Structural damage localization using modal macro strain-based flexibility methods with long-gage FBG sensors

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ABSTRACT: The performance of change in modal flexibility and flexibility curvature methods of damage identification using global quantities of structure such as acceleration or displacement responses from the translational degrees-of-freedom is limited by its vulnerability to measurement noise and inability to accurately localize closely spaced or multiple-damages in structures. However, recent advances in smart sensors and sensing technology, material sciences and information technologies have significantly improved the performance of various vibration-based structural damage identification techniques. This paper presents modal macro strain-based flexibility and flexibility curvature methods for damage identification in flexural structures using long-gage FBG sensors. A simply supported beam is investigated through numerical simulation to verify the applicability of the proposed methods to practical civil structures where measurement noise is inevitable. The findings of the study show that distributed strain-flexibility methods are very sensitive to small local damages, robust to as much as 5% measurement noise level and can reliably locate both single and closely distributed damages.

1 INTRODUCTION

Early damage detection and localization in structure is very essential to ensure its safety and functionality. The development of structural health monitoring techniques has advanced the need for prompt and reliable assessment of the performance and safety of civil structures which hitherto receive due attention only immediately after natural disaster or accidental damage. Vibration-based damage identification (VBDI) methods have increasingly become an essential field of research due to their flexibility of measurement, cost-effectiveness and nondestructive approach of damage identification in global structure. During the past few decades, a significant amount of research has been conducted in the area of non-destructive damage detection via changes in modal responses of a structure such as natural frequencies, mode shapes and modal damping. Most VBDI techniques utilize changes in modal features such as natural frequencies, mode shape curvatures or strain mode shapes, modal strain energy, frequency response functions, modal flexibility and flexibility curvatures. Although most of the VBDI techniques have demonstrated various degrees of success in identifying damages in small structures; the inherent complexity, the diversity as well as heterogeneity of material properties, the influence of noise and the variability of environmental conditions are basic challenges facing civil SHM. Also, the type and placement of sensors are very crucial for reliable damage identification and

successful implementation of SHM programs. Extensive literature reviews on a range of different VBDI techniques have been reported by Doebling *et al.* (1996) and Sohn *et al*. (2003).

Specifically, VBDI techniques using structural flexibility are receiving remarkable attention in civil engineering field because the flexibility matrix can be easily and accurately estimated from a few low frequency modes. Several damage detection methods based on the modal flexibility have been proposed. The earliest damage localization technique based on measured flexibility is presented by Pandey and Biswas (1994). However, notable among the drawbacks of the conventional change in flexibility method is its inability to distinctly locate multiple or closely spaced damages and the necessity for a sufficiently fine spatial resolution of measurement for its effective performance. Toksoy and Aktan (1994) utilize the anomaly in the uniform load surface calculated by modal flexibility as a damage feature. Zhang and Aktan 1995 also propose a damage detection method based on changes in curvatures of uniform flexibility shape. The damage locating vector (DLV) was proposed to localize damages based on changes in the flexibility with the intact finite element model (Bernal 2002). However, the reliability of the method for practical application depends primarily on the accuracy of the flexibility matrices of the intact and damaged structures. Lu et al. (2002) show that the most practical advantage of flexibility curvature method is its high sensitivity to closely distributed damages without the need for the parameters of intact structure as baseline. However, flexibility curvature is very sensitive to the errors of mode shapes and numerical differentiation approximations. Adewuyi et al. (2009) also find that flexibility change and flexibility curvature methods using displacement or acceleration data are very vulnerable to noise and hence not reliable for civil SHM.

Until recently, most of the researchers have considered only the displacement mode shape as a reference to analyze the dynamic behavior of the structures. Moreover, strain has been considered the most sensitive response to local damage. Sohn *et al*. (2003) document few improvements in the modal curvature method by using strain measurement data. However, besides the problems associated with stability, durability and long-term reliability, traditional foil strain gages cannot effectively reflect the influence of damage in large-scale civil structures unless when attached to the damaged region because they are “point” sensors and damage is usually localized and its location is unknown a priori.

