Statistical moment-based damage detection of building structures

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ABSTRACT: This paper presents a novel structural damage detection method based on the statistical moments of dynamic responses of building structures under random excitation. The principle of the method is put forward in terms of a single-degree-of-freedom (SDOF) system and then extended to multi-degree-of-freedom (MDOF) systems with resort to the least-squares method. The numerical studies on a shear building structure manifest that the proposed method is sensitive to local structural damage but insensitive to measurement noise. Furthermore, the proposed method was experimentally examined to assess its reliability in practical applications. Eight damage scenarios were designed and a series of shaking table tests were conducted on three shear building models before and after damage under white noise or color noise ground excitations. The experimental results matched the theoretical values well and the statistical moment-based damage detection method is demonstrated to be feasible and accurate.

1 INTRODUCTION

Vibration-based structural damage detection methods have attracted considerable attention in recent years for the assessment of health and safety of civil engineering structures. Although these methods have demonstrated various degrees of success, the damage detection of complex civil engineering structures still remains a challenging task. The main obstacles include the insensitivity to local structural damage for the methods based on modal properties and the high sensitivity to measurement noise for the methods based on mode shape changes and those derivatives and model updating methods (Alvandi *et al*. 2006, Doebling *et al*. 1998). Even for the modal strain energy method which is less affected by measurement noise, a 3% noise level has already been considered as a high level of noise which makes it difficult to identify damage locations of a structure (Worden *et al*. 2005). In this regard, a new structural damage detection method based on the statistical moments of dynamic responses of a structure under random excitation is proposed by the authors. Its efficiency and effectiveness in sensitivity to local structural damage but insensitivity to measurement noise are first demonstrated numerically and then verified experimentally.

2 STATISTICAL MOMENT-BASED DAMAGE DETECTION METHOD (SDOF)

Structural damage such as stiffness reduction in a structure under random excitation will cause changes in both probability density function (PDF) and statistical moments of structural responses. The odd moments, in general, relate to the information about the position of the peak of PDF while the even moments indicate the shape characteristics of PDF. Therefore, the

changes in statistical moments, particularly higher even statistical moments, may be sensitive to structural damage. The principle of the statistical moment-based damage detection method is put forward in terms of a SDOF system in this section.

2.1 Statistical moments

Let us consider a one-story shear building subjected to zero-mean Gaussian white-noise ground acceleration. Only linear-elastic structural system is considered. The equation of motion of the shear building can be expressed as

$$m\ddot{x} + c\dot{x} + kx = -m\ddot{x}_g \quad \text{or} \quad \ddot{x} + 2\xi\omega_o\dot{x} + \omega_o^2 x = -\ddot{x}_g \tag{1}$$

where m , c and k are respectively the mass, damping and stiffness coefficient of the building; x , $\dot{x}$ and $\ddot{x}$ are respectively the relative displacement, velocity and acceleration response of the building to the ground; $\ddot{x}_g$ is the Gaussian white noise ground acceleration with a constant power spectral density (PSD) S_0 ; ξ is the damping ratio of the building; and ω_o is the circular frequency. For a linear structural system, if the excitation is a stationary Gaussian random process, the structural response is also a stationary Gaussian random process. The second-, fourth- and sixth-order moments of displacement, velocity and acceleration responses can be derived accordingly and listed in Table 1.

Table 1 Statistical moments of displacement, velocity and acceleration

M	$M_2(=\sigma^2)$	$M_4(=3\sigma^4)$	$M_6(=15\sigma^6)$
M^{dis}	$\frac{\pi S_0}{2\xi\sqrt{mk^3}}$	$\frac{3\pi^2 {S_0}^2}{4m\xi^2 k^3}$	$\frac{15\pi^3 {S_0}^3}{8\xi^3\sqrt{m^3k^9}}$
M^{v}	$\frac{\pi S_0}{2\xi\sqrt{m^3k}}$	$\frac{3\pi^2 {S_0}^2}{4m^3\xi^2 k}$	$\frac{15\pi^3 {S_0}^3}{8\xi^3\sqrt{m^9k^3}}$
M^{a}	$\frac{\pi S_0(1-4\xi^2)\sqrt{k}}{2\xi\sqrt{m^5}}$	$\frac{3\pi^2 {S_0}^2}{4m^5\xi^2}(1-4\xi^2)^2 k$	$\frac{15\pi^3 {S_0}^3}{8\xi^3\sqrt{m^{15}}}(1-4\xi^2)^3\sqrt{k^3}$

