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Optimal Placement of Sensors for Structural Health Monitoring of Buildings under Seismic Actions

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ABSTRACT: Optimal Placement of sensors in a structure is essential for accurately capturing its response to the seismic actions and identifying the system parameters. A simplified methodology using Covariance Matrix Representation and Parametric Model has been proposed for that purpose. The response of the frame is recorded by using a predefined number of sensors at different story level and the difference of the measurement of the response from the ideal condition is evaluated. A set of five and ten story steel moment resisting building frames located in Vancouver, Canada, and a five storey reinforced concrete hypothetical building have been used in this study. The building frames are analyzed for a set of earthquake ground motion records to generate the floor acceleration response, which is considered as the target response of the building to be recorded for the evaluation of the performance of different combination of the sensors. The study shows that the proposed method is viable and practical.

1 INTRODUCTION

The response of structure to the earthquake is an uncertain phenomenon. So, it is very difficult to record of response of the structure accurately. The accuracy of recorded response depends on the appropriateness of the methodology used in capturing the response. Therefore, the seismic analysis/design of a structure is a probabilistic approach. There are different approaches to evaluate the seismic behavior such as Laboratory testing, computerized analysis and Natural laboratory of the earth (Celibi, 2000). In the evaluation approach the modeling of the parameters is very important usually the mathematical models are used in the prediction of the response of the structure. Therefore, the structure may not behave under seismic action as it supposes to be. The monitoring of structure under seismic force can provide some real design parameters of the structure. Placements of sensors are necessary in the different locations of the structure to collect the information about the structure's response to the seismic action. These information can be used later to evaluate the level of performance of the structure under seismic action. In selection of the appropriate location of the sensor to evaluate the capabilities of the structure the optimal instrumentation methodology can provides important information (Yuen et al, 2001).

The factors influencing the selection methodology for the optimal location of sensors include (a) the stiffness of the system, (b) nature of parameter of the structure to be specified (c) availability of the number of sensors (d) length of time over which the recording of response would be done. In placing the sensors it is also important to place the sensors at the relevant degree of freedom of the structure which maximize the measure of the collected information of the recorded data.

2 METHODOLOGY

In the methodology for optimizing the locations of sensors, the selection of a model for response measurement is very important. Each model has several uncertainties in their parameter. The optimal sensor location minimizes the uncertainty of the model parameters (Yuen et al, 2001). The reliability of the measured response of the structure depends on the model used in the methodology of response determination. Although various type of models are available to identify the parameters that can used in the determination of optimal sensor location, the Covariance Matrix Representation and Parametric Models using target acceleration response are selected for this study. The response data of the structure depends on the characteristics of the instrument and the position of the instrument in the structure (Udwadia, 1994). To study the response at ideal condition the response of the frame is recorded by placing the sensors at all story level. But as the target of the study is to find the optimize location of the sensors, the response of the frame is recorded by using two sensors at different story level and variation of the measurement of the response with ideal condition is evaluated.

2.1 Covariance Matrix Representation

When two datasets vary together than these two datasets is called as covariance. The covariance describes the variability of the product of the averages of the deviation of the data points around the mean value of the dataset. Therefore, the covariance matrix is nothing but collection of several covariances and presented in the form of BxB matrix i.e. equal number of row and column. The dispersion of the variables around their mean is measured by the variance of the variable. The element of a covariance matrix can be defined by the Equation 1 (Datta *et al*, 2001).

$$C_{ij} = \frac{1}{N-1} \sum_{k=1}^{N} (a_i(k) - \mu_i)\ (a_j(k) - \mu_j) \tag{1}$$

Where, i,j =1,2,3, the number of element, N is the total number of data points, a_i and a_j denote the two orthogonal components of acceleration response, μ_i and μ_j is the temporal mean of the ith and jth component of the response. Shah and Udwadia (1978) used the approach of covariance matrix to the problem of optimal sensor location in the structure; they used the locations in the structure where a positive definite scalar norm of the covariance matrix of the parameter estimates is minimized for a given number of sensors. Following this approach, the optimal location of two sensors for identifying stiffness parameters of a shear building was found to be at base and at the floor immediately above. However, the reliability of these parameters cannot be determined unless additional information a *priori* probability distribution of the unknown parameters is provided. A new methodology has been proposed by Udwadia (1994) for optimum sensor locations based on the concept of efficient estimators. For such an estimator, e.g. the maximum likelihood estimator, the covariance of the parameter estimates is a minimum.

