Damage Detection in Beam-like Structures Using Deflections Obtained by Modal Flexibility Matrices

Ki Young Koo[1], James M.W. Brownjohn[1], and Chung-Bang Yun[2]

[1] Dept. of Civil & Structural Engineering, University of Sheffield, Sheffield, UK
[2] Dept. of Civil & Environmental Engineering, KAIST, Daejeon, Korea

ABSTRACT: In bridge structures, damage may induce an additional deflection which may naturally contain essential information about the damage. However, inverse mapping from the damage-induced deflection to the actual damage location and severity is generally complex, particularly for statically indeterminate systems. In this paper, a new load concept, called the positive-bending-inspection-load (PBIL) is proposed to construct a simple inverse mapping from the damage-induced deflection to the actual damage location. A PBIL for an inspection region is defined as a load or a system of load which guarantees bending moment to be positive in the inspection region. From the theoretical investigations, it was proven that the damage-induced chord-wise deflection (DI-CD) has the maximum value with the abrupt change in its slope at the damage location under a PBIL. Hence, a novel damage localization method is proposed based on the DI-CD under a PBIL. Experimental verification study was also carried out on a two-span continuous steel box-girder model with a steel box-girder. It was found that the proposed method clearly identified the damage existence and damage location for small damage cases with narrow cuts at the bottom flange.

1 INTRODUCTION

Structural health monitoring (SHM) is an emerging subject in civil engineering, offering potential to prevent catastrophic structural failures, to increase cost effectiveness in maintenance, and to prolong the service life by continuous and/or periodic assessments of the structural integrity of civil infrastructures. During the past three decades, vibration-based SHM methods have been extensively studied and developed utilizing various vibration characteristics such as natural frequencies, mode shapes, modal curvatures, modal flexibility, etc. An excellent review can be found in a report by Doebling et al. (1996). In spites of in-depth research efforts in vibration-based SHM and damage detection methods, there still remain several challenges to be resolved for practical and reliable applications. Especially, from the viewpoint of online SHM frameworks, it is very important to detect damage or abnormality at the earliest stage to establish proper maintenance and/or retrofit plans on time. Therefore, it is highly required to adopt an appropriate damage detection method for the continuously or periodically incoming measurement data. More research efforts are also needed to enhance the applicability of the SHM technologies to real structure systems with a large number of degrees of freedom.

Recently, the modal flexibility has become one of the promising damage descriptors owing to its high sensitivity to damage. Through numerical studies for a spring-mass system with five degrees of freedom, the modal flexibility was shown to be more sensitive for damage detection than the natural frequencies and mode shapes Zhao et al.(1999). Several damage detection methods have been proposed based on the modal flexibility (Pandey and Biswas (1994), Bernal (2002), Toksoy and Aktan (1994), Zhang and Aktan (1995) and Ni et al. (2008)). The methods have a few drawbacks such as the high noise sensitivity due to curvature calculations (Zhang and Aktan (1995), the no explicit relationship between damage and damage features (Pandey and Biswas (1994)), and the requirement of an intact finite element model (Bernal (2002)). In this study, the proposed method originated from the same idea with the method (Toksoy and Aktan (1994)), but is based on the explicit relationship between the damage and damage-induced deflection. The load requirement for deflection-based damage detection is also investigated theoretically.

In this paper, at first, the background theory is explained including the modal flexibility, the deflection estimation by the modal flexibility, the Positive-Bending-Inspection-Load (PBIL), and the deflection-based damage localization method. Then, experimental verification studies were shown for a two-span continuous steel box-girder model. Finally, the conclusion is followed.

2 THEORY

2.1 Deflection Estimation by Modal Flexibility

The dynamic characteristic equation for a structure can be written as

$$\mathbf{M\Psi\Lambda = K\Psi} \tag{1}$$

where $\mathbf{M}$ and $\mathbf{K}$ are the mass and stiffness matrices; $\mathbf{\Psi}$ is the un-scaled mode shape matrix which may be obtained by the output-only modal analysis (Brinker and Andersen 2002); and $\mathbf{\Lambda}$ is the modal frequency matrix with ω_i^2 on its diagonal. When the un-scaled mode shape matrix $\mathbf{\Psi}$ is scaled to the mass-normalized mode shape matrix $\mathbf{\Phi}$ ($\mathbf{\Phi}^T\mathbf{M\Phi} = \mathbf{I}$), the stiffness matrix $\mathbf{K}$ and the flexibility matrix $\mathbf{G}$ can be expressed as

