Delta - \frac{d_{tot}(t)}{A} = 0 \quad \textbf{Eq. 3}$$

where

Δ = Palmgren-Miner's critical damage index (resistance)
$D(t)$ = Palmgren-Miner's damage accumulation index during the service life of the bridge (loading)
$d_{tot}(t)$ = total accumulated fatigue damage

Palmgren-Miner's critical damage accumulation index (Δ) accounts for the uncertainty from using Palmgren-Miner's rule. For fatigue in offshore structures, the critical damage accumulation index is modeled by a lognormal distribution with a median value (x_Δ) of 1.0 and coefficient of variation (δ_Δ) of 30%, which is based on a survey of fatigue tests on structural sections [10]. For most tests, the median is 1.0 and the coefficient of variation ranges between 30-60% [11]. Because fatigue damage accumulation in offshore structures is similar to fatigue in bridges, x_Δ = 1.0 and δ_Δ = 0.3 was assumed for this paper. The lognormal parameters can be calculated using Eqs. 4 and 5.

$$\lambda_\Delta = \ln(x_\Delta) \quad \textbf{Eq. 4}$$

$$\zeta_\Delta = \sqrt{\ln(1+\delta_\Delta^2)} \quad \textbf{Eq. 5}$$

The fatigue constant (A) comes from the experimental tests conducted to determine the number of cycles corresponding to failure for a given stress range. Fisher et al. [12] concluded that $\log_{10}(N_{f,i})$ can be assumed to follow a normal distribution. As such, $\log_{10}(A)$ can also be assumed to follow a normal distribution, while A follows a lognormal distribution. The parameters for the lognormal distribution for A are determined by Eqs. 6 and 7. The derived parameters of the lognormal distribution for each fatigue category are summarized in Table 1.

$$\lambda_A = \ln(10) * \log_{10}(\mu_A) \quad \textbf{Eq. 6}$$

$$\zeta_A = \ln(10) * \log_{10}(\sigma_A) \quad \textbf{Eq. 7}$$

where

μ_A = mean of fatigue constant (A)
σ_A = normalized standard deviation of fatigue constant (A)

While random variables are used to describe Δ and A, the cumulative damage can be estimated from measurements. Most measurement periods are typically between two and eight weeks and corresponding to year (k) after the bridge was first put in service. To calculate the cumulative fatigue damage, the previous amount of damage has to be estimated. Assuming the average traffic volume grows geometrically at a constant rate each year, the cumulative damage can be estimated by Eq. 8.

$$d_{tot}(t) = \mu_{d_{yr,1}} \frac{(1+r)^t - 1}{r} \quad \textbf{Eq. 8}$$

where

$\mu_{d_{yr,1}}$ = mean damage in first year of service
r = annual rate of growth (constant)

t = year evaluated

Table 1. Variation in fatigue constant (A) from [13] and derived parameters for lognormal distribution.

AASHTO Fatigue Category	Fatigue constant		Normalized standard deviation for fatigue constant (σ_A) [13]	Parameters for lognormal distribution	
	A [1] (code value)	μ_A [13] (mean value)		λ_A	ζ_A
A	$250*10^8$	$689*10^8$	1.66	10.84	0.22
B	$120*10^8$	$235*10^8$	1.40	10.37	0.15
B′	$61*10^8$	$145*10^8$	1.54	10.16	0.19
C	$44*10^8$	$59*10^8$	1.16	9.77	0.06
C′	$44*10^8$	$59*10^8$	1.16	9.77	0.06
D	$22*10^8$	$35*10^8$	1.26	9.54	0.10
E	$11*10^8$	$17*10^8$	1.26	9.24	0.10
E′	$3.9*10^8$	$9.9*10^8$	1.59	8.99	0.20

As seen from Eq. 8, the cumulative damage depends on the mean damage in the first year of service. The mean fatigue damage from the current year (k) of service, obtained from measurements, can be used to approximate the damage from the first year by using Eq. 9.

$$\mu_{d_{yr,1}} = \frac{\mu_{d_{yr,k}}}{(1+r)^{k-1}} \quad \textbf{Eq. 9}$$

where

$\mu_{d_{yr,k}}$ = mean damage in year k

k = current year

Because Δ and A are lognormally distributed and the limit state function involves products, the reliability factor (β) can be determined using Eq. 10. This equation can be modified to model uncertainty in $d_{tot}(t)$; however, it was found that uncertainty in $d_{tot}(t)$ did not significantly change the reliability factor (β) for the particular bridge discussed in the next section.