2.2 Parametric Model

The parametric model deals with various structural parameters that are collected by locating the instrument in the different place of the structure. Parametric model is mainly a mathematical procedure based on the statistical hypothesis where the principle of probability distribution is applied. Bayesian theory is a good statistical approach that can be used in the parametric modeling. The parametric methods of time series modeling can also be used to model a large variety of physical phenomena. These models are developed on the assumption of a *finite* memory of the system so that the response of the system to an excitation can be expressed in terms of a linear difference equation involving the present and some past values of the system response, input excitation, and a random noise. Depending on the way the noise is represented in the model two different types of model structures are possible, *viz.*, (*i*) the equation error model, and (*ii*) the output error model. A generic parametric model incorporating both of these types of model structures is shown in the Equation 2 as described by Safak and Celibi (1991)

$$[A(q)]\{y(t)\} = [F(q)]^{-1}[B(q)]\{u(t)\} + [D(q)]^{-1}[C(q)]\{e(t)\} \tag{2}$$

where, [A], [B], [C], [D], and [F] are matrices of polynomials of the "shift operator" q, and {$y(t)$}, {$u(t)$}, and {$e(t)$} denote the vectors of output, input data and random noise, respectively.

Table 1: Some commonly used parametric models

POLYNOMIAL TERMS	NAME OF MODEL STRUCTURE
B	FIR
A, B	ARX
A, B, C	ARMAX
A, C	ARMA
A, B, D	ARARX
A, B, C, D	ARARMAX
B, F	OE
B, F, C, D	BJ

Some of the commonly used parametric models which can be derived from the generic model of Eq. (1) are shown in Table 1. The first six of the above mentioned models can be grouped together as the *equation error* model structure because the noise term *e(t)* is used to model the error in linear difference equation. The last two models correspond to the *output error* family of model structures. It may be noted that the output error (OE) model can be developed as a special case of Box-Jenkins (BJ) model. The primary difference between equation error and output error model structures is the way error is modeled. In the equation error model structures, the white noise error term is assumed to go through the dynamics of the system, while in the output error models the noise term is directly added to the output of the system.

In these models, FIR represents the Finite Impulse Response of a system to external excitation, and being the impulse response function it is independent of all the previous output values. In ARX model the system response at time *t* is predicted by means of a linear regression on a finite history of the response of system (auto-regression, AR) and also on the instantaneous and some past values of the external input. Further, the error in the equation is directly modeled as a white noise process. In ARMAX model, the equation error is modeled as the moving average (MA) of white noise and other structure remains same as that for ARX model. The auto-regression (AR) of the equation error of a ARX model is directly taken as white noise in the case of ARARX model. Similarly, the auto-regression of the equation error of an ARMAX model is modeled as white noise in the ARARMAX structure. In ARMA models, the instantaneous response of the system is predicted using a finite set of previous response history of the system and the equation error is modeled as the moving average of white noise. The ARX model turns out to be the simplest of all in the family of equation error model structures.

In the development of parametric models for a pair of input-output data, it is desirable to include minimum number of parameters for adequate representation. The use of more sophisticated model structures, as mentioned in Table1, can be justified if the mechanics of the dynamical system to be modeled is known. In the absence of sufficient information about the system dynamics, however, simple model structures are preferable. Out of all possible model structures, the ARX and OE structures require the smallest number of parameters. Further, the ARX model involves solution of a linear regression problem for estimation of model parameters. On the other hand, OE model structure leads to a non-linear regression problem which may be difficult to solve in the case of identification of multiple input – multiple output (MIMO) systems. Since reliable estimates of the parameters of ARX model can be obtained with little computational effort it is deemed to be suitable for use in identification of MIMO systems.

3 EXAMPLE APPLICATIONS

3.1 Five-story reinforced concrete building

A hypothetical building is assumed to be subjected by real type earthquake ground excitation. The beams and columns cross-sections of the shear frame building (see Fig. 1) are 300 mm by 300 mm. The slab thickness is 150mm. The crushing strength of concrete is 20 MPa, and the yield strength of steel is 415 MPa. All other details are as per standard design and practice in India. The response records are generated for three dimensional Finite Element Model due to excitation of the Michoacan Earthquake of 19 Sept., 1985 in Mexico recorded at CDAO station, 400 km from the epicentre, shown in Fig.2 (Datta et al., 2000). The ground excitation is proportionately scaled up (eight and half times the original time histories) to compute the non-linear system response. The accelerometers are assumed to be located at the centre of the diaphragm as shown in Fig. 1 and the diaphragm is considered to be rigid.

