On-line damage detection using autoregressive time series models

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ABSTRACT: In structural health monitoring (SHM) and damage detection studies two issues occupy a central spot: the selection of damage sensitive feature(s) and subsequent analysis of these features in order to detect, localize and assess the extent of damage. In this study, the two issues are addressed via the application of time series analysis and artificial neural networks (ANNs), respectively. While many existing damage detection and SHM techniques analyze the whole available data in an off-line manner, on-line damage detection would allow damage to be detected and tracked as it accumulates over time. In this study an on-line damage detection method is proposed. The method fits the acceleration time histories with autoregressive (AR) time series models and takes the coefficients of these models as damage sensitive features. The AR coefficients are identified on-line using recursive algorithms. Damage detection is based on the fact that changes in the system dynamics caused by damage will be reflected in the values of the AR coefficients. Initially, a linear MDOF model representing a building subjected to seismic excitation was investigated. Damage in the structure was defined as the ratio of remaining lateral stiffness to initial stiffness and was varied over time. Two recursive identification techniques were chosen to identify the AR models: the forgetting factor and the Kalman filter approaches, and their ability to identify damage on-line was evaluated. The AR coefficients identified recursively at each time step were used as inputs into a back-propagation (BP) ANN that was trained to relate these coefficients to the damage at each storey. This allowed damage to be quantified and localized at each time step. Next, detection of damage from nonlinear responses was investigated on systems with either elastoplastic or Bouc-Wen hysteretic characteristics. Intervention analysis was used to analyze the AR coefficients that changed abruptly at the onset of yielding. This allowed yielding events in both models to be detected. Overall, the performance of the proposed damage detection method was promising.

1 INTRODUCTION

Time series analysis techniques were originally developed for analyzing long sequences of regularly sampled data, and have been extensively used in various fields of science and engineering (Wei 2006). Although they appear to be inherently suited to structural health monitoring (SHM) their application in the field can still be regarded as emerging and relatively unexplored. In a study by Sohn et al. (2000) time series model coefficients were chosen to be damage sensitive features. The authors fitted the dynamic response of a concrete bridge pier using autoregressive (AR) models. By performing statistical analysis on the AR coefficients, responses coming from several damage states could be distinguished. Omenzetter and Brownjohn (2006) used a vector seasonal autoregressive integrated moving average model to detect abrupt changes in strain data collected from the continuous monitoring of a major bridge structure. Nair et al. (2006) used an autoregressive moving average (ARMA) time series to model the vibration signals from an experimental structure and defined a damage sensitive feature used to discriminate between the damaged and undamaged states of the structure based on the first three AR coefficients. Nair and Kiremidjian (2007) investigated Gaussian mixture modeling to model the first AR coefficients obtained from fitting ARMA time series to an analytical structural model. The extent of damage was shown to correlate well with the Mahalanobis distance between undamaged and damaged feature clusters. Localization of damage was achieved by introducing another feature, also based on the AR coefficients, found to increase from a baseline value when damage was near.

While the majority of the aforementioned references use AR coefficients identified via off-line approaches, in this research damage detection is achieved using AR coefficients identified using recursive techniques. Such an approach enables changes and/or damage in the system to be continuously traced in real time as it develops and accumulates. The outline of the paper is as follows: In Section 2 an outline and flow of the proposed damage detection method is provided, analytical tools required are identified and their theory explained. Section 3 presents application of the method to three numerical models: a linear 3DOF structure with sudden stiffness losses, an elastoplastic SDOF oscillator, and a 3DOF nonlinear Bouc-Wen model. These models are used to gain insight into the method efficiency and performance.

2 OUTLINE OF PROPOSED DAMAGE DETECTION METHOD AND THEORY

2.1 Outline of proposed damage detection method

The proposed damage detection methods consist of the following steps:

1. Time histories of responses, such as accelerations, of a structure subjected to dynamic excitation are recorded continuously and modeled as AR time series, where the coefficients of those AR models are identified recursively using algorithms such as forgetting factor and/or Kalman filter.
2. The AR coefficients are considered damage sensitive features. For structures where damage can be assumed as a permanent stiffness loss a relationship between remaining stiffness and values of the AR coefficients is established using training data from different damage states and modeled using an artificial neural network (ANN). The AR coefficients identified recursively from continuous monitoring are then presented to the ANN, which identifies remaining stiffness.
3. For structures where damage cannot be easily described as permanent stiffness loss but takes form of plastic deformations, the AR coefficients show sudden changes when yielding occurs. Monitoring these changes using intervention analysis enables detection of yielding events.