$$\mathbf{K} = \mathbf{\Phi}^{-T}\mathbf{\Lambda\Phi}^{-1} \tag{2}$$

$$\mathbf{G} = \mathbf{\Phi\Lambda}^{-1}\mathbf{\Phi}^T \tag{3}$$

The influence of the ith mode on the stiffness matrix $\mathbf{K}$ increases with the squares of the natural frequencies ω_i^2, whereas the influence on the flexibility matrix $\mathbf{G}$ decreases with ω_i^{-2}. Therefore, the flexibility matrix can be constructed more accurately by using only a few lower modes than the stiffness matrix (Gao, Spencer et al. 2006). The modal flexibility matrix $\mathbf{G}_m$ using m-lower modes can be obtained as

$$\mathbf{G}_m = \mathbf{\Phi}_m\mathbf{\Lambda}_m^{-1}\mathbf{\Phi}_m^T \tag{4}$$

where $\mathbf{\Lambda}_m = \left[\begin{smallmatrix}\omega_1^2 & & \\ & \ddots & \\ & & \omega_m^2\end{smallmatrix}\right]$, ω_i is the ith natural frequency, $i = 1, \cdots, m$; $\mathbf{\Phi}_m = [\phi_1 \cdots \phi_m]$; and ϕ_i is the ith mode shape with the mass-normalization which can be carried out by added mass methods (Brinker and Andersen 2002; Bernal 2004).

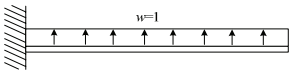
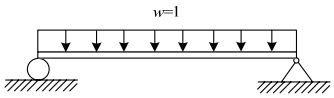
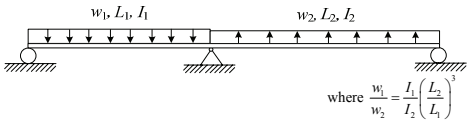
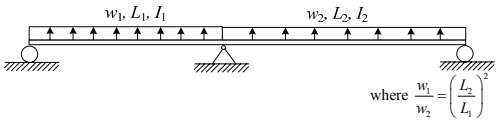
When the modal flexibility matrix is evaluated using Eq. (4), any deflection profile under an arbitrary load $\mathbf{f}$ can be estimated by a simple matrix multiplication as

$$\mathbf{u} = \mathbf{G}_m \mathbf{f} \tag{5}$$

2.2 *Positive Bending Inspection Load*

The inflection point is the point where the curvature of the deflection and the bending moment are zero. It is worthy to note that damage near an inflection point may not induce significant additional deflection to the beam structure because the bending moment on the damaged portion is negligibly small. Therefore, it is almost impossible to detect damages near the inflection points using the damage-induced deflection. To detect the damage in a region of interest, a load or a system of loads producing no inflection point in the region is desirable. For this end, a positive-bending-inspection-load (PBIL) is proposed in this study, which guarantees the positive bending moment in the inspection region, so that no inflection point may occur in that region as shown in Table 1.

Table 1. Examples of PBILs for various inspection regions.

Beam Types	Inspection Regions Ω	PBILs	Definitions of Inspection Loads
Cantilever Beam	Whole structure	$f^{1,*)}$	(f^1) 
Simply Supported Beam	Whole structure	f^1	(f^1) 
2-Span Continuous Beam	1st span-region 2nd span-region	f^1 $-f^1$	(f^1) 
	Intermediate support-region**)	$f^{2,*)}$	(f^2) 

*) f^1 and f^2: Span-region inspection load and intermediate support-region inspection load.

**) Intermediate support-region may cover the 1/4 span regions on both sides of the support

2.3 *Damage localization By Damage-Induced Deflection.*

To make the damage localization procedure as simple as possible, a damage-induced chord-wise deflection (DI-CD) $\Delta u_\Omega(x)$ is introduced, which can be defined as the additional deflection due to damage measured along with the chord connecting two points at $x = x_a$ and x_b defined as

$$\Delta u_{\Omega}(x) = \Delta u(x) - \left(\frac{\Delta u(x_b) - \Delta u(x_a)}{x_b - x_a}(x - x_a) + \Delta u(x_a) \right) \tag{6}$$

where $\Omega = [x_a, x_b]$ is the inspection region.

From the theoretical investigation on the DI-CD, the damage localization algorithm can be established as follows.

