Feasibility investigations for long-term health monitoring of short span bridges using poles of AR model

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ABSTRACT: This study intends to investigate the feasibility of modal parameter identification for health monitoring of short span bridges using the vibration data of the bridges induced by a moving vehicle. The idea of this study using the traffic-induced vibration of short span bridges for their modal identification is that the modal parameters identified repeatedly under a moving vehicle provide a statistical pattern which may give useful information for the current bridge condition. The autoregressive (AR) model is adopted for identifying modal parameters. Feasibility investigations for health monitoring of short span bridges are performed through a moving vehicle laboratory experiment.

1 INTRODUCTION

Bridge health monitoring (BHM) at a global level using dynamic system parameters has been one of the most important approaches, and has been intensively studied to minimize the safety risk of bridge structures (e.g. Doebling, *et al*. (1996)). The basic idea behind BHM using the dynamic system parameters is that frequency and damping characteristics as well as mode shapes may provide useful information for decision making on the current health condition of bridges. The fundamental concept of this technology is that modal parameters are functions of a structure's physical properties. Therefore, a change in physical properties, such as reduced stiffness resulting from damage, will detectably change these modal properties. Of course, applying sensors around expected or suspected damage members is one of the best approaches to detect damage. This is only possible, however, when the bridge structure has well defined damage models. For real bridge structures defining a damage model is difficult differently from other structures such as automobiles, aerial vehicles, etc. Most precedent studies focusing on BHM thus have specifically examined the global change of modal properties and quantities of bridge structures.

This study intends to investigate feasibility of BHM especially focused on short span bridges using modal parameters identified using traffic-induced vibration data of bridges. Short span bridges' great contribution to bridge infrastructure and their insensitivity to ambient vibration such as wind are the major background of this study. The idea of this study even using the traffic-induced vibration of short span bridges for their modal identification is that the modal parameters identified repeatedly under moving vehicles can provide a pattern or even a

statistical one which may give useful information for the current bridge condition despite of the possibility of the identified parameters to be contaminated by the nonstationarity of the traffic-induced vibration (e.g. Kim & Kawatani (2008)). For the short span bridges in local areas where we cannot expect vibration whenever we want because of their light traffic volumes, an inspection car may be a solution to excite the bridge. This paper covers feasibility investigations for health monitoring of short span bridges by means of modal parameter identification using traffic-induced vibrations through a moving vehicle laboratory experiment. The experiment includes the scenario of the damaged bridge. This study adopts AR model for the modal parameter identification, which has been applied to identify structural parameters since the 1970s (e.g. Gersch *et al*. (1973)).

2 AR MODEL AND MODAL IDENTIFICATION

If $\mathbf{y}_c(t)$ denotes output of a bridge structure taken from an observation point, then the corresponding state equation of a continuous-time system is described as

$$\dot{\mathbf{x}}_c(t) = \mathbf{A}_c\mathbf{x}_c(t) + \mathbf{B}_c\mathbf{f}_c(t),\ \mathbf{y}_c(t) = \mathbf{C}_c\mathbf{x}_c(t) \tag{1}$$

where,

$$\mathbf{x}_c(t) = \begin{bmatrix} z(t) \\ \dot{z}(t) \end{bmatrix} \tag{2}$$

$$\mathbf{A}_c = \begin{bmatrix} 0 & 1 \\ -k_b/m_b & -c_b/m_b \end{bmatrix} \tag{3}$$

$$\mathbf{B}_c = \begin{bmatrix} 0 \\ -1/m_b \end{bmatrix} \tag{4}$$

$$\mathbf{C}_c = \begin{bmatrix} 1 & 0 \end{bmatrix} \tag{5}$$

Therein, m_b, k_b, c_b, respectively indicate the mass, damping, and stiffness of the bridge. $z(t)$ denotes the displacement response, and $\mathbf{f}(t)$ represents an external force vector. If the observability matrix is defined as $\mathbf{L}^T$=[C; CA_c; …; CA_c^{n-1}], then the discrete state equation for the system from the equation of motion for a bridge through an observability transformation can be described (Ljung (1999)) as