2.2 Theory

2.2.1 Autoregressive time series models

In this study, AR time series models (Wei 2006) were used to describe the acceleration time histories. A univariate AR model of order p, or AR(p), for the time series $\{x_t\}$ (t = 1, 2, …, n) can be written as

$$x_t = \phi_1 x_{t-1} + \phi_2 x_{t-2} + \ldots + \phi_p x_{t-p} + a_t \tag{1}$$

where x_t, x_{t-1} , …, x_{t-p} are the current and previous observations of the series $\{x_t\}$, $\{a_t\}$ is a Gaussian white noise error time series with a zero mean and constant variance, and ϕ_1, ϕ_2, …, ϕ_p

are the AR coefficients. Equation (1) can alternatively be expressed using a polynomial in the backshift operator $\phi(B)=1-\phi_1 B-\phi_2 B^2-\ldots-\phi_p B^p$ as follows:

$$\phi(B)x_t = a_t \quad (2)$$

where the backshift operator B is defined as $B^s x_t = x_{t-s}$. In this investigation, the AR coefficients were allowed to change with time reflecting changes in the structure caused by damage.

2.2.2 Recursive identification of autoregressive time series

In the proposed method, the time varying AR coefficients are selected as damage sensitive feature. In order to track their values with time they need to be recursively identified. Recursive parameter estimation techniques for the identification of times series models used in this study include the forgetting factor and Kalman filter approaches (Ljung 1999).

The forgetting factor approach (Ljung 1999) is a recursive algorithm that gives a weighted least squares estimate. The forgetting or weighting factor λ, usually taken to be $0.98<\lambda<0.995$, determines the method's ability to track changes over time by assigning more weight to the most recent observations. The algorithm estimates the model parameters $\boldsymbol{\theta}_t$ at time step t from recursive application of the following equations:

$$\boldsymbol{\theta}_t = \boldsymbol{\theta}_{t-1} + \mathbf{L}_t\left[x_t - \boldsymbol{\varphi}_t^T \boldsymbol{\theta}_{t-1}\right], \quad \mathbf{L}_t = \frac{\mathbf{P}_{t-1}\boldsymbol{\varphi}_t}{\lambda + \boldsymbol{\varphi}_t^T \mathbf{P}_{t-1}\boldsymbol{\varphi}_t}, \quad \mathbf{P}_t = \frac{1}{\lambda}\left[\mathbf{P}_{t-1} - \frac{\mathbf{P}_{t-1}\boldsymbol{\varphi}_t \boldsymbol{\varphi}_t^T \mathbf{P}_{t-1}}{\lambda + \boldsymbol{\varphi}_t^T \mathbf{P}_{t-1}\boldsymbol{\varphi}_t}\right] \quad (3)$$

where $\boldsymbol{\varphi}_t$ is a vector containing previous observations of time series $\{x_t\}$, and superscript T denotes transposition.

The Kalman filter (Harvey 1989) is an optimal linear recursive estimator. Consider the following state space model:

$$\mathbf{y}_t = \mathbf{Z}_t \boldsymbol{\alpha}_t + \boldsymbol{\varepsilon}_t, \quad \boldsymbol{\alpha}_t = \mathbf{T}_t \boldsymbol{\alpha}_{t-1} + \boldsymbol{\eta}_t \quad (4)$$

where, $\mathbf{y}_t$ is the vector of outputs and $\boldsymbol{\alpha}_t$ is the state vector. Matrices $\mathbf{Z}_t$ and $\mathbf{T}_t$ are the measurement and transition matrices, respectively. Vectors $\boldsymbol{\varepsilon}_t$ and $\boldsymbol{\eta}_t$ represent noises with zero mean multivariate Gaussian distributions and covariance matrices $\mathbf{H}_t$ and $\mathbf{Q}_t$, respectively. These noises can also be contemporaneously correlated with covariance matrix $\mathbf{G}_t$. The optimal estimation $\mathbf{a}_t$ of the state vector $\boldsymbol{\alpha}_t$ conditional on the information available at time t can be obtained through recursive application of the following prediction equations