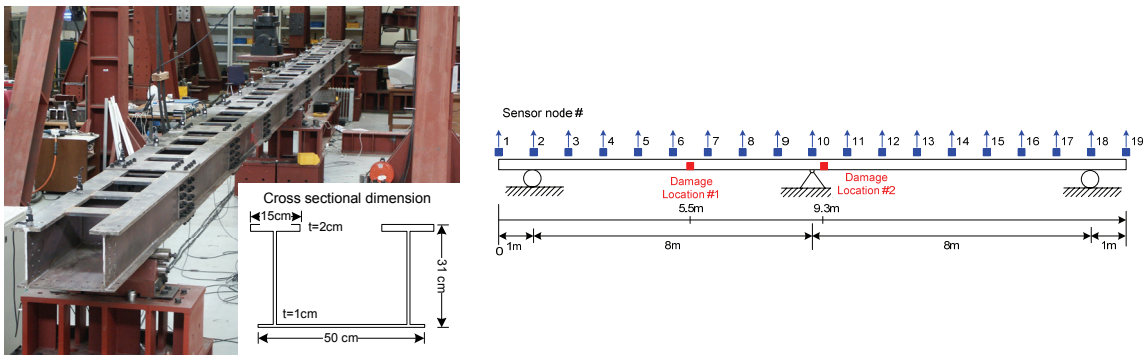
$$\text{Damage occurs at } x = x_D \Leftrightarrow \begin{cases} \Delta u_{\Omega}(x) \text{ has the maximum at } x = x_D \text{ and} \\ \Delta u'_{\Omega}(x) \text{ changes abruptly at } x = x_D \text{ under a PBIL} \end{cases} \tag{7}$$

where a PBIL is a positive-bending-inspection-load in the inspection region Ω which guarantees the moment positiveness in Ω, as defined in the previous section and in Table 1.

Eq. (11) states that the damage can be localized at the location with the maximum DI-CD and the abrupt change in its slope. The proof of Eq. (7) is found in (Koo, Lee et al. 2008).

3 EXPERIMENTAL STUDY

Experimentally validation of the proposed method was carried out on a steel box-girder model shown in Fig. 1. The test model is a two-span continuous box-girder model of 18m long, which is composed of 9 segmental boxes connected by bolts and plates. Detailed dimensions of the test model were given at Fig. 1-(a). Nineteen accelerometers were evenly placed on the upper surface of one side of the box-girder as shown in Fig. 1-(a) and (b). Small damages of narrow cuts were imposed at two locations on the bottom flange by saw cuts as described in Tables 2 and 3. Vibration responses were induced by successive impact loads during 4 minutes and the acceleration responses were measured with 200 Hz sampling rate. The acceleration measurements were repeated 18 times for the intact case and 7 times for each damage case. The experiments were performed during about 6 hours when the temperature remains nearly steady in order to make temperature effects negligible.



(a) Overview of test set-up (b) Accelerometer layout and damage locations

Fig. 1 Laboratory test on a steel-box girder model

Modal parameters were extracted using the enhanced frequency domain decomposition (EFDD) (Brincker, Zhang et al. 2001) which can estimate natural frequencies accurately using curve fitting in time domain. Modal flexibility matrices were constructed using three lower mode shapes and natural frequencies. Deflections were estimated using Eq. (5) with the PBILs f_1 and f_2 as shown in

Table 4. The eighteen intact deflections were scaled with a constant so that the absolute maximum value is to be 1.0 and were shown in Fig. 2. The eighteen deflections almost coincide with each other and the deviation levels are less than $\pm 0.8 \times 10^{-3}$, which indicates the excellent repeatability of the estimated deflections. These repeatable deflection estimations contribute to high probability of damage detection.

Table 2. Damage severity on test section

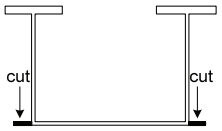
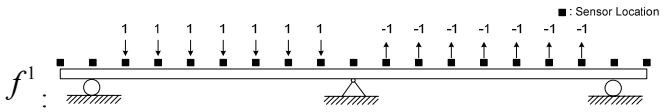
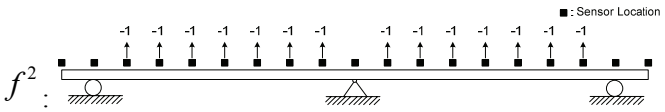
Item	Description
Section Shape	
Description	a cut of 5.0cm long and 1mm wide on each side
Bending Rigidity (EI)	12% Reduction in the cut-section
Area	4.6% Reduction

Table 3. Damage scenarios

Damage Cases	Damage Scenario
Damage 1	at Location 1
Damage 2	at Location 1 and 2

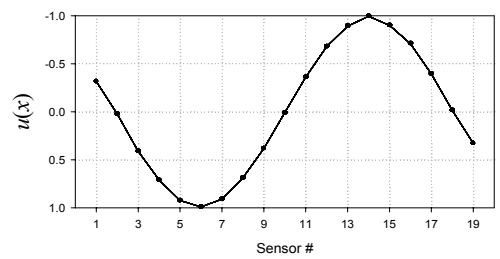
Table 4. PBILs for test model

Inspection Regions	PBILs	Definition of PBIL
1st span-region	f^1	
2st span-region	$-f^1$	
Intermediate support-region	f^2	

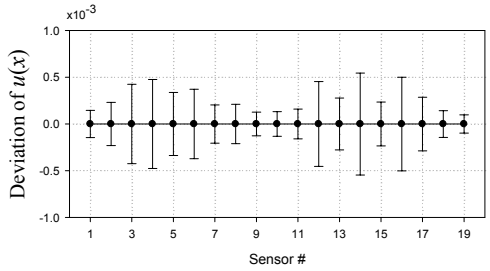
For Damage Cases 1 and 2, the DI-CD in each inspection region are shown in Figures 3 and 4. For Damage Case 1 with a small narrow cut at $x_D = 5.5$ in the 1st span, the DI-CD in the 1st span increased very significantly with an abrupt change in the slope near the damage location, while small change occurred in the other two regions without damages as shown in Fig. 3.