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{f}(k) \tag{6}$$

$$\mathbf{y}(k) = \mathbf{C}\mathbf{x}(k) \tag{7}$$

where $\mathbf{x}(k) = \mathbf{L}\mathbf{x}_d(k)$, and the subscript d indicates discretized one from the continuous system. The state transition matrix $\mathbf{A}$, input influence matrix $\mathbf{B}$ and output influence matrix $\mathbf{C}$ through an observability transformation are described as

$$\mathbf{A} = \mathbf{L}\mathbf{A}_d\mathbf{L}^{-1} = \left[\{0\,1\,0\cdots 0\};\{0\,0\,1\cdots 0\};\cdots;-\{a_n\ a_{n-1}\cdots\ a_1\}\right] \tag{8}$$

$$\mathbf{B} = \mathbf{L}\mathbf{B}_d = \left[b_1;\cdots;b_n\right] \tag{9}$$

$$\mathbf{C} = \mathbf{C}_d\mathbf{L}^{-1} = \left[1\,0\cdots 0\right] \tag{10}$$

where $\mathbf{A}_d = e^{\mathbf{A}_c\Delta}$;and Δ represents the sampling period.

If the coefficient from a_1 to a_n of the system matrix $\mathbf{A}$ is known, then eigenvalues of the matrix $\mathbf{A}$ provide the modal information of the system. Therefore, the next step is to derive linear

equations for the coefficient using the measurements (or observation) $\mathbf{y}(k)$. The state equation in Eq.(6) is rewritable as

$$\begin{aligned}
&x_1(k+1) = x_2(k) + b_1 f(k) \\
&\vdots \\
&x_{n\text{-}1}(k+1) = x_n(k) + b_{n\text{-}1} f(k) \\
&x_n(k+1) = -a_n x_1(k) - a_{n\text{-}1} x_2(k) - \cdots - a_1 x_n(k) + b_n f(k)
\end{aligned} \tag{11}$$

Using the relation of the observability in Eq. (7), Eq. (11) can be rearranged as

$$\begin{aligned}
&y(k) = x_1(k) \\
&y(k+1) = x_1(k+1) = x_2(k) + b_1 f(k) \\
&\vdots \\
&y(k+n-1) = x_1(k+n-1) = x_{n-1}(k+1) + b_{n-2} f(k+1) + \cdots\cdots + b_1 f(k+n-2) \\
&\qquad\qquad = x_n(k) + b_{n-1} f(k) + \cdots\cdots + b_1 f(k+n-2)
\end{aligned} \tag{12}$$

$\mathbf{x}(t)$ of Eq.(12) is rewritable as

$$\mathbf{x}(k) = \mathbf{y}(k) - \mathbf{b}\mathbf{f}(k) \tag{13}$$

where,

$$\mathbf{x}(k) = \left[x_n(k); x_{n-1}(k); \cdots; x_1(k)\right] \tag{14}$$

$$\mathbf{y}(k) = \left[y(k+n-1); \cdots; y(k+1); y(k)\right] \tag{15}$$

$$\mathbf{f}(k) = \left[f(k+n-1); \cdots; f(k+1); f(k)\right] \tag{16}$$

$$\mathbf{b} = \left[\overline{\mathbf{b}}_n; \cdots; \overline{\mathbf{b}}_1\right] \left(\overline{\mathbf{b}}_n = \{0 \quad b_1 \quad \cdots \quad b_{n-1}\}; \ \ldots\ ; \ \overline{\mathbf{b}}_1 = \{0 \quad \cdots \quad 0\}\right) \tag{17}$$

The last relation in Eq.(11) gives

$$x_n(k+1) = -\mathbf{a}\mathbf{x}(k) + b_n f(k) \tag{18}$$

where $\mathbf{a} = \{a_1 \cdots a_n\}$.