$$\beta = \frac{\lambda_\Delta + \lambda_A - \ln(d_{tot}(t))}{\sqrt{\zeta_\Delta + \zeta_A}} \quad \textbf{Eq. 10}$$

The probability of failure, ($p_{failure}$) can be determined by using a standard normal cumulative distribution function (Eq. 11).

$$p_{failure} = \Phi(-\beta) \quad \textbf{Eq. 11}$$

where

$\Phi()$ = standard normal cumulative distribution function

4. Application to a Bridge

A three-span, twin plate-girder bridge was instrumented with strain gages to evaluate its fatigue performance. The bridge is along a major transportation corridor with significant truck traffic. The critical connections were added to the bridge when it was widened more than 35 years ago. The bridge is considered to be fracture critical because the twin-girder system is non-redundant and the failure of a flange from a longitudinal girder would be expected to cause collapse of the entire bridge. The total length is 272 ft with 73.5-ft end spans and a 125-ft center span. The longitudinal girders in the end spans are continuous

over the interior supports and extend 30.6 ft into the center span (Figure 2). Because the center section is suspended by hangers between the cantilevers in the middle span, the bridge is statically determinate.

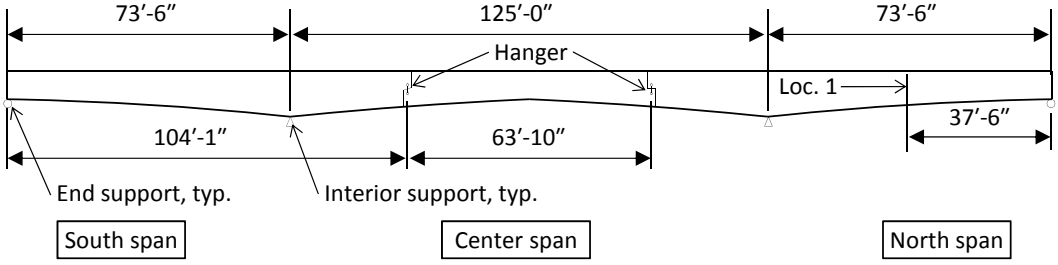

Figure 2. Elevation of the bridge and location of the strain gages. Strain gages were installed at Location 1 on both sides of the top flanges of the longitudinal girders.

Strain gages were installed at multiple locations along the length of the bridge on the top and bottom flanges. However, the focus of this paper is on data gathered from gages installed on the west side of the top flanges of the longitudinal girders at Location 1 (Figure 2). This location corresponds to an AASHTO Category E detail. To measure the nominal response of the girder (matches assumption of AASHTO S-N curve), the gages were installed 2 feet away from the stiffener angles. The nomenclature used to identify the gages is Girder Location-Flange. So, East 1-W is a gage installed along the west side of the top flange of the east longitudinal girder at Location 1.

Seventy-one days of data were captured at the site. Because the data acquisition system did not run continuously, the distribution between days was different (nine Mondays, eight Tuesdays, etc.). As such, the mean weekly fatigue damage (Eq. 2) in year k can be determined by summing the mean damage from each day of the week, as seen in Eq. 12.

$$\mu_{d_{week,k}} = \mu_{d_{Mon,k}} + \mu_{d_{Tue,k}} + \mu_{d_{Wed,k}} + \mu_{d_{Thu,k}} + \mu_{d_{Fri,k}} + \mu_{d_{Sat,k}} + \mu_{d_{Sun,k}} \quad \textbf{Eq. 12}$$

The means and standard deviations for gage locations East 1-W and West 1-W for each day of the week are summarized in Table 2. The daily, weekly, and yearly mean for year 37 is also summarized in the table.

Table 2. Summary of fatigue data obtained in year 37 from strain gage measurements.

	East 1-W		West 1-W	
	$\mu_{d_{37}}$ (ksi^3)	$\sigma_{d_{37}}$ (ksi^3)	$\mu_{d_{37}}$ (ksi^3)	$\sigma_{d_{37}}$ (ksi^3)
Monday (9 days)	562,000	90,000	83,000	17,000
Tuesday (8 days)	666,000	58,000	98,000	12,000
Wednesday (12 days)	657,000	44,000	92,000	9,300
Thursday (10 days)	639,000	41,000	89,000	11,000
Friday (10 days)	619,000	45,000	91,000	7,100
Saturday (13 days)	442,000	33,000	63,000	6,100
Sunday (9 days)	299,000	24,000	37,000	4,400
Daily mean in year 37	555,000	48,000	79,000	9,500
Weekly mean in year 37	3,880,000	336,000	553,000	66,000
Yearly mean in year 37	203,000,000	17,500,000	28,800,000	3,400,000

Following the equations presented in the previous section, the probability of failure can be plotted on a yearly basis, as shown in Figure 3. Figure 3(b) plots the probability of failure for the east girder, which corresponds to the right lane for this bridge. Because more trucks cross the bridge in the right lane as compared to the left lane, the east girder is expected to have more damage than the west girder. Comparing Figures 3(a) and 3(b), the east girder has a higher probability of failure than the west girder for any given year.

Because the annual rate of traffic growth was not known for the bridge, a range of growth rates (2%-6%) were considered. If the 2% model of annual growth were correct, the probability of failure for the current age of the bridge is nearly 100% for Location 1 along the east girder. If 6% growth were correct, the probability of failure is at 95%. Using a deterministic method to estimate the remaining fatigue life, the design fatigue life (5% percentile) would have been exceeded in year 10 for a 2% annual growth and in year 22 for a 6% annual growth in the east girder [2]. The values are comparative to the probabilistic model (year 9 for 2% growth and year 20 for 6% growth) because the uncertainty in the damage was small.

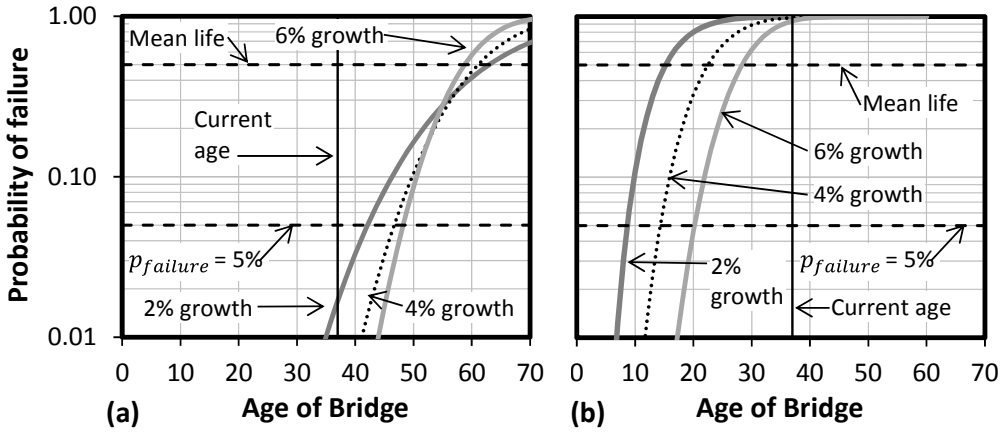

Figure 3. Probability of failure at Location 1 for the (a) west girder and (b) west girder.

Considering the gage installed on the west girder, the estimates for the deterministic and probabilistic approaches are similar. For the deterministic approach, the 5% percentile would have been exceeded in year 48 for 2% annual growth and year 51 for 6% annual growth [2]. In contrast, the probability of failure approaches 5% for the probabilistic approach between years 41-48. Thus, the probabilistic method indicates that the probability of failure reaches 5% prior to the deterministic approach. Some restraint should be exercised when considering such a long length into the future: will traffic growth continue to increase at a constant rate? But the difference between 2% and 6% growth rates is minor for the west girder as compared to the difference of 2% and 6% in the east girder. The main benefit of these graphs is that it shows that the east girder is much more sensitive to fatigue damage than the west girder. As such, more attention during inspections can be focused on the east girder.

5. Conclusions

Measured strain data can be useful for estimating the remaining fatigue life of fracture-critical bridges. Dynamic strain data can be captured on a continuous or semi-continuous basis, analyzed by a rainflow counting method, and then the probability of failure can be estimated by using the probabilistic model. As additional data are captured over the life of

the bridge, the confidence of the fatigue life estimate will be improved by determining the actual rate of traffic growth and the impact of increases in legal vehicle loads.

A probabilistic approach to calculating the remaining fatigue life handles the uncertainty directly and provides advantages compared with the deterministic approach. For the bridge considered in the example, the deterministic approach produced a similar fatigue life for the east girder because the damage was so large and the uncertainty in the damage was minimal. For the west girder, the damage was not as significant and the uncertainty had more of an influence. As such, the probabilistic approach better characterized the fatigue life because it addressed the uncertainty using Palmgren-Miner's rule (Δ). Only a few levels for the probability of failure of the fatigue data (two standard deviations from mean (design S-N curve), one standard deviation from mean, and mean) can be calculated using a deterministic approach. In contrast, many different levels for the probability of failure can be calculated for various growth models (2% to 6%) and sources of uncertainty (fatigue data and Palmgren-Miner's rule) using a probabilistic approach.

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