2.2 Sensitivity analysis

Based on Table 1, the following sensitivity equations can be derived:

$$\frac{dM_2^{dis}}{M_2^{dis}} = -\frac{3}{2}\frac{dk}{k}, \quad \frac{dM_2^{v}}{M_2^{v}} = -\frac{1}{2}\frac{dk}{k}, \quad \frac{dM_2^{a}}{M_2^{a}} = \frac{1}{2}\frac{dk}{k} \tag{2}$$

$$\frac{dM_4^{dis}}{M_4^{dis}} = -3\frac{dk}{k}, \quad \frac{dM_4^{v}}{M_4^{v}} = -\frac{dk}{k}, \quad \frac{dM_4^{a}}{M_4^{a}} = \frac{dk}{k} \tag{3}$$

$$\frac{dM_6^{dis}}{M_6^{dis}} = -\frac{9}{2}\frac{dk}{k}, \quad \frac{dM_6^{v}}{M_6^{v}} = -\frac{3}{2}\frac{dk}{k}, \quad \frac{dM_6^{a}}{M_6^{a}} = \frac{3}{2}\frac{dk}{k} \tag{4}$$

It can be observed that the ratio of the relative change of the second-, fourth- and sixth-order moments of displacement to the relative change of stiffness is three times of the counterparts of velocity and acceleration either positively or negatively. This result reflects that the relative change of the statistical moment of displacement is two times more sensitive to the relative change of stiffness than that of velocity or acceleration. Thus, the statistical moment of displacement is selected. Furthermore, based on the numerical results the fourth-order moment instead of the second-order or the sixth-order moment is selected for the proposed method to

balance the sensitivity of an index to structural damage and its stability to random excitation.

2.3 Damage detection

The fourth statistical moments of a measured structural displacement response whenever it fits the Gaussian distribution or not can be directly calculated using the following equation:

$$M_4 = \frac{1}{N}\sum_{i=1}^{N} x_i^4 - \frac{4}{N^2}\sum_{i=1}^{N} x_i \sum_{i=1}^{N} x_i^3 + \frac{6}{N^3}(\sum_{i=1}^{N} x_i)^2 \sum_{i=1}^{N} x_i^2 - \frac{3}{N^4}(\sum_{i=1}^{N} x_i)^4 \quad (5)$$

where N is the number of the sampling points of the structural response time history. The stiffness of the structure can then be obtained by

$$k = \sqrt[3]{\frac{3\pi^2 S_0^2}{4m\xi^2 M_4^{dis}}} \quad (6)$$

In summary, the statistical moment-based damage detection of a SDOF system can be carried out in the following steps: (1) measure or compute the displacement response of the SDOF system under Gaussian white noise ground acceleration before and after damage; (2) calculate the fourth-order statistical moments before and after damage, $\hat{M}_4^u$ and $\hat{M}_4^d$, respectively, using Eq. (5); (3) calculate the structural stiffness of the undamaged and damaged structure $\hat{k}^u$ and $\hat{k}^d$ using Eq. (6); and (4) the structural damage severity $\hat{\mu}$ can then be identified by

$$\hat{\mu} = \frac{\hat{k}^d - \hat{k}^u}{\hat{k}^u} \times 100\% \quad (7)$$

2.4 Effect of measurement noise

To investigate the effect of measurement noise on damage detection quality, a white noise is added to the displacement response. The measurement noise intensity is defined as the ratio of the root mean square (RMS) of measurement noise ε to the RMS of displacement response x.

$$\alpha = \frac{RMS(\varepsilon)}{RMS(x)} \times 100\%. \quad (8)$$

By defining the structural response with measurement noise as y, there is a relationship: $y = x + \varepsilon$, where x is the actual structural response; ε is the measurement noise independent of x. Then, by taking a noise intensity of 15% for example, one may have: $\sigma_y^2 = \sigma_x^2 + \sigma_\varepsilon^2 = 1.0225\sigma_x^2$. The identified stiffness with considering measurement noise k^n can then be derived as

$$k^n = \sqrt[3]{\frac{3\pi^2 S_0^2}{4m\xi^2 M_{4y}}} = \frac{1}{\sqrt[3]{1.0225^2}} \sqrt[3]{\frac{3\pi^2 S_0^2}{4m\xi^2 M_{4x}}} = 0.9853k \quad (9)$$

where M_{4y} and M_{4x} are respectively the fourth-order moments of y and x; k is the real stiffness. It can be seen that the theoretical noise effect ratio is only 1.47% even at the noise intensity of 15%. This result indicates that the fourth-order moments of displacement response are not sensitive to measurement noise.

3 STATISTICAL MOMENT-BASED DAMAGE DETECTION METHOD (MDOF)

Let us consider an N-story linear shear building subjected to ground acceleration $\ddot{x}_g$ in this section. The lumped mass, lateral stiffness and structural damping coefficients of the ith story of the building are denoted as m_i, k_i and c_i, respectively, where $i = 1, 2,\ldots,N$. The equation of

motion in the matrix form for this shear building can be expressed as

$$\mathbf{M}\ddot{\mathbf{x}}(t)+\mathbf{C}\dot{\mathbf{x}}(t)+\mathbf{K}\mathbf{x}(t)=-\mathbf{M}\mathbf{I}\ddot{x}_g(t) \tag{10}$$

where $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are the mass matrix, damping matrix and stiffness matrix of the building structure, respectively; $\ddot{x}_g$ is the ground excitation whose power spectral density is denoted as $S_0(\omega)$ $(\omega_l \le \omega \le \omega_u)$. Because story drifts are directly related to horizontal stiffness, the statistical moments of story drifts are considered in this section. By adopting the Rayleigh damping assumption, the Fourier transform of the story drift of the *i*th floor can be obtained

$$\Delta X_i(\omega)=X_i(\omega)-X_{i-1}(\omega)=\sqrt{S_o(\omega)}\sum_{k=1}^{N}\Delta\alpha_{ik}(\omega)m_k,\ \ X_0(\omega)=0,\ i=1,2,3...N \tag{11}$$

$$\Delta\alpha_{ik}(\omega)=\alpha_{ik}(\omega)-\alpha_{(i-1)k}(\omega),\quad \text{and}\quad \alpha_{0k}(\omega)=0 \tag{12}$$

$$\alpha_{ik}(\omega)=\sum_{j=1}^{N}\frac{\phi_{ij}(\mathbf{K})\cdot\phi_{kj}(\mathbf{K})}{\omega_j(\mathbf{K})^2-\omega^2+2i\omega\omega_j(\mathbf{K})\xi_j} \tag{13}$$

where $\phi_{ij}(\mathbf{K})$ is the *i*th component of the *j*th theoretical mode shape and $\omega_j(\mathbf{K})$ is the *j*th theoretical circular natural frequency. They both are only dependent on the given $\mathbf{K}$ if the mass matrix is assumed to be invariant. ξ_j is the *j*th modal damping ratio. In this paper, the first two modal damping ratios of the structure are estimated form the measured acceleration responses, while the higher modal damping ratios are derived according to the Rayleigh damping assumption. Based on Eq.(11), the fourth-order moment of the *i*th story drift can be expressed as

$$M_{4i}=3\sigma_{\Delta x_i}{}^4=3\left[\int_{\omega_l}^{\omega_u}S_o(\omega)\Big(\sum_{k=1}^{N}\Delta\alpha_{ik}(\omega)m_k\Big)\Big(\sum_{k=1}^{N}\Delta\alpha_{ik}(\omega)^*\cdot m_k\Big)d\omega\right]^2 \tag{14}$$

where $\alpha_{ik}(\omega)^*$ is the conjugate of $\alpha_{ik}(\omega)$. It can be seen that the theoretical fourth-order moment of the story drift is the function of the lateral stiffness, denoted as $\mathbf{M}_4(\mathbf{k})$. In addition, the fourth-order moment vector of the measured quantities can be directly calculated by Eq. (5), denoted as $\hat{\mathbf{M}}_4$. The residual vector between $\mathbf{M}_4(\mathbf{k})$ and $\hat{\mathbf{M}}_4$ can be written as

$$\mathbf{F}(\mathbf{k})=\mathbf{M}_4(\mathbf{k})-\hat{\mathbf{M}}_4 \tag{15}$$

Practically, the optimal stiffness vector can be identified by the least-squares method, that is, giving $\mathbf{k}$ an initial value $\mathbf{k}_0$ and minimizing $\|\mathbf{F}(\mathbf{k})\|^2$. Thus, the statistical moment-based damage detection method for a MDOF system can be carried out in three steps: (1) the fourth-order statistical moments of story drifts are estimated from the measured displacements for undamaged and damaged building respectively; (2) the structural stiffness vector is identified by the least-squares method for the undamaged and damaged building respectively. Since it is physically impossible that the lateral stiffness values of damaged models are larger than those of corresponding undamaged models, the constrained optimization method can be utilized to identify the lateral stiffness values of damaged models. The upper limit values of the lateral stiffness of damaged models in the constrained optimization method are the identified stiffness values of corresponding undamaged models; and (3) the structural damage locations and severity values can be detected by comparing the identified stiffness vector $\hat{\mathbf{k}}^u$ of the undamaged building with the identified stiffness vector $\hat{\mathbf{k}}^d$ of the damaged building.

4 A NUMERICAL EXAMPLE

In this section, the robustness of the statistical moment-based damage detection method is numerically demonstrated using a three-story shear building. The mass and horizontal stiffness

coefficients of the three-story shear building are respectively 350250 kg and 4728400 kN/m for the first story, 262690 kg and 315230 kN/m for the second story, and 175130 kg and 157610 kN/m for the third story. The mass of each floor is assumed to be invariant. The first, second and third modal damping ratios are taken as 1%, 2.14% and 5.56%, respectively. The ground acceleration is simulated as a colored white noise corresponding to the Kanai-Tajimi spectrum having parameters ω_g = 15.6 rad/sec and ζ_g = 0.6. The magnitude is chosen such that the maximal absolute value of acceleration is 5.0 m/sec^2. To simulate the colored white noise excitation, a method of digital simulation random processes