With the development of measurement device and signal processing techniques, structural dynamic properties can be measured more conveniently and precisely, and the various damage identification methods can be improved. The advent of fiber optic sensors (FOS), for instance, has provided flexibility and practicality where the conventional sensors are not capable to make the desired measurements. It has been established that long-gage FOS sensors have the potential to integrate both local and global information due to the fact that strain is a typical local response and distributed sensor placement helps to record the data covering the large region of a structure. The development of FOS for civil engineering infrastructure can be found in Sohn *et al*. (2003) and Inaudi (2001). In addition, the authors’ laboratory has dedicated special effort towards developing a feasible distributed long-gage strain sensing system for practical adaptation to civil structural engineering (Wu and Li, 2007; Li and Wu, 2007). Adewuyi et al. (2009) prove that the adaptation of distributed strain measurements from long-gage FBG sensors to typical damage identification methods can significantly improve their sensitivity and robustness to noise.

This paper presents modal macro strain-based flexibility and flexibility curvature methods for damage identification in flexural structures practicable with long-gage FBG sensors. The relationship between the rotational displacements and strain measurement is formulated; and the practicability of modal macro-strain for direct structural flexibility evaluation and damage localization is investigated through numerical simulation for practical applications in civil structures where measurement noise is inevitable.

2 THEORETICAL BACKGROUND

2.1 Dynamic Macro-strain measurements from long-gage FBG sensors

Most vibration-based damage identification methods are premised on linearity of system behavior in the pre- and post-damage states. Consider a long-gage fiber optic sensor attached onto the surface of a structural element in a configuration parallel to the neutral axis of the beam element as shown in Figure1.

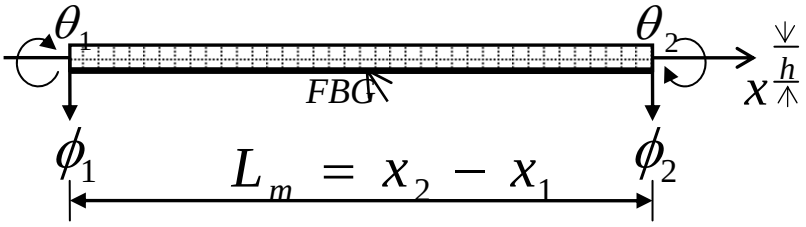

Figure 1. Fiber Bragg Grating sensor attached onto the surface of a beam element

The change in length of FBG sensor can be obtained by integrating the axial infinitesimal strain, ε_m, along the length of the sensor. For the FBG sensor attached onto the bottom surface of one beam element, the sensor deformation, Δl, is theoretically given by

$$\Delta l = \int_{x_1}^{x_2} \varepsilon_m dx = \frac{h}{2}\int_{x_1}^{x_2} \theta_x dx = \frac{h}{2}\left(\frac{d\phi_x}{dx}\right) \tag{1}$$

where θ_x and ϕ_x are rotation and vertical translational displacement of the beam element along the sensor. x_1 and x_2 are the coordinates of the two nodes corresponding to the ends of the beam element. The macro-strain of the FBG sensor can then be described as

$$\varepsilon_m = \frac{\Delta l}{x_2 - x_1} = \frac{h}{2}\frac{(\theta_2 - \theta_1)}{(x_2 - x_1)} = \frac{h}{2}\frac{(\theta_2 - \theta_1)}{L_m} \tag{2}$$

The curvature mode shape κ_m can be measured directly from modal macro-strain (MMS) measurement δ_m from the m^{th} long-gage sensor as

$$\kappa_m = \frac{\bar{\varepsilon}_m}{h_m} = \frac{\delta_m}{h_m} \tag{3}$$

2.2 Distributed strain-based flexibility methods

2.2.1 Overview of dynamic flexibility methods using conventional measurement techniques

Pandey and Biswas (1994) present a damage identification technique that uses the changes in measured flexibility of the structure from first few measured modes. The flexibility matrix, $[F]$, of a structure can be easily estimated as follows:

$$[F_{ij}] = \Phi\Omega^{-1}\Phi^T \approx \sum_{j=1}^{n} \frac{1}{\omega_i^2}\{\phi_i\}\{\phi_i\}^T \tag{4}$$

where Φ is the modal matrix and Ω is the diagonal matrix containing eigenvalues ω_i^2 on its diagonal; n is number of measured modes; ω_i and ϕ_i are the modal frequency and mass-normalized mode shapes respectively for the i^{th} mode. Denoting undamaged and damage states by superscripts u and d respectively, the change in flexibility caused by damage can be expressed as

$$[\Delta F] = [F^d] - [F^u] \tag{5}$$

$$\delta_i = \max|\Delta F| \tag{6}$$

The diagonal elements or column elements of matrix, $[\Delta F]$ corresponding to the largest δ_i indicates the i^{th} DOF with damage.