Moreover the following relation is obtainable from Eq.(12)

$$x_n(k+1) = y(k+n) - \sum_{i=1}^{n-1} b_i f(k+n-i) \tag{19}$$

Substituting Eq.(19) and Eq.(13) into Eq.(18), and assuming $\mathbf{f}(k)$ as a white noise input $\mathbf{e}$(k) gives the following ARMA model.

$$\mathbf{y}(k) = -\mathbf{a}\mathbf{y}(k-n) + \tilde{\mathbf{b}}\mathbf{e}(k-n) \tag{20}$$

where,

$$\tilde{\mathbf{b}} = \begin{bmatrix} b_1 & \cdots & b_n \end{bmatrix} \begin{bmatrix} 1 & a_1 & \cdots & \cdots & a_{n-1} \\ 0 & 1 & a_1 & \cdots & a_{n-2} \\ & & \ddots & & \vdots \\ & & & 1 & a_1 \\ & & & & 1 \end{bmatrix} \tag{21}$$

Fortunately, the AR model with infinite order is equivalent to the ARMA model, which means that one can express the responses of a linear system subjected to white-noise input using the AR model with sufficient large order (Wang and Fang (1986)).

$$y(k)+\sum_{i=1}^{p} a_i y(k-i) = e(k) \tag{22}$$

To estimate the AR parameter, the autocorrelation function of $y(k)$ is used, which is obtainable by multiplying each term of Eq. (22) with $y(k\text{-}s)$ and taking mathematical expectation. This process yields the following Yule-Walker equation

$$\mathbf{Ra} = -\mathbf{r} \tag{23}$$

where $\mathbf{R}$ is a Toeplitz matrix about $R(p, s) = \mathrm{E}[y(k\text{-}p)y(k\text{-}s)]$ which is the autocorrelation function of the signal. $\mathbf{r}^T = [R_1; \ldots; R_n]$. The Levinson-Durbin algorithm is adopted to solving Eq.(23) for a (Ljung (1999)). It is noteworthy that the coefficient a_p terms the poles of the system because the z-transformation of Eq. (22) is

$$y(k) = \frac{1}{H(z)} e(k) = \frac{1}{1+\sum_{i=1}^{p} a_i z^{-1}} e(k) \tag{24}$$

where $H(z)$ is the transfer function of the system in discrete-time complex domain, and z^{-i} denotes a forward shift operator.

The denominator of the transfer function is the characteristic equation of the dynamic system and encapsulates modal information of each vibration mode of the system. The frequency and damping constant of each mode can be estimated from the complex valued pole as (e.g. Inman (2008))

$$z = \exp\left(h_k \omega_k \pm j\omega_k \sqrt{1-h_k} \right) \tag{25}$$

Therein, h_k and ω_k indicate the damping constant and circular frequency of k-th mode of the system, respectively j represents the imaginary unit.

3 LABORATORY EXPERIMENT

3.1 Outline of the experiment

Laboratory experiments are performed to investigate feasibility of the proposed method. The experiment setup is shown in Figure 1 with geometry and structural properties of the girder model which is modeled from a real bridge. Three points at 1/4, 1/2, and 3/4 of the span length are the observation points for acceleration responses. A damaged girder is realized by applying saw cuts at both sides of web plates.

The sampling rate of signals is 100Hz. Two different vehicle speeds such as 0.93m/s and 1.63m/s are used in the experiment. Two types of vehicle models, of which natural frequency for the bounce motion is changeable using a different set of mass and spring, are used in the experiment: those frequencies are 2.93Hz and 3.76Hz for two vehicles. The fundamental frequencies and damping constants with and without damage are summarized in Table 1. Those fundamental frequencies are obtained by free vibrations. Especially, damping constants are estimated from the free vibration time histories after the vehicle leaving the model girder because of their dependence on the amplitude.