$$\mathbf{a}_{t|t-1} = \mathbf{T}_t \mathbf{a}_{t-1}, \quad \mathbf{P}_{t|t-1} = \mathbf{T}_t \mathbf{P}_{t-1} \mathbf{T}_t^T + \mathbf{Q}_t \quad (5)$$

and updating equations

$$\begin{aligned} \mathbf{a}_t &= \mathbf{a}_{t|t-1} + \left(\mathbf{P}_{t|t-1}\mathbf{Z}_t^T + \mathbf{G}_t\right)\mathbf{F}_t^{-1}\left[\mathbf{y}_t - \mathbf{z}_t\left(\mathbf{a}_{t|t-1}\right)\right] \\ \mathbf{P}_t &= \mathbf{P}_{t|t-1} - \left(\mathbf{P}_{t|t-1}\mathbf{Z}_t^T + \mathbf{G}_t\right)\mathbf{F}_t^{-1}\left(\mathbf{Z}_t \mathbf{P}_{t|t-1}^T + \mathbf{G}_t^T\right) \end{aligned} \quad (6)$$

with

$$\mathbf{F}_t = \mathbf{Z}_t \mathbf{P}_{t|t-1} \mathbf{Z}_t^T + \mathbf{Z}_t \mathbf{G}_t + \mathbf{G}_t^T \mathbf{Z}_t^T + \mathbf{H}_t \quad (7)$$

In Equations (5)-(6), $\mathbf{P}_t$ is the covariance matrix of the estimation error.

2.2.3 Back-propagation artificial neural networks

ANNs are computational structures capable of pattern recognition, classification and function approximation. In this research, they are used to map changes in the AR coefficients into changes in structural stiffness. ANNs utilizing the supervised error back-propagation (BP) training algorithm (Rumelhart et al. 1986) are commonly referred to as BP ANNs. A BP ANN consists of interconnected neurons, or basic computational units, arranged in several layers: an input layer, hidden layer(s) and output layer. Outputs from a preceding layer become inputs into the following layer. The output y of a single neuron is calculated using the weighted sum of all its inputs as

$$y = f\left(\mathbf{v}^T\mathbf{x}\right) \tag{8}$$

where $\mathbf{x}$ is the vector of inputs into the neuron, $\mathbf{v}$ is the vector of weights for the neuron, and f is the neuron's activation function. The error E at the output layer is a function of the weights for the entire network, denoted by $\mathbf{w}$, and can be written as

$$E(\mathbf{w}) = \frac{1}{2}\mathbf{e}(\mathbf{w})^T\mathbf{e}(\mathbf{w}) \tag{9}$$

where $\mathbf{e}(\mathbf{w})$ is an error vector quantifying the difference between the desired outputs and actual outputs. In the training phase, the network calculates the outputs for given inputs and the error is propagated backwards from the output layer to the preceding layers using the back-propagation algorithm. The weights can be updated through the application of an iterative process until the total error of Equation (9) is minimized.

2.2.4 Intervention analysis

Intervention analysis is used to detect the existence and timing of external events that cause disturbances in time series referred to as outliers (Pankratz 1991). In the context of this study, yielding of members was assumed to be external events and source of outliers in the AR coefficients.

Consider a time series $\{y_t\}$ which can be described by the following AR(1) process:

$$(1-\phi_1 B)\, y_t = a_t \tag{10}$$

However, instead of observing $\{y_t\}$ suppose a contaminated series $\{z_t\}$ is available:

$$z_t = f(t) + y_t \tag{11}$$

where $f(t)$ is a function describing the external event and is assumed to have the form

$$f(t) = \omega(B)/\delta(B)\, X_t \tag{12}$$

where X_t is a deterministic binary pulse variable that for an intervention to series $\{y_t\}$ at time i assumes the value X_t=1 at t=i and X_t=0 for $t \neq i$. The $\omega(B)$ and $\delta(B)$ are the transfer function numerator and denominator operators, respectively. This formulation allows several different types of interventions including additive outliers, level shifts and innovational outliers to be described. In this study, the innovational outliers model was used. This type of outlier consists of a temporary impulse whose effect decays with time as dictated by the memory of the AR process it affects.