2.2.2 Distributed strain-based flexibility method approach

In general, the definition of flexibility is not limited to the translational displacement pattern of the structure associated with a unit force but also covers the rotation pattern associated with a bending moment applied at the corresponding DOF. However, with the recent major advances in the field of structural dynamics and mechanical vibration measurements, the modal flexibility can be modified to accommodate rotation measurements since rotation of mode shapes may be feasible to be measured via modal macro-strain data.

Consider a simply supported $m-$element beam of equal length l_m and connected to $N = m+1$ nodes as shown in Figure 2. Each intermediate node has two DOF, a vertical translation and a rotation; while the two support nodes have only the rotational DOF.

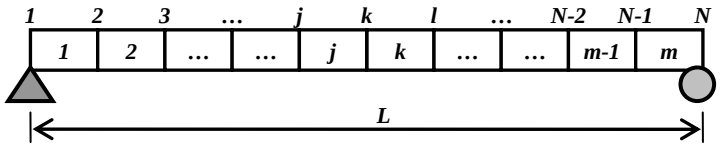

Figure 2. Formation of macro strain-based flexibility

The flexibility matrix whose columns represent the rotational displacement pattern of the structure when a unit moment is applied at the corresponding DOF as obtained from the mass-normalized rotational mode shapes and frequencies is given by

$$F_\theta = \sum_{r=1}^{N} \frac{1}{\omega_r^2} \{\theta_i\}\{\theta_i\}^T \tag{7}$$

However, by attaching long-gage FBG sensor of equal gage length l_m to each element of the beam and each measuring section is separately fixed on its ends, the angle between string and horizon is the relative rotation between the two end nodes of the element when a unit moment is applied on the structure. Hence, the flexibility matrix due to the end rotations of each element can be assembled as

$$F_{\bar{\theta}} = \sum_{r=1}^{N} \frac{1}{\omega_r^2} \begin{Bmatrix} \theta_{2r} - \theta_{1r} \\ \vdots \\ \theta_{nr} - \theta_{n-1,r} \end{Bmatrix} \begin{Bmatrix} \theta_{2r} - \theta_{1r} \\ \vdots \\ \theta_{nr} - \theta_{n-1,r} \end{Bmatrix}^T \tag{8}$$

It is noteworthy that although the gage length and element length are assumed to be equal, the algorithm is also appropriate for unequal lengths. The strain flexibility matrix can be simplified in terms of r^{th} mass-normalized modal macro-strain mode shape δ_r as

$$F_{\bar{\theta}} = F_{\bar{\varepsilon}} = \sum_{r=1}^{m} \frac{1}{\omega_r^2} \begin{Bmatrix} \delta_{1r} \\ \delta_{2r} \\ \vdots \\ \delta_{m-1,r} \\ \delta_{mr} \end{Bmatrix} \begin{Bmatrix} \delta_{1r} \\ \delta_{2r} \\ \vdots \\ \delta_{m-1,r} \\ \delta_{mr} \end{Bmatrix}^T = \Phi^{\bar{\varepsilon}} \Omega^{-1} \Phi^{\bar{\varepsilon}T} \tag{9}$$

where $\Phi^{\bar{\varepsilon}}$ and $\Phi^{\bar{\varepsilon}T}$ are the mass-normalized modal macro-strain matrix and its transpose, respectively. Denoting undamaged and damage states by superscripts u and d respectively, the change in macro-strain flexibility caused by damage can be expressed as

$$\left[\Delta F_{\bar{\varepsilon}}\right] = \left[F_{\bar{\varepsilon}}^{d}\right] - \left[F_{\bar{\varepsilon}}^{u}\right] \tag{10}$$

$$\delta_i = \max\left|\Delta F_{\bar{\varepsilon}}\right| \tag{11}$$

The column or the diagonal elements of matrix $\left[\Delta F_{\bar{\varepsilon}}\right]$ corresponding to the largest δ_i indicates the i^{th} DOF with damage.