Examples of acceleration time histories of the bridge at the 1/4 of the span length L and their Fourier amplitude spectra before and after damage are shown in Figure 2. It shows that the amplitude of acceleration responses drastically decreases due to the damage. The strong damping constant due to the saw cuts (see Table 1) may be a major reason of the drastic decreasing of the amplitude. Additional peaks of dominant frequencies due to the saw cuts in comparison with those of the intact bridge are observed. Another interesting point is that dominant frequencies near 2.5Hz and 23.4Hz are weakened by the damage.

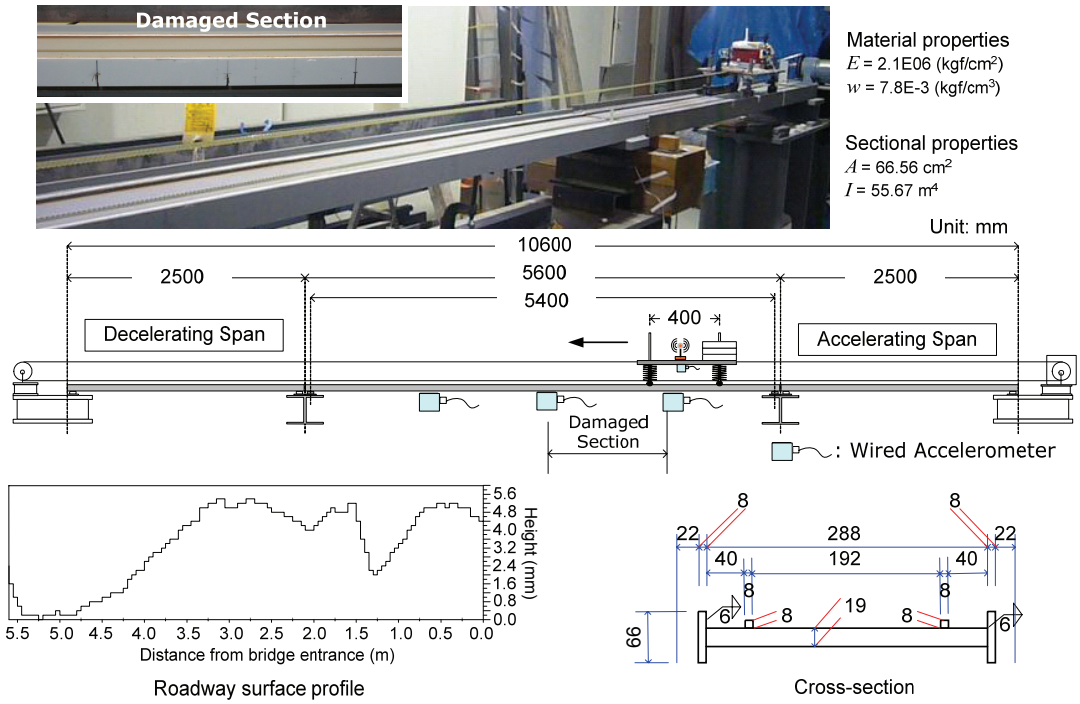


Figure1. Experiment setup.