For innovational outliers the contamination term $f(t)$ can be modeled with $\omega(B)=\omega_0$ and $\delta(B)=1-\phi_1 B$. The contaminated series can then be written as

$$z_t = \frac{1}{1-\phi_1 B}\left(\omega_0 X_t + a_t\right) \tag{13}$$

The residual time series $\{\zeta_t\}$ defined as

$$\zeta_t = \omega_0 X_t + e_t \tag{14}$$

can be used to detect the presence of interventions in $\{z_t\}$. The coefficient ω_0 can be estimated using a least squares approach. To determine the presence of an intervention at time $t=i$ the value of ω_0 can be compared with the standard deviation of the residual series $\{e_t\}$ denoted by σ_e. If ω_0 exceeds a threshold value, usually taken as a multiple of σ_e, an intervention has occurred.

3 APPLICATION OF PROPOSED DAMAGE DETECTION METHOD

3.1 Application to a linear 3DOF model with sudden stiffness loss

In the first application, a linear 3DOF lumped-mass building model was used. The lateral stiffness of each storey was set to 1.0×10^7 N/m and the lumped storey masses were set to 1.0×10^4 kg. A Rayleigh damping model was adopted and damping was set at 5% critical for the 1st and 2nd modes. The natural frequencies of the structure were 2.24 Hz, 6.28 Hz and 9.07 Hz for the 1st, 2nd, and 3rd modes, respectively. The model was excited using Gaussian white noise ground excitation.

Three univariate AR(30) models were identified from the acceleration of the three stories using either the forgetting factor or Kalman filter approaches. In order to compare the two recursive identification methods, a common training and testing procedure was adopted. Damage was defined as the ratio of remaining stiffness to the initial stiffness at each storey. For training of the BP ANN, damage severity assumed seven discrete values of 0.4, 0.5, 0.6, 0.7, 0.8, 0.9 and 1.0. A set of 100 AR coefficients were obtained at these seven discrete damage states for each storey giving a total of 7^3=343 states to be calculated. For both approaches, a single hidden layer BP ANN with 3 hidden layer neurons was used.

To test the method on-line, the stiffness in the 1st storey was reduced from 1.0 to 0.8 and afterwards from 0.8 to 0.4 at preset times as shown in Figure 1. Stiffness in the 2nd storey was reduced from 1.0 to 0.7, and in the 3rd storey from 1.0 to 0.9. In all cases, the stiffness was linearly interpolated over a 2 second interval.

Using the forgetting factor approach, the ability of the method to track changes can be tuned using the forgetting factor λ. After some initial trails a value of λ=0.99 was adopted for all three models. Figure 1 shows the detected remaining stiffness in each storey compared to the simulated value. The figure shows that the forgetting factor approach was effective at tracking damage and scatter about the simulated values was small.

The tracking abilities of the Kalman filter can be tuned by selection of the matrix **Q**. After initial trails **Q**=1.5**I** was adopted for all three models. Figure 2 shows the detected stiffness at each storey compared to the simulated values. The figure shows generally a slower response to changes in the structure than in Figure 1. Also, in the 3rd storey the detected damage appeared to not track the simulated damage. One problem with the Kalman filter was the number of parameters required to define the filter for efficient tracking. In this case, three 30×30 matrices were required while the forgetting factor approach required only three forgetting factors.

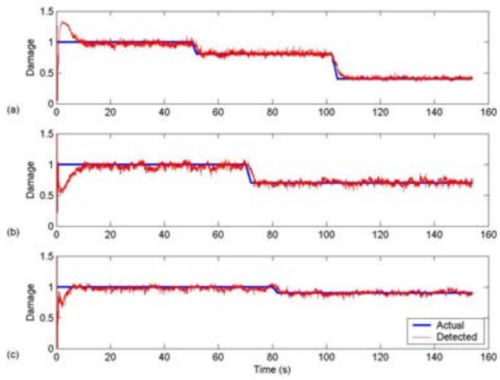

Figure 1. Stiffness detection in 3DOF linear model using forgetting factor: (a) 1st storey, (b) 2nd storey, and (c) 3rd storey.

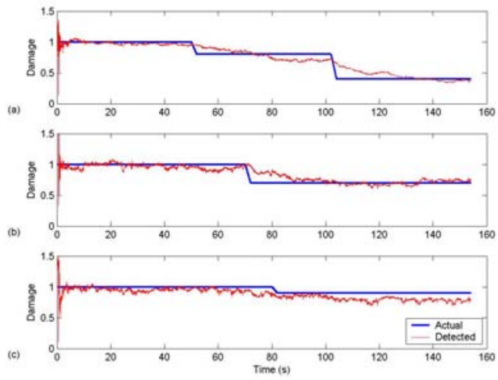

Figure 2. Stiffness detection in 3DOF linear model using Kalman filter: (a) 1st storey, (b) 2nd storey, and (c) 3rd storey.