2.2.3 Change in distributed strain-based flexibility curvature method

Similar to the change in flexibility curvature method proposed by Zhang and Aktan (1995), the change in distributed strain flexibility curvatures between the intact and damaged structures can be expressed as

$$\{\Delta F_{\bar{\varepsilon}}^{''}\} = \sum_{j=1}^{n} \left| \{F_{\bar{\varepsilon}}^{''d}\} - \{F_{\bar{\varepsilon}}^{''u}\} \right| \tag{12}$$

where $\{\Delta F''\}$ is the absolute flexibility curvature change and n is number of DOFs. The elements of $\{\Delta F''\}$ that have comparatively large values correspond to damage location.

3 NUMERICAL SIMULATION

Numerical simulations are carried out on a $20 \times 10 \times 2000$ mm simply supported beam modeled by 32 equal elements connected to 33 nodes as shown in Figure 1. Each intermediate node has two DOFs, translational and rotational displacements while the two support nodes have only the rotational DOF. Damage is simulated by reduction of bending stiffness at the affected elements. Two damage scenarios are considered: (1) 10% reduction in bending stiffness at element 16 designated SC1 and (2) multiple damages with 10% bending stiffness reduction at elements 12 and 16 designated SC2. The damage scenarios, which represent moderate damage levels, are sited at sections of high flexural moment. The element strain measurements are the basic response data used to study the techniques. A practical condition is simulated by introducing 2% and 5% normally distributed random noise into the distributed strain mode shapes; and 1% random noise is added into the natural frequencies.

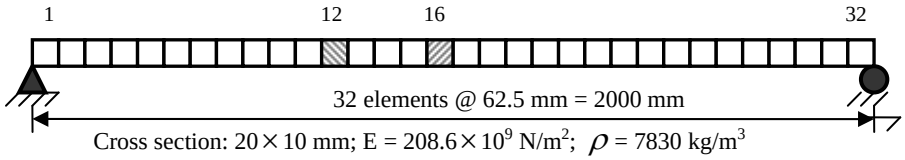

Figure 3. Simply supported beam model

3.1 Discussion of numerical investigation results

3.1.1 Natural frequency shifts

The first five natural frequencies for the intact and damage cases SC1 and SC2 are listed in Table 1. It is obvious that the simulated damage is small that it does not translate to a significant change in the natural frequencies. The percentage drops in the modal frequencies from intact to damage cases are marginal, and hence could not clearly and confidently indicate damage or its location.

Table 1. Natural frequencies for the intact and damage cases SC1 and SC2

Mode No.	Natural frequency (Hz)			% Frequency drop	
	Intact	SC1	SC2	SC1	SC2
1	5.8258	5.8058	5.7895	0.3433	0.6231
2	23.303	23.302	23.254	0.0043	0.2103
3	52.429	52.254	52.242	0.3338	0.3567
4	93.201	93.185	92.883	0.0172	0.3412
5	145.62	145.16	144.97	0.3159	0.4464

3.1.2 Change in distributed strain flexibility

Curves of the distributed strain-flexibility and its curvature for the two damage scenarios using the first 3 and 5 modes are plotted in Figure 4. Although the column or the diagonal elements of flexibility change matrix corresponding to the largest δ_i indicates the i^{th} DOF with damage, the former is used this analysis. It can be clearly seen that the change in strain-based flexibility method can correctly and clearly locates single and multiple damages unlike the conventional flexibility method based on displacement mode shapes. It is also apparent from Figure 4 that distributed strain-based flexibility is more robust to measurement noise as the method proved effective up to 5% normally distributed random noise added into the strain mode shapes. Moreover, similar to the conventional flexibility approach, it is very clear that the flexibility matrix converges rapidly with the first 3 modes. Whereas Lu et al (2002) find that displacement-based flexibility change is not a good damage indicator for closely distributed damages, even when the damage extent of the element is increased to 50% stiffness loss; distributed strain-flexibility method has proved very reliable even with 10% damage.