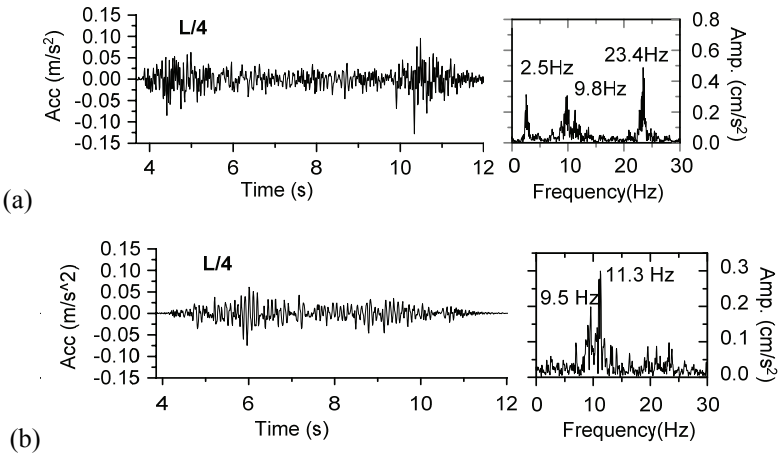
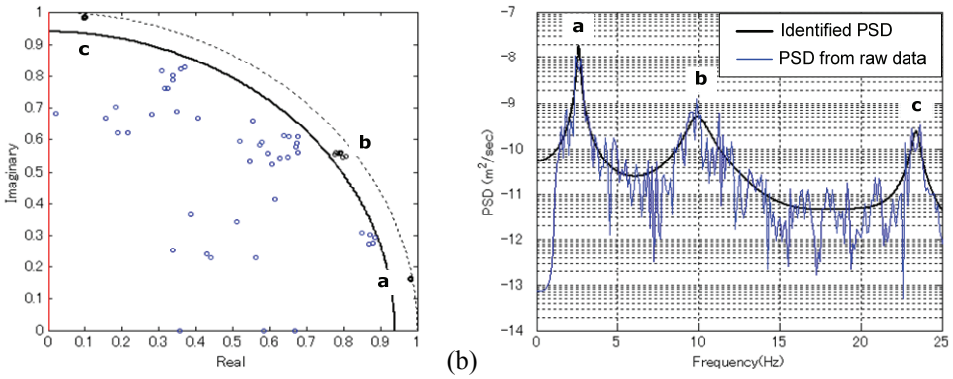


Figure 2. Accelerations of bridge under vehicle with natural frequency of 2.93Hz for bounce motion and speed of 0.93m/s: (a) intact; and (b) damage.

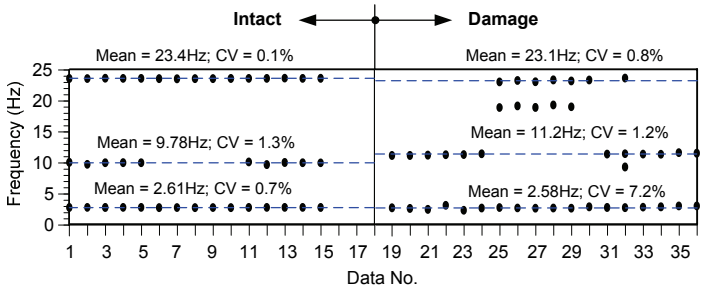
Table 1 Fundamental frequencies and damping constants of the bridge model.

Mode	Intact		Damage	
	Frequency (Hz)	Damping constant	Frequency (Hz)	Damping constant
First	2.6	0.058	2.5	0.185
Second	10.6	-	9.7	-
Third	23.3	-	23.1	-

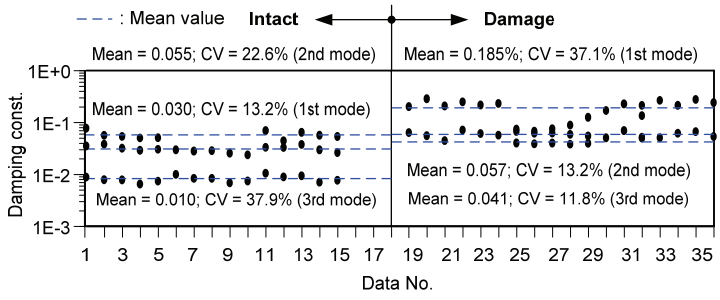


(a) (b)

Figure 3. Poles and identified PSD curve of the bridge system using data under vehicles with speed of 0.93m/s: (a) poles on imaginary space; and (b) identified PSD curve.