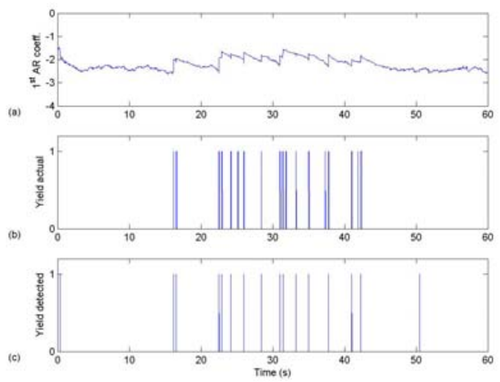

Figure 3. Yielding detection in a SDOF elastoplastic oscillator: (a) 1st AR coefficient, (b) actual yielding, and (c) detected yielding.

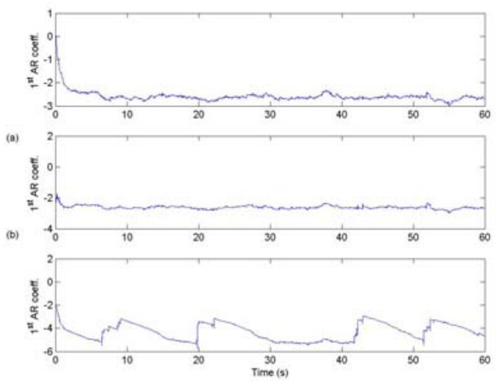

Figure 4. 1st AR coefficient for the 3DOF Bouc-Wen structure: (a) 1st storey, (b) 2nd storey, and (c) 3rd storey.

3.2 Application to a SDOF elastoplastic oscillator

The previous simulation has dealt with the detection of damage in a linear structure in which damage was defined as a change in stiffness. However, the response of a damaged structure during a strong earthquake would be most likely nonlinear. To investigate the implications of nonlinearity on damage detection a SDOF elastoplastic oscillator was studied. The following model properties were adopted k=5×10^5 N/m, m=1×10^4 kg, 5% viscous damping and a yield restoring force of 2×10^4 N. The model was subjected to Gaussian white noise ground excitation with a peak acceleration of 0.5g. A univariate AR(20) model was identified using the forgetting factor method with λ=0.99 from the acceleration time history. Figure 3a shows the 1st AR coefficient identified using the forgetting factor approach. Figure 3b shows when the structure was actually yielding, adopting the value 1. The 1st AR coefficient appeared to change abruptly when the structure yielded. Similar behavior was observed for the remaining AR coefficients, however, analysis was restricted to the 1st AR coefficient in this study. These abrupt changes in the 1st AR coefficient could be utilized to detect yielding events. The time series formed by the 1st AR coefficient was analyzed as a random walk model (Wei 2006) using intervention analysis. Yielding events in the structure were considered to be innovational outliers and the procedure outlined previously was used. A threshold value of $\pm 3\sigma_e$ was adopted and once exceeded the structure was assumed to be yielding. Figure 3c shows the instances where an innovational outlier was detected and yielding was thought to occur, assuming the value 1. In two instances a false yielding event was detected, once in the initial start-up phase, which is just an error caused by the assumed zero initial conditions, and again, shortly after 50 seconds. Of the 136 instances in which the structure was yielding, only 22 of these events were correctly detected. However, most of the yielding events lasted for several time instances or occurred very closely in time. Figures 3b and 3c clearly show that the initial yield in a group of closely spaced yields was successfully detected.

3.3 Application to a 3DOF Bouc-Wen hysteretic system

More realistically, the transition between elastic and plastic behavior in a structure would be smoother. The popular Bouc-Wen hysteretic model (Yang et al. 2007) can be used to model a broad range of hysteretic restoring forces and was adopted in this study for a 3DOF building model. The lateral stiffness of each storey was set to $k_1=k_2=k_3=1.0\times10^7$ N/m, lumped storey masses to 1.0×10^4 kg and the viscous damping coefficients to 1.0×10^4 Ns/m. The maximum restoring forces $Rm_1=1.2\times10^5$ N, $Rm_2=1.0\times10^5$ N and $Rm_3=1.0\times10^5$ N were adopted. The following Bouc-Wen parameters were adopted $A=k_1/Rm_1$, $\gamma=0.5k_1/Rm_1$, $\beta=0.5k_1/Rm_1$ and $n=20$. The structure was excited with Gaussian white noise ground motion with a peak acceleration of 0.5g. During the simulation yielding occurred on the 1st and 2nd stories while the 3rd storey remained elastic.