3.1.3 Change in uniform distributed strain flexibility curvatures

Figure 5 shows the plots of change in distributed strain flexibility curvatures for the two damage cases SC1 and SC2 using the first 3 and 5 modes. By using Equation (12), a quantity of higher derivatives than the modal curvature is obtained because strain mode shapes are essentially curvature mode shapes. It is apparent that the change in distributed strain flexibility curvature can accurately and distinctly localize both single and multiple damages even with as much as 5% random noise level in strain mode shape. As stated above, there is a rapid convergence in the flexibility matrix converges with the first three modes. Another remarkable practical quality of this method is that it can localize damage in any determinate flexural structures without any reference to the parameters of the intact structure as baseline. In contract with the flexibility curvature method using translational displacements at the DOFs that is not sensitive to small damages less than 30% stiffness loss (Lu et al. 2002); distributed strain-flexibility curvature method is very sensitive to small damages. Furthermore, it is evident from Figure 6 that the curvature of macrostrain-flexibility of flexural structures is a non-baseline feature for damage localization. In other words, the information of the intact state is not required to correctly locate single and multiple damages in the structure.

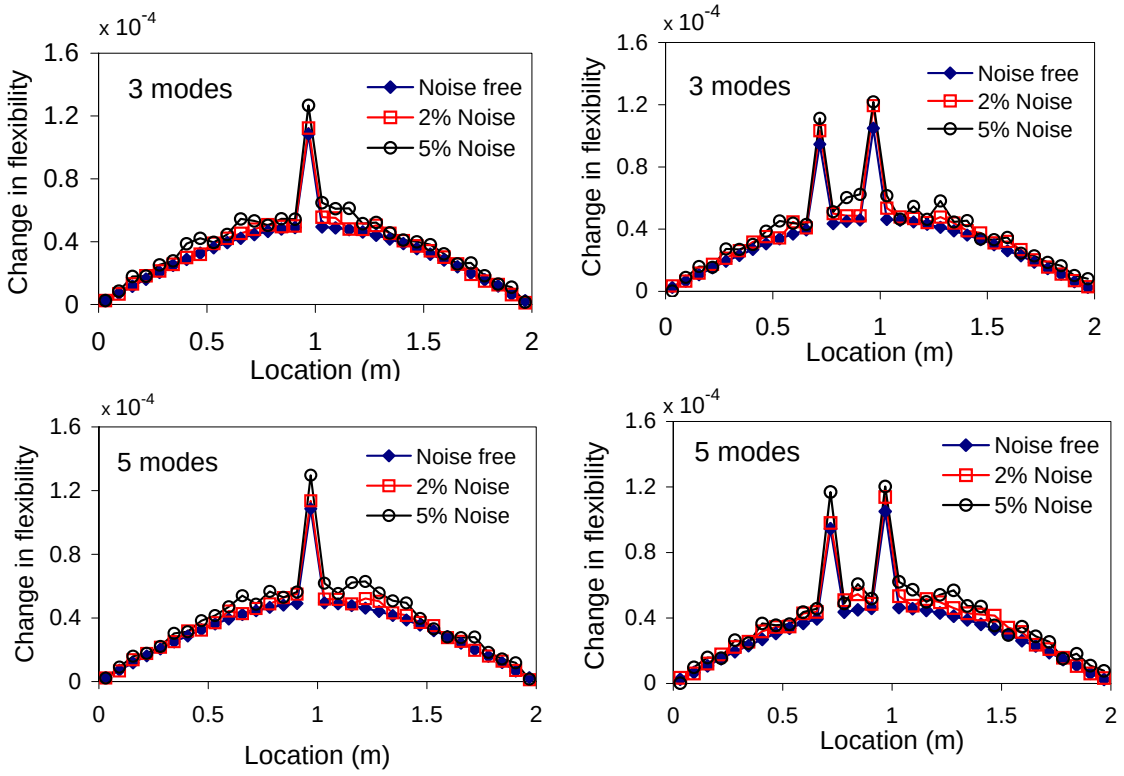

Figure 4. Maximum change in strain-flexibility due to single and multiple damages using 3 and 5 modes.