(a)



(b)

Figure 4. Identified modal parameters using data under vehicles with speed of 0.93m/s; (a) dominant frequencies; and (b) damping constants.

3.2 Modal parameter identification

An example for identified pole plots and PSD curve reconstructed using identified AR coefficients is shown in Figure 3. Frequencies and damping constants of each mode are estimated using identified poles and Eq. (25), and are summarized in Figure 4. Therein the data

numbers of the second frequency missing correspond to observation data at span center which seldom provide the second bending mode.

The data under the moving vehicle of the low speed (0.93m/s) are used in the modal identification because higher the vehicle speed stronger the vehicle effect on bridge response would be was observed from a preliminary investigation. The mean values of identified frequencies of the intact girder are 2.61Hz, 9.78Hz and 23.4Hz, which link with 2.6Hz, 10.6Hz and 23.3Hz for the first, second and third modes obtained from the free vibration respectively. Those mean frequencies after introducing the damage are 2.58Hz, 11.2Hz and 23.1Hz which are corresponding to 2.5Hz, 9.7Hz and 23.1Hz from the free vibration of the damage girder respectively. It shows that the first frequency of 2.58Hz is smaller than that of the intact girder. However the second frequency of 11.2Hz is greater than that of the intact girder. This is different from the well known fact that decreasing stiffness results in low fundamental frequencies. One of the reasons of the result is that the dominant frequency near 9.8Hz of the intact girder is disturbed by damages as shown in PSD curve of Figure 2(b). As a result the dominant frequency near 11Hz of the damaged girder is detected. An interesting result is that the damage gives another dominant frequency near 20Hz except 23.1Hz which is near the third dominant frequency of the intact girder.

The identified damping constants using the data under the vehicle moving with the low speed (0.93m/s) are summarized in Figure 4(b). For the identified mean damping constants corresponding to the first mode, the value of 0.030 of the intact girder grows by 0.185 due to the damage, which are very similar to those estimated from free vibrations such as 0.058 and 0.185 respectively for intact and damage girders (see Table 1). An interesting point is that the pattern

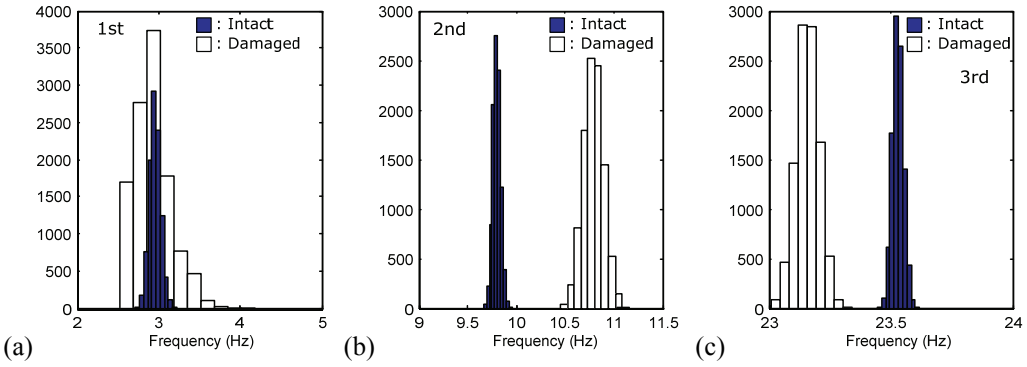


Figure 5. Bootstrap distribution for 10000 resample means from the identified frequency sample: (a) first mode (b) second mode; and (c) third mode.