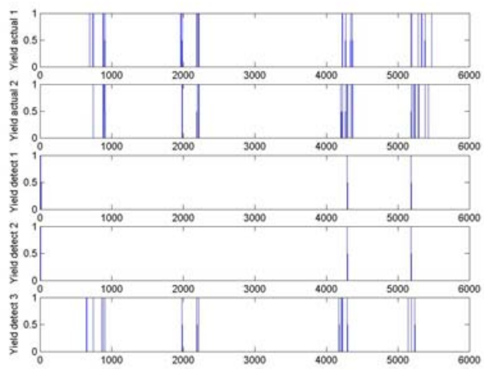

Figure 5. Detection of yielding in a 3DOF Bouc-Wen model: (a) actual 1st storey yield, (b) actual 2nd storey yield, (c) yield detected from 1st storey AR model, (d) yield detected from 2nd storey AR model, and (e) yield detected from 3rd storey AR model.

A univariate AR(20) model was identified from the interstorey acceleration of each storey using the forgetting factor approach with $\lambda=0.988$ adopted for all three models. Figure 4 shows the 1st AR coefficient for all three models. In the figure, the value of all three coefficients, particularly the 1st AR coefficient for the 3rd storey model, appeared to jump at various time instants. This could be used to detect a departure from linear behavior. Due to the smooth transition between elastic and plastic behavior, the structure was assumed to sustain damage when the restoring force was greater than 99% of the maximum restoring force. Figure 5a and 5b shows the time instances at which the 1st and 2nd storey was actually yielding, assuming the value 1. The time series formed by the 1st AR coefficients for all three models were analyzed as random walk models using intervention analysis. As previously for the elastoplastic model, yielding events in the structure were considered to be innovational outliers and were identified using the procedure outlined in Section 2.2.4. A threshold of $\pm3\sigma_e$ was adopted. The detected yielding from analysis of the 1st AR coefficients time series for the three stories (see Figure 4), has been shown in Figures 5c-e, assuming a value of 1 for yielding. A total of 115 yielding events were present and overall 15 of these were correctly identified. However, Figure 5 shows that the first initial yield in a group of closely spaced yields could successfully be detected. Localization of the yielding to either the 1st or 2nd storey proved to be difficult. A majority of yielding was detected from analysis of the 1st AR coefficient time series originating from the 3rd storey (Figure 5e).

4 CONCLUSIONS

An on-line method of damage detection using recursive identification of the AR time series models has been presented. Firstly, a linear 3DOF lumped-mass shear structure with damage introduced as a loss in stiffness was studied. Two recursive system identification algorithms, the forgetting factor and Kalman filter, were investigated for AR model identification. A BP ANN was trained to relate changes in the AR coefficients to the remaining stiffness at each storey and was used to track time dependent stiffness. The forgetting factor approach allowed for easier adjustment of the tracking properties than the Kalman filter and showed better results.

Nonlinear damage detection was investigated on a elastoplastic SDOF oscillator and a 3DOF Bouc-Wen hysteretic system. For the elastoplastic model, yielding in the structure was observed to cause abrupt changes in the AR coefficients. Intervention analysis was used to identify outliers in AR coefficient time series, allowing yielding to be detected. Studies on the 3DOF Bouc-Wen hysteretic system showed that yield detection was achievable in a system in which elastic and plastic response regions were less clearly defined. The use of recursive identification techniques for on-line nonlinear damage detection appears to be promising but further efforts are required such as experimental verification and more rigorous analytical treatment of data.

One of the challenges of using ANNs is that they require training data from damage states, which in practical applications may be very difficult to obtain. Computer models may need to be resorted to for obtaining such data. On the other hand, the approach using intervention analysis is much less dependent on extensive data base of structural responses but in the present form can only be used to detect yielding. These challenges will be addressed in future research.

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