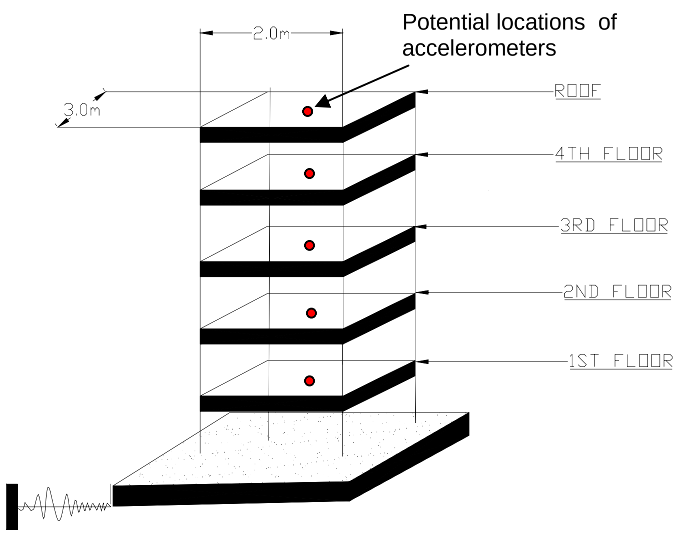

Figure 1 : Five-Storeyed R.C. Building of Storey Height 3.0m each

For each component of motion, the response time history is obtained by concatenating the response of all floors together. The 3x3 covariant matrix, C computed from these time histories is considered to be exact representation of the zero-mean Gaussian random process characterizing the building response. For finding the optimal location of sensors using two triaxial aaccelerometer placed at two floors, all possible combinations as shown in Table 2 are considered. The approximate covariance matrix has been computed for each of these combinations, and the determinant and Trace of the covariance martix are listed in Table 2. It is noted from Table 2 that the records from the two sensors located at either the first floor and roof, or the second floor and the fourth floor provide the best approximation for the covariance matrix.

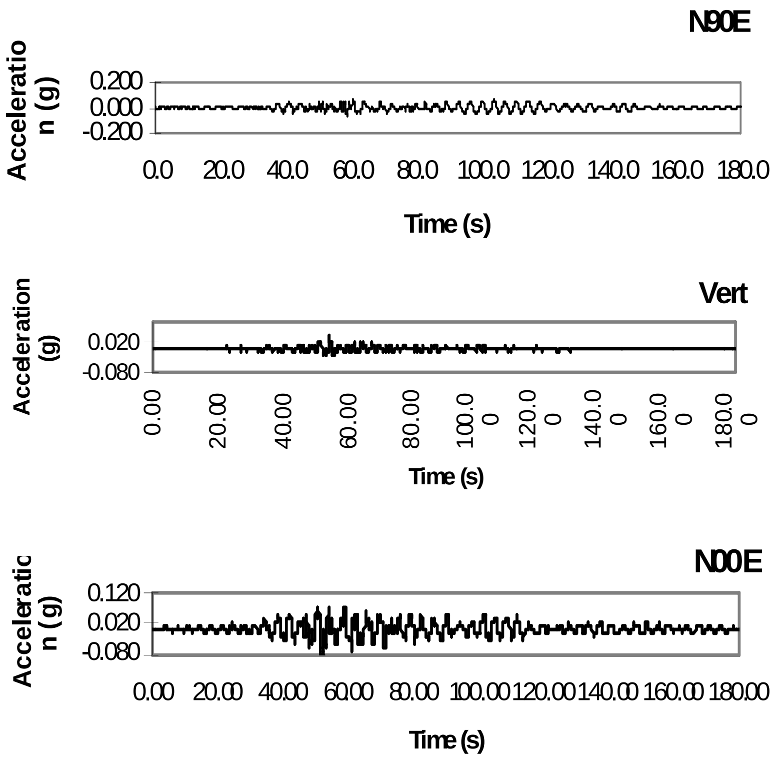

Figure 2 : Time histories of Michoacan Earthquake

Table2: Element of the Covariance Matrix for two sensors for five story building.