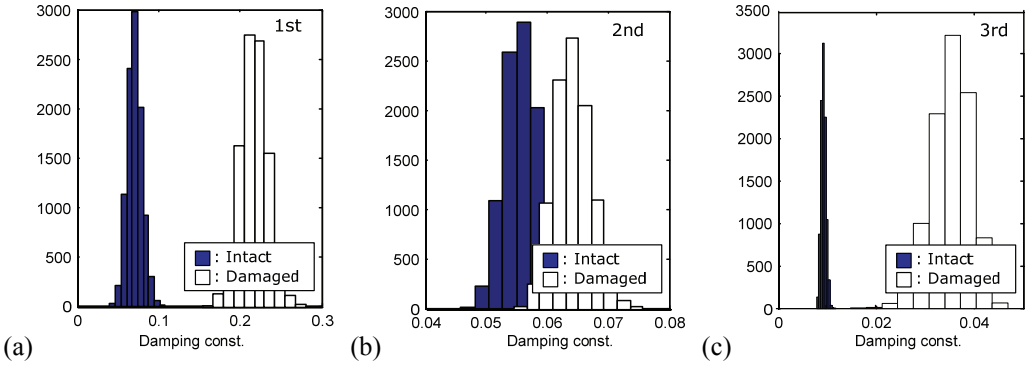


Figure 6. Bootstrap distribution for 10000 resample means from the identified damping constant sample: (a) first mode (b) second mode; and (c) third mode.

change of the identified damping constant due to the damage is very apparent comparing to that of the dominant frequency.

Noteworthy point is that the vertical scale of the plot for identified modal damping constants adopts log-scale because of the big variation of the identified value. Therefore, in this study, the bootstrap technique (Efron & Tibshirani (1993)) is applied to resample the identified data. Figure 5 shows bootstrap distribution of the resample frequency for 10000 resample means from the identified data shown in Figure 4(a). It demonstrates that apparent difference of the bootstrap distribution between intact and damaged bridges is appeared for the second and third mode. It is also observed that the variation of the distribution increases due to the damage. Those distributions for modal damping constants are summarized in Figure 6, which provides very apparent change of the distribution due to the damage.

Observations from the experimental investigation on the intact girder demonstrate the feasibility of the AR model for modal identification of short span bridges. Moreover comparing the identification results between intact and damage girders such as pattern changes of identified parameters encourages the use of the method for long term health monitoring even for short span bridges.

4 CONCLUSIONS

In this study, feasibility of the modal identification for health monitoring of short span bridges from traffic-induced vibration is investigated through a moving vehicle laboratory experiment. Observations demonstrate that the dominant frequency and damping constant of the bridge are well identified using the measurement data under vehicle running with lower speed.

The bootstrap distribution of the resample means from the confined number of identified data provides a clear difference of the distribution between with and without damages.

Observations through the study demonstrates that comparing the identification results between intact and damage girders such as pattern change of identified parameters encourages the use of the method for long term health monitoring even for short span bridges.

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6 REFERENCES

Efron, B. and Tibshirani, RJ. 1993. *An introduction to the Bootstrap*, New York: Chapman & Hall.

Doebling, SW, Farrar, CR, Prime, MB and Shevitz, DW. 1996. Damage identification and health monitoring of structural and mechanical systems from changes in their vibration characteristics: A literature review. Los Alamos National Laboratory Report LA-3070-MS.

Gersch, W, Nielsen, NN and Akaike, H. 1973. Maximum likelihood estimation of structural parameters from random vibration data. *J. of Sound and Vibration*, 31(3): 295-308.

Inman, DJ. 2008. *Engineering vibration*. 3rd ed., Upper Saddle River, N.J.: PTR Prentice Hall.

Kim, CW and Kawatani, M. 2008. Pseudo-static approach for damage identification of bridges based on coupling vibration with a moving vehicle. *Structure and Infrastructure Engineering*, 4(5): 371-379.

Ljung, L. 1999. *System identification - Theory for the user*. 2nd ed., Upper Saddle River, N.J.: PTR Prentice Hall.

Wang, Z and Fang, T. 1986. A time-domain method for identifying modal parameters. Journal of Applied Mechanics, ASME, 53(3): 28-32.