For Damage Case 2 with an additional small cut near the intermediate support, very visible increase can be observed in the DI-CD under the PBIL in the intermediate support region, while no significant increases in the other regions without damages.

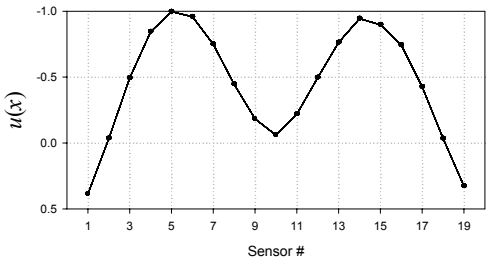
In summary, it was found that the damage locations were successfully localized by Eq. (7) for the two damage locations in the intermediate support and the 1st span region.



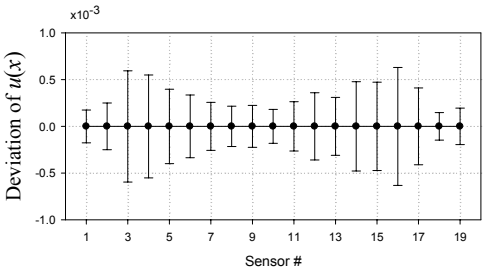
(a) Intact deflections under f^1



(b) Deviation of intact deflections under f^1



(c) Intact deflections under f^2



(d) Deviation of intact deflections under f^2

Fig. 2 Eighteen sets of the intact deflections and their deviation level under PBILs

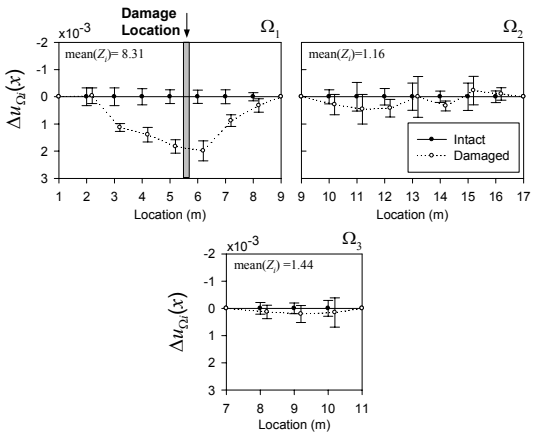


Fig. 3 Means and 2σ deviations in the DI-CDs for Damage Case 1 from Intact Case

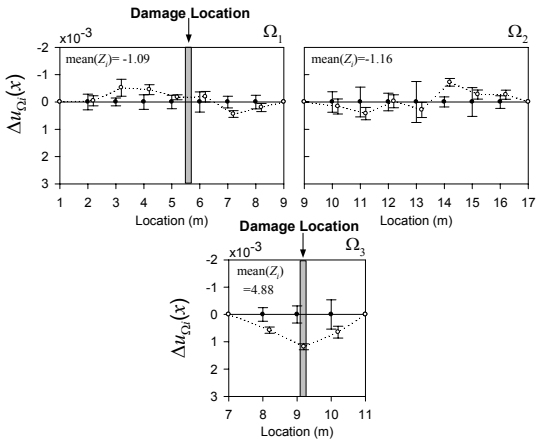


Fig. 4 Means and 2σ deviations in the DI-CDs for Damage Case 2 from Damage Case 1

4 CONCLUSION

In this paper, a novel damage detection method was proposed based on the damage-induced chord-wise deflections which were estimated using the modal flexibility matrices before and after damage.

A new load concept, so called Positive Bending Inspection Load (PBIL) was proposed to simplify the proposed damage detection and localization procedures A PBIL in an inspection region is defined as a load or a system of loads which guarantees positive bending moment in the inspection region of a structure. The relationship between the damage location and the damage-induced chord-wise deflection (DI-CD) was theoretically investigated, and it was proved analytically that the DI-CD has the maximum value at the true damage location with the abrupt change in its slope under a PBIL. Hence, a novel damage detection method was proposed based on the DI-CD estimated by the modal flexibility matrices before and after damage and the PBILs designed for a specific structure.

Experimental verifications were carried out for a two-span continuous steel-box girder model. It has been founded that the proposed method can detect small narrow cuts inflicted on the bottom flange which caused 12% reduction in the bending rigidity (EI) on the narrow segment of the box-girder.

5 REFERENCES

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