Combination	$\|C\|$	Trace [C]
All level	1.18	2.79
First + Second	0.04	0.77
First + Third	0.21	1.46
First + Fourth	0.64	2.25
First + Fifth	1.25	2.95
Second + Third	0.48	1.93
Second + Fourth	1.14	2.71
Second + Fifth	2.01	3.42
Third + Fourth	2.18	3.40
Third + Fifth	3.47	4.11
Fourth + Fifth	5.56	4.89

Three orthogonal component of ground motion excitations (input) and three orthogonal component of acceleration response computed at first floor and roof level (output) are used for the development of ARX model. The entire time histories (180s) of input-output are segmented in ten second window lengths. Corresponding to each time window using input-output pair (three inputs and six outputs) mathematical models are developed. From the frequency response function plots of the time window models the frequencies are identified. The identified first mode frequencies are 1.80 Hz and 1.54 Hz (see Fig. 3) for first time window and last time window respectively, while that computed using the finite element analysis is 1.92 Hz.

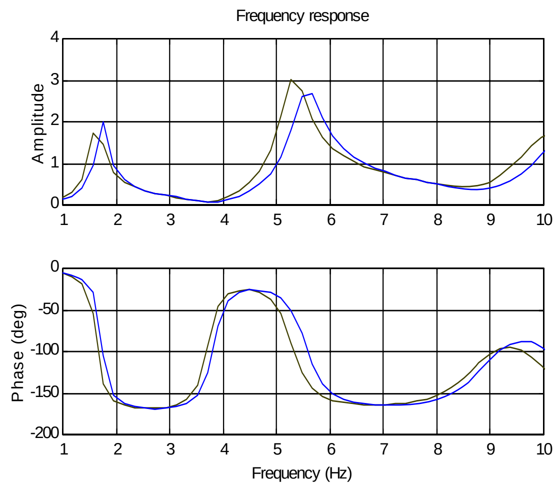

Figure 3 : Frequency Response Function Plots for First and Last Time Window Model

3.2 Five and ten story steel frame buildings

Two five and ten story steel moment resisting frame has been used in this study. The typical plan and typical elevation the buildings frame is shown in the Figure 4. The building frames have been designed according to seismic design criteria of the National Building Code of Canada, NBCC 2005. The accelerometers are assumed to be located at the right corner at each floor of the frame Fig. 4. The first mode of vibration has been used in the earthquake design of

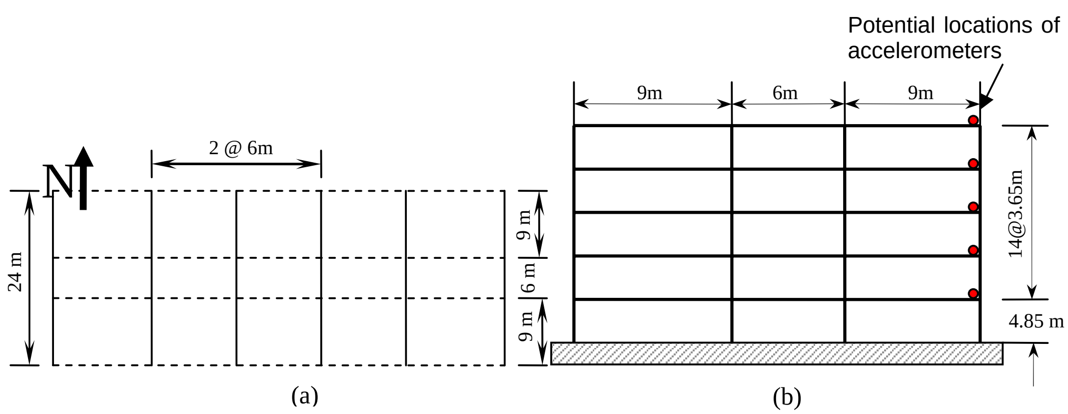

Figure 4: Geometric details of the builings (a)Typical plan, and (b) Typical elevation (five story).

the frames. In the design 5% viscous damping has also been considered. For the five story building all internal and external columns have same sections of W310x179 and the size of the beams is W310x253. The details of different elements of ten story building is shown in the Table 3.

The element of the temporal covariance matrix is calculated for different combination of the two sensors and the exact condition (Sensors at all level). In this case, the ground motion and the floor acceleration reponsonse are considered in only one direction (*i.e.*, horizontal or *x* direction), the covarience martrix has only one element. Four sets of synthesized ground motion, compatible to the seismic characteristics of Vancouver, Canada, are used to calculate the response of the building at the different level. In this study two dimensional finite element model of the buildings is considered. The acceleration is considered as the targeted response of the building to be recorded to evaluate the performance of different combination of the sensors. Among these four ground motion records two of which are of short period (SP) and the other two is long period (LP). Samples of the time history of the ground motion records are shown in the Figure 5.

Table 3: Sections of Ten story building.