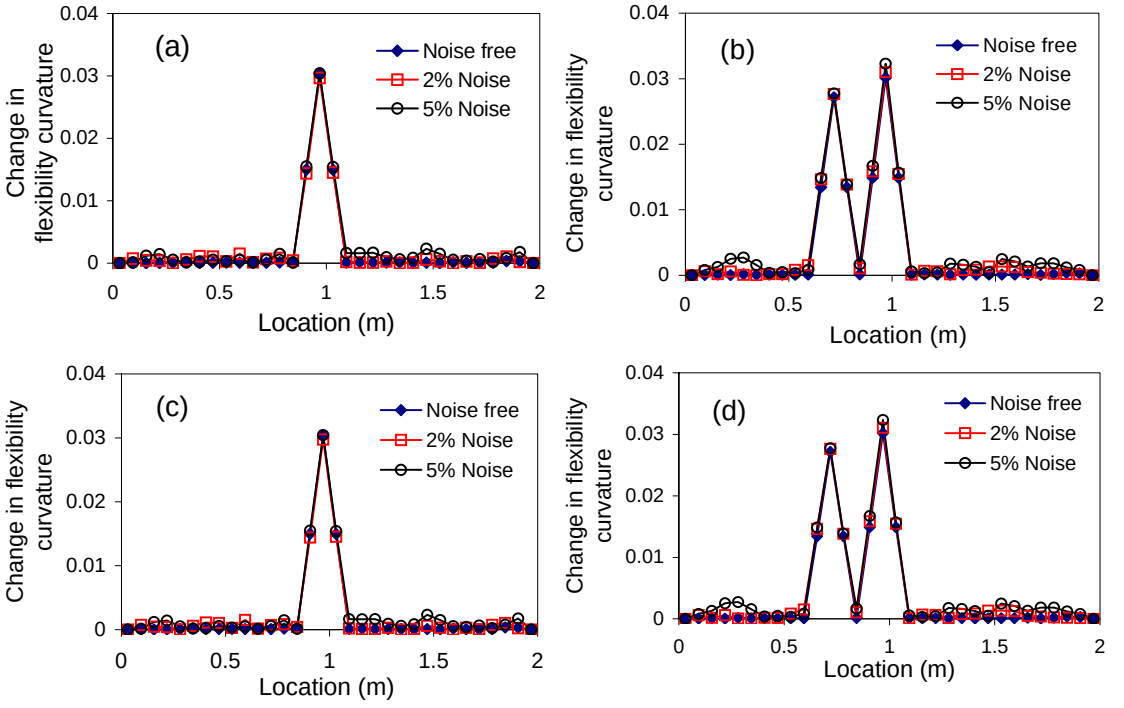

Figure 5. Change in strain-flexibility curvatures due to single and multiple damages using 3 and 5 modes.

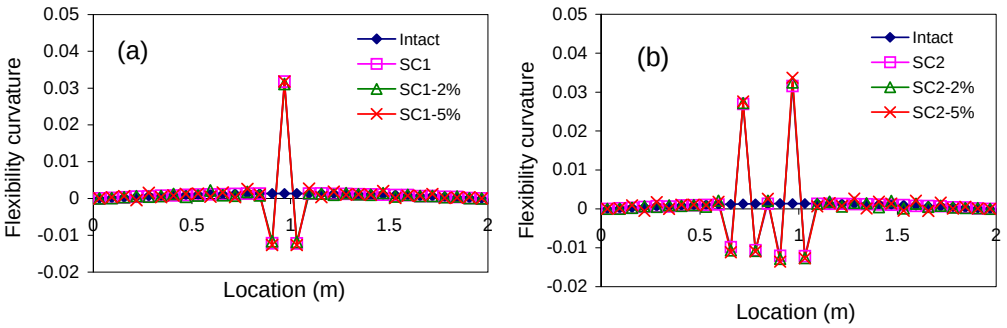

Figure 6. Macrostrain flexibility curvatures of intact, single and multiple damage scenarios from 3 modes.

4 CONCLUSIONS

Based on distributed fiber optic sensing technique typical of long-gage FBG strain sensors, the applicability of change in distributed strain-flexibility and uniform distributed strain-flexibility curvature change methods has been investigated. The following are the main conclusions:

(1) Direct replacement of displacement mode shapes with distributed or modal macrostrain mode shapes produces more robust flexibility matrices.
(2) The distributed strain-flexibility methods are very sensitive to small local damage.
(3) The methods can reliably locate both single, multiple and closely distributed damages.
(4) Like the typical flexibility curvature method, the curvature of distributed-strain flexibility matrix of a damaged simply supported flexural structure can correctly localize damage without the for any baseline information.

They are very robust to as much as 5% measurement noise level and hence suitable for civil SHM.

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