4th International Conference on Structural Health Monitoring of Intelligent Infrastructure (SHMII-4) 2009
22-24 July 2009, Zurich, Switzerland

Damage Detection via Substructural Analysis

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ABSTRACT: This paper proposes a method to assess the conditions of a substructure by simultaneously identifying the interface forces between substructures and detecting local damages in a selected substructure from the dynamic response with the structure under support excitation. The substructure is subject to a set of interfacing forces at the interface degrees-of-freedom. These interfacing forces between substructures are represented by orthogonal functions, and a sensitivity-based method is then used for identifying the interface forces and detecting local damages of substructure. A five-storeys plane frame structure is taken as an example for validating the proposed method. Numerical simulations with noisy measured accelerations show that the proposed method can accurately identify the interface forces and detect local damages of substructure without the inclusion of other substructures.

1 LITERATURE REVIEW

It is well known that the structural identification is inherently an ill-conditioned problem and an optimal and/or iterative procedure is always necessary to solve this inverse problem. The difficultly to achieve convergence will increase dramatically with the number of unknown structural parameters. In addition, the model error between the finite element model and the real structure it represents will also increase with an increase in the number of unknown structural parameters. Since most complex systems can be seen as an assembly of subsystems (Gontier & Bensaibi, 1995), it is therefore feasible and advantageous to reduce the size of the problem by adopting a “substructure” strategy. Many researchers have proposed methods for substructural identification. Most of these methods require interface response measurements, which are then treated as input to the substructures (Koh et al. 2003). However, it is not always possible to obtain all interface measurements, especially if rotational response is required for beam or frame structures. Other researchers proposed methods for parameter identification of substructures without the need of interface measurements. Yun & Bahng (2000) adopted natural frequencies and mode shapes as input parameters of neural networks to identify the stiffness of substructure. Kol & Shankar (2003) proposed a substructure identification method in frequency domain based on the genetic algorithms approach, and the fitness function is defined to minimize the difference between the estimates of interface forces obtained using different sets of response measurements. Yuen & Katafygiotis (2006) used a probability substructure identification method based on Bayes’ theorem to identify damage in the substructure of a multi-story plane frame building. These methods have been proved to be effective in identifying the system parameters of substructures in a plane beam, frame and/or truss structures.

It should be noted that interface forces between substructures can be taken as excitations when each substructure is taken as an independent system. However, the interface forces cannot be measured directly unless the structure has been instrumented at the time of construction. Fortunately, the technique of dynamic load identification has been developed rapidly in the last two decades as a kind of inverse problems in dynamics (Kammer, 1998; Lu & Law 2007; Zhu & Law, 2000). If the global structure is divided into several substructures, the interface forces become external excitation forces for the substructure of concern, and they can be identified from the dynamic response of the substructures using dynamic load identification methods.

This paper addresses the problem of system identification of a structure with a large number of degrees-of-freedom and thousands of structural components. A substructuring technique is developed for the inverse problem of damage detection. It may be used in the local health monitoring of critical components without the global information of the structure. The new substructuring technique takes into account the adjacent substructures in the form of interfacing forces between substructures which are represented explicitly in the equation of motion of the substructure under studied. A time domain sensitivity-based algorithm is incorporated in the study whereby measurement from only a few sensors is required for the identification. Numerical results with a multi-storeys plane frame structure show that the identification of local damages in the form of changes in the stiffness of member could be conducted with good accuracy.

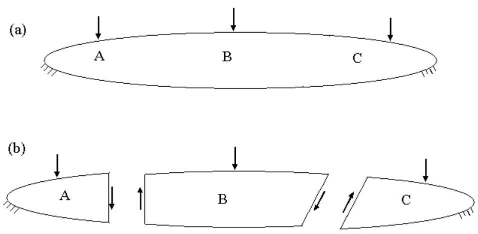

Figure 1. Sketch map for the substructuring
(a) global structure (b) substructures

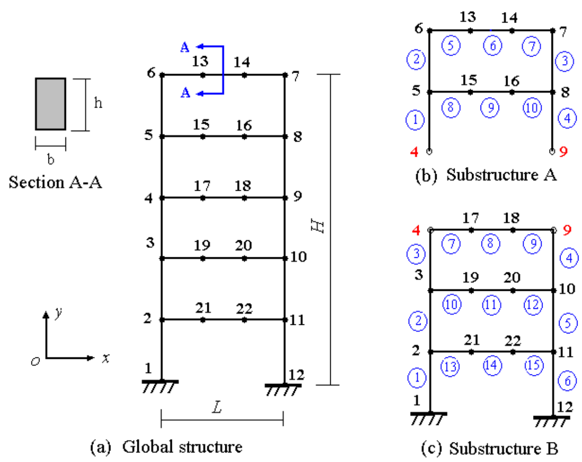

Figure 2. A five-story plane frame structure

2. SUBSTRUCTURAL APPROACH

The equation of motion of an *N*-DOFs damped substructural system with both external and support excitations as shown in Fig. 1(a) can be expressed as

$$M(\ddot{x} + G\ddot{x}_g) + C\dot{x} + Kx = L \cdot F \tag{1}$$

where M , C , K are the mass, damping and stiffness matrices of the structure respectively. Rayleigh damping is adopted which is of the form $C = a_1 M + a_2 K$, where a_1 and a_2 are constants to be determined from two modal damping ratios. L and G are the mapping matrices relating the external forces F and the support excitation $\ddot{x}_g$ to the corresponding DOFs of the structural system. $x, \dot{x}, \ddot{x}$ are respectively the nodal displacement, velocity and acceleration vectors of the structure.

Assume that the global structure is divided into three substructures *A*, *B* and *C* as shown in Fig. 1(b). All substructures can be taken as independent system excited by the interface forces, external forces and the inertia forces caused by support excitation. The equation of motion of the whole structure can be rearranged as

$$\begin{bmatrix} M_{uu}^{A} & M_{uf}^{AB} & & & \\ M_{fu}^{AB} & M_{ff}^{AB} & M_{fr}^{AB} & & \\ & M_{rf}^{AB} & M_{rr}^{B} & M_{rg}^{BC} & \\ & & M_{gr}^{BC} & M_{gg}^{BC} & M_{gv}^{BC} \\ & & & M_{vg}^{BC} & M_{vv}^{C} \end{bmatrix} \left\{ \begin{pmatrix} \ddot{x}_u^A \\ \ddot{x}_f^{AB} \\ \ddot{x}_r^B \\ \ddot{x}_g^{BC} \\ \ddot{x}_v^C \end{pmatrix} + \begin{pmatrix} G_u^A \ddot{x}_g \\ G_f^{AB} \ddot{x}_g \\ G_r^B \ddot{x}_g \\ G_g^{BC} \ddot{x}_g \\ G_v^C \ddot{x}_g \end{pmatrix} \right\} + \begin{bmatrix} C_{uu}^{A} & C_{uf}^{AB} & & & \\ C_{fu}^{AB} & C_{ff}^{AB} & C_{fr}^{AB} & & \\ & C_{rf}^{AB} & C_{rr}^{B} & C_{rg}^{BC} & \\ & & C_{gr}^{BC} & C_{gg}^{BC} & C_{gv}^{BC} \\ & & & C_{vg}^{BC} & C_{vv}^{C} \end{bmatrix} \begin{Bmatrix} \dot{x}_u^A \\ \dot{x}_f^{AB} \\ \dot{x}_r^B \\ \dot{x}_g^{BC} \\ \dot{x}_v^C \end{Bmatrix}$$

$$+ \begin{bmatrix} K_{uu}^{A} & K_{uf}^{AB} & & & \\ K_{fu}^{AB} & K_{ff}^{AB} & K_{fr}^{AB} & & \\ & K_{rf}^{AB} & K_{rr}^{B} & K_{rg}^{BC} & \\ & & K_{gr}^{BC} & K_{gg}^{BC} & K_{gv}^{BC} \\ & & & K_{vg}^{BC} & K_{vv}^{C} \end{bmatrix} \begin{Bmatrix} x_u^A \\ x_f^{AB} \\ x_r^B \\ x_g^{BC} \\ x_v^C \end{Bmatrix} = \begin{Bmatrix} L_u^A F_u^A \\ L_f^{AB} F_f^{AB} \\ L_r^B F_r^B \\ L_g^{BC} F_g^{BC} \\ L_v^C F_v^C \end{Bmatrix} \quad (2)$$

where subscripts '*u*', '*f*', '*r*', '*g*' and '*v*' denote DOFs in different sets including internal DOFs of substructure or interface DOFs between substructures, and these DOF sets are marked by superscripts '*A*', '*AB*', '*B*', '*BC*' and '*C*' respectively.

Substructure *B* is selected as the target substructure to illustrate the substructrual approach. Let *i* denotes all interface DOFs (including '*f*' and '*g*') for substructure *B*. The equation of motion of substructure *B* can be extracted from Equation (2) to yield

$$\begin{bmatrix} M_{rr}^B & M_{ri}^B \end{bmatrix} \left\{ \begin{pmatrix} \ddot{x}_r^B \\ \ddot{x}_i^B \end{pmatrix} + \begin{pmatrix} G_r^B \ddot{x}_g \\ G_i^B \ddot{x}_g \end{pmatrix} \right\} + \begin{bmatrix} C_{rr}^B & C_{ri}^B \end{bmatrix} \begin{Bmatrix} \dot{x}_r^B \\ \dot{x}_i^B \end{Bmatrix} + \begin{bmatrix} K_{rr}^B & K_{ri}^B \end{bmatrix} \begin{Bmatrix} x_r^B \\ x_i^B \end{Bmatrix} = L_r^B F_r^B \quad (3)$$

The interaction effects between the substructures are treated as 'input forces' to the substructures. These forces together with the external forces and inertia forces from earthquake excitation form the force vector in the Equation (3) which can subsequently be rewritten as

$$M_{rr}(\ddot{x}_r + G_r \ddot{x}_g) + C_{rr}\dot{x}_r + K_{rr}x_r = L_E F_E \quad (4)$$

where $F_E = F_r - M_{ri}(\ddot{x}_i + G_i \ddot{x}_g) - C_{ri}\dot{x}_i - K_{ri}x_i$ is the load vector of substructure *B* including the interface forces between adjacent substructures, and L_E is the mapping matrix relating the forces F_E to the corresponding DOFs of substructure *B*. The superscript *B* has been removed for conveinence.

2.1 The Interface Forces

Equation (4) can be rewritten for substructure *B* as

$$M^s(\ddot{x}^s + G^s \ddot{x}_g) + C^s \dot{x}^s + K^s x^s = L^s \cdot F^s \quad (5)$$

where M^s, C^s, K^s are the mass, damping and stiffness matrices of the substructure respectively. L^s and G^s are the mapping matrices relating the external forces F^s and support excitation $\ddot{x}_g$ to the corresponding DOFs of the substructural system. x^s, $\dot{x}^s$, $\ddot{x}^s$ are respectively the nodal displacement, velocity and acceleration vectors of the substructure.

By using Chebyshev polynomial approximation, the *i*th external force can be represented by

$$F^{si}(t) = \sum_{m=1}^{Nm} c_m^i T_m^i(t) \qquad (i = 1,2,\cdots, Nf\,) \tag{6}$$

where c_m^i and $T_m^i(t)$ are the orthogonal coefficients and orthogonal functions of the *i*th external force. *Nm* is the number of terms of orthogonal functions for the *i*th external force, and *Nf* is the total number of the external forces.

Substituting Equation (6) into Equation (5), we have

$$M^s(\ddot{x}^s + G^s\ddot{x}_g) + C^s\dot{x}^s + K^s x^s = L^s \cdot \left(\sum_{i=1}^{Nf}\sum_{m=1}^{Nm} c_m^i T_m^i(t) \right) \tag{7}$$

where the orthogonal coefficients c_m^i are taken as the unknown force parameters to be identified in the inverse problem.

2.2 *The Sensitivities*

Assuming the local structural damages are in the form of $\Delta K^s = \sum_{i=1}^{Ne} \alpha_i k_i^e$ with $0 \le \alpha_i \le 1.0$, where α_i is the fractional change in the stiffness k_i^e of an element and N_e is the number of elements in the substructure. Performing differentiation to both sides of Equation (7) with respect to the unknown parameters of forces and the physical parameters of structures respectively, we have

$$M^s \frac{\partial \ddot{x}^s}{\partial c_m^i} + C^s \frac{\partial \dot{x}^s}{\partial c_m^i} + K^s \frac{\partial x^s}{\partial c_m^i} = L^s \cdot T_m^i(t) \tag{8}$$

$$M^s \frac{\partial \ddot{x}^s}{\partial \alpha_i} + C^s \frac{\partial \dot{x}^s}{\partial \alpha_i} + K^s \frac{\partial x^s}{\partial \alpha_i} = -\frac{\partial K^s}{\partial \alpha_i} x^s - a_2 \frac{\partial K^s}{\partial \alpha_i} \dot{x}^s \tag{9}$$

Responses x^s, $\dot{x}^s$ are obtained from Equation (5). The right-hand side of Equations (8) and (9) can be considered as equivalent forcing functions. Therefore the sensitivities $\frac{\partial \ddot{x}^s}{\partial c_m^i}, \frac{\partial \dot{x}^s}{\partial c_m^i}, \frac{\partial x^s}{\partial c_m^i}, \frac{\partial \ddot{x}^s}{\partial \alpha_i}, \frac{\partial \dot{x}^s}{\partial \alpha_i}$ and $\frac{\partial x^s}{\partial \alpha_i}$ can be calculated from Equations (8) and (9) using the Newmark method.

2.3 *Identification of the Forces and Damages*

When accelerations are used, the identification equation can be expressed as

$$\ddot{x}_d^s - \ddot{x}^s = [S_F \quad S_K] \begin{Bmatrix} c \\ \alpha \end{Bmatrix} \tag{10}$$

where $\ddot{x}_d^s$ and $\ddot{x}^s$ are respectively the measured and calculated acceleration responses.

$S_F = \left[\frac{\partial \ddot{x}^s}{\partial c_1^1} \quad \frac{\partial \ddot{x}^s}{\partial c_2^1} \quad \cdots \quad \frac{\partial \ddot{x}^s}{\partial c_{Nm}^{Nf}} \right]$ and $S_K = \left[\frac{\partial \ddot{x}^s}{\partial \alpha_1} \quad \frac{\partial \ddot{x}^s}{\partial \alpha_2} \quad \cdots \quad \frac{\partial \ddot{x}^s}{\partial \alpha_{Ne}} \right]$ are the sensitivity matrices of the dynamic response to the orthogonal parameters of external forces $c = \left(c_1^1 \quad c_2^1 \quad \cdots \quad c_{Nm}^{Np} \right)^T$ and the physical parameters of the structure $\alpha = \left(\alpha_1 \quad \alpha_2 \quad \cdots \quad \alpha_{Ne} \right)^T$ where the superscript T denotes the transpose of the matrix. Equation (10) can be rewritten in a more concise form as

$$\delta\ddot{x}^s = S\beta \tag{11}$$

where $\delta\ddot{x}^s = \ddot{x}_d^s - \ddot{x}^s$, $S = [S_F \quad S_K]$ and $\beta = (c \quad \alpha)^T$.

The unknown parameters can be calculated from Equation (11) by the straightforward least-squares methods, but this is an ill-posed problem like many other inverse problems. In order to provide bounds to the solution, the damped least-squares method (DLS) proposed by Tikhonov (1963) is used to solve the inverse problem. Equation (11) can be written in the following form in the DLS method as

$$[S^T S + \lambda I]\beta = S^T \delta\ddot{x}^s \tag{12}$$

where λ is the non-negative damping coefficient governing the participation of least-squares error in the solution. For $\lambda > 0$, the matrix operator $[S^T S + \lambda I]$ is mathematically unique, and its inverse is continuous in the pseudo-inverse solution of Equation (12). When the parameter λ equals zero, the estimated parameter vector β approaches to the solution obtained from the straightforward least-squares methods. An L-curve method (Hansen, 1992) is used in this study to obtain the optimal regularization parameter λ_{opt}. The convergence of this solution has been proved (Chen & Li, 2004).

A two-stage identification strategy is adopted. When measurements from the intact state of a structure are obtained, the physical parameters together with the force parameters are firstly identified from Equation (11) based on the initial analytical model of the structure. The initial analytical model is then updated and the sensitivities of structural responses to both force parameters and physical parameters are again computed for the next iteration. Convergence is considered to be achieved when $\|\beta_{k+1} - \beta_k\| / \|\beta_{k+1}\| \le T_{olerance}$ is met, where k denotes the kth iteration. When measurement from the damaged state is obtained, the updated analytical model is used in the iteration in the same way as that using measurement from the intact state. The final vector of physical parameter increments corresponds to the local changes between the two states of the structure, and the identified force parameters can be used to calculate the external forces from Equation (6). The convergence criteria $T_{olerance}$ is set equal to 1.0×10^{-6} in this study.

3. NUMERICAL SIMULATIONS

A five-storeys plane frame structure as shown in Fig. 2(a) is studied to illustrate the proposed method. It consists of 22 nodes each with three DOFs. The frame is fixed at nodes 1 and 12 which are modeled with large translational and rotational stiffnesses of 1.5×10^{12} N/m and 1.5×10^{12} N·m/rad respectively. The height and width of the frame are respectively 15m and 6m, and the cross-sectional dimensions of members are b=0.3m and h=0.6m with the second moment of inertia in the plane of bending equals 0.0054 m^4. The mass density of material is

2.7×10^3 kg/m, and the elastic modulus of material is 33GPa. The first twelve undamped natural frequencies of the intact frame range from 3.923 Hz to 75.638 Hz. Rayleigh damping model is adopted with damping ratios of the first two modes taken equals to 0.01. The equivalent Rayleigh coefficients a_1 and a_2 are 0.3779 and 1.8902×10^{-4} respectively.

The structure is subject to the east-west El-centro earthquake acceleration with a reduction coefficient of 0.286 corresponding to a site with seven degree seismic intensity. The reduced ground acceleration acts on the two supports along the *x*-axis. The responses of the system are calculated using a time increment of 0.004 second, and the earthquake data is interpolated to have the same time increment, namely, with a sampling frequency of 250Hz.

The frame is divided into two substructures *A* and *B*, connected in six interface DOFs at nodes *4* and *9*, as shown in Figs. 2(b) and 2(c). The six interface forces at these interface DOFs become external excitation for each substructure. Substructure *A* is taken as an example to validate the proposed method. Error in the identified interface forces can be defined as the relative percentage errors (RPE) between the identified F^e and true interface forces F^{tr} as

$$e_F = \frac{\left\| F^e - F^{tr} \right\|}{\left\| F^{tr} \right\|} \times 100\% \qquad (15)$$

where $\|\bullet\|$ is the norm of vector. The interfacing forces are modelled with 760 terms of orthogonal functions.

Table 1. Simulated damage scenarios

Damage scenarios	Damage elements	Damage extent	Noise level	Model error
1	6th	5%	no	no
2	5th, 6th	5%, 10%	no	no
3	1st, 5th, 9th	10%, 5%, 10%	no	no
4	5th, 6th	5%, 10%	5%	no
5	5th, 6th	5%, 10%	no	3% reduction of stiffness in all elements
6	1st, 5th, 6th	5%, 5%, 10%	10%	5% reduction of stiffness in all elements

3.1 Validation of the Proposed Method

The first three damage scenarios without noise and model errors listed in Table 1 are firstly studied to validate the accuracy of the proposed method. The transverse and vertical accelerations at nodes *4*, *5*, *8* and *9* are recorded for a period of 2 seconds after the excitation starts. Results shown in Figs. 3(a) to 3(c) show that the proposed method can accurately identify local damages in the concerned substructure without considering any information outside the target substructure. The errors listed in the columns 2 to 4 of Table 2 indicate that the six interface forces can be accurately identified for each of these damage scenarios.

3.2 Effect of Measurement noise and Model Errors

The acceleration response is assumed to suffer from the measurement noise modeled with white noise added to the calculated responses to simulate the polluted measurements. Damage scenario 4 as listed in Table 1 is studied with 5% noise in the measured responses. The presence of noise seems not greatly affect the identified result on the damage element as shown in Fig. 3(d). Results not shown indicate that a minimum of eight acceleration responses is necessary in the simultaneous identification of structural physical parameters and the six interface forces.

The errors in the identified interface forces with 5% noise, listed in the fifth column of Table 2, however, show that measurement noise has large effect on the identified forces. The errors of the transverse forces and the torsional moments are larger than those of the vertical forces.

Table 2. Errors of identified results of interface forces for different damage scenarios

	Damage Scenarios	1	2	3	4	5	6
Node 4	Transverse force	4.91×10^{-7}	1.36×10^{-6}	3.77×10^{-6}	13.43	5.51×10^{-6}	28.04
	Vertical force	5.26×10^{-7}	4.99×10^{-6}	1.02×10^{-6}	1.20	4.85×10^{-6}	2.41
	Torsional moment	1.35×10^{-6}	4.86×10^{-6}	8.40×10^{-6}	26.84	1.43×10^{-5}	46.33
Node 9	Transverse force	3.59×10^{-7}	1.21×10^{-6}	3.84×10^{-6}	13.45	3.65×10^{-6}	27.57
	Vertical force	4.26×10^{-7}	5.13×10^{-6}	1.06×10^{-6}	1.20	4.68×10^{-6}	2.40
	Torsional moment	7.91×10^{-6}	2.21×10^{-6}	6.32×10^{-6}	20.62	8.44×10^{-6}	42.77

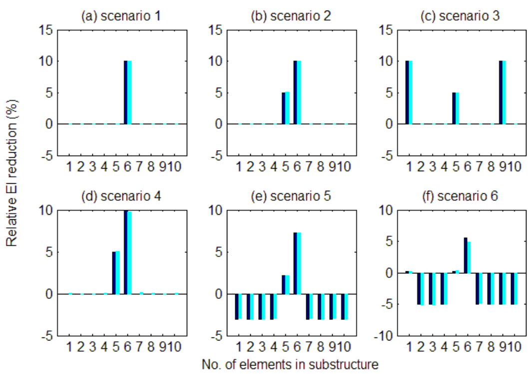

Figure 3 - Identified results for damage scenarios 1-6 in substructure *A* (true identified)

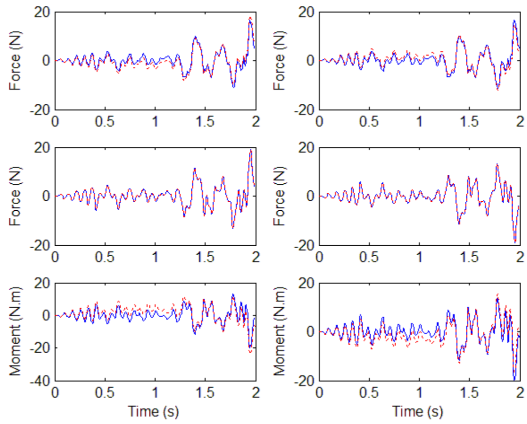

Figure 4. Six interface forces between substructures *A* and *B* for scenario 6 (calculated identified with 10% noise)

Scenario 5 includes a model error of 3% under-estimation in the elastic modulus of all element of the substructure in the FEM plus two adjacent damages without noise in the measured accelerations. The same measured acceleration responses for the above studies are used. Fig. 3(e) shows that the identified results are very accurate with an increase of approximately 3% in elements 1 to 4 and 7 to 10, and a reduction of 2% and 7% for the damaged elements 5 and 6. The identified 2% and 7% reductions are relative to the original elastic modulus without under-estimation. The interface forces can be very accurately identified for this case as shown in the 6th column in Table 2.

The coupling effect of model errors and measurement noise on the identified results are studied in Scenario 6 with a bigger model error of 5% under-estimation in the elastic modulus of all element of the substructure in the FEM plus two adjacent damages and one local damage adjacent to the interface with 10% noise in the measured accelerations. The same measured accelerations for the above studies are used. The identified damages shown in Fig. 3(f) show that the decrease of approximately 5% in elements 2 to 4 and 7 to 10, and an increase of approximately zero, zero and 5% for the damaged elements 1, 5 and 6. These results are noted to be relative to the original model with 5% under-estimation and are relatively insensitive to noise. The errors in the identified interface forces are slightly larger as listed in the 7th column of Table 2. However, a comparison of the identified interface forces with the true forces in Fig. 4 shows that the identified interface forces can still reflect the true forces except in a period in the middle of the force time histories.

4. CONCLUSIONS

A new substructural analysis method is proposed for the identification of local damages whereby both the interface forces of a substructure and its local damages are identified simultaneously. Only a finite element model of the selected substructure is necessary without the inclusion of other parts of the structure. This approach may be used for the monitoring of certain critical components of a large structure without the inclusion of the full structural model in the inverse identification. The presence of measurement noise has some effect on the identified interface forces. The identified results of local damages are insensitive to both measurement noise and initial model errors. All these features of the method are made possible with the fact that the problem of identification is now confined to a small portion of the structure with a small number of components.

ACKNOWLEDGEMENTS

The work described in this paper was supported by a research grant from the Hong Kong Polytechnic University.

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