	Sections of Beam	Sections of Column	
		External	Internal
Story 1 to 5	W310x107	W310x283	W310x314
Story 6 to 9	W310x107	W310x158	W310x202
Story 10	W310x79	W310x158	W310x202

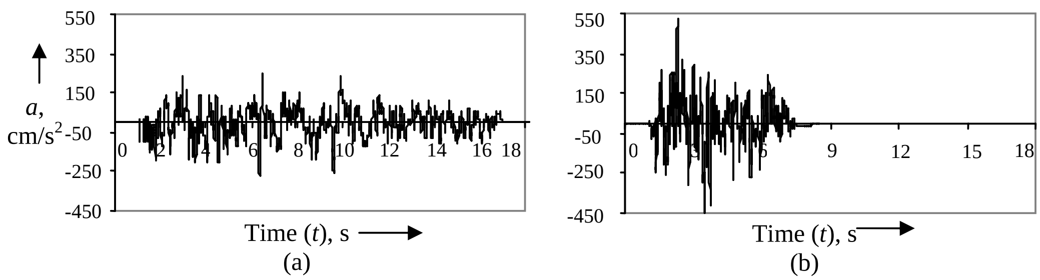

Figure 5: Samples of the synthesized ground motion records used in the study: (a) short period (SP) event, (b) long period (LP) event.

Table 4: Element of the Covariance Matrix for two sensors for five story building.

Combination	C_{ij}			
	LP1	LP2	SP1	SP2
All level	3.35	4.39	1.65	1.28
First + Second	3.23	2.99	1.22	0.96
First + Third	3.07	3.48	1.27	1.02
First + Fourth	2.61	3.78	1.31	1.20
First + Fifth	3.50	4.45	1.84	1.48
Second + Third	3.61	3.99	1.49	1.06
Second + Fourth	3.15	4.29	1.52	1.16
Second + Fifth	4.04	4.97	2.05	1.52
Third + Fourth	2.99	4.77	1.58	1.22
Third + Fifth	3.88	5.45	2.11	1.58
Fourth + Fifth	3.42	5.78	2.14	1.68

The element of the covarient matrix obtained by assuming that the response at each floor level is avialable, is compared with that obtaioned from the response from the response of the two floors where the sensors are assumed to be installed. All combinations of the pairs of floors are studied. From Table 4, it is observed that for the five story building, the sensor locations at the first and fifth, or second and fourth floor levels provide their optimum placement. On the other hand, the results in Table 5 indicate that for the ten story building, the optimal locations are the first and tenth floor levels, or the second and ninth floor levels. The Transfer functions from the ARX models have been obtained for all combinations of sensor locations. The identification of modal parameters in this case is not possible because of the short duration of records and presence of noise. Loger duration ground motion needs to be used for that purpose.

Table 5: Element of the Covariance Matrix for two sensors for ten story building.

Combination	C_{ij}			
	LP1	LP2	SP1	SP2
All level	2.81	3.04	1.37	1.18
First + Second	1.92	2.03	0.91	0.76
First + Third	2.22	2.03	0.87	0.80
First + Fourth	2.00	2.15	0.90	0.80
First + Fifth	2.00	2.52	1.03	0.81
First + Sixth	2.05	2.29	1.10	0.93
First + Seventh	2.20	2.65	1.11	0.90
First + Eighth	2.24	2.61	1.14	0.89
First + Ninth	2.40	2.46	1.59	0.95
First + Tenth	2.85	3.07	1.18	1.37
Second + Third	2.69	2.40	1.10	0.99
Second + Fourth	2.47	2.52	1.10	0.98
Second + Fifth	2.47	2.90	1.23	0.99
Second + Sixth	2.52	2.67	1.25	1.12
Second+ Seventh	2.67	3.03	1.33	1.08
Second + Eighth	2.71	2.99	1.34	1.08
Second + Ninth	2.87	2.84	1.37	1.14
Second + Tenth	3.32	3.45	1.37	1.55

4 CONCLUSIONS

In this study a methodology has been proposed to optimize the placing of sensors (two numbers) in the structure to record the seismic response of a structural system**.** The proposed methodology can be used to capture sufficient information for system identification applied in structural health monitoring. The system parameters such as the natural frequencies can be identified with minimum number of optimally placed sensors when ground records used is suufficiently long. The methodology for identifying the optimal sensor locations works well with both three and two dimensional models. The five story buildings studied here with three and two dimesional models provide similar locations for optimal placement of sensors. In case of the ten story building, the optimal locations of sensors are compatible with those in the other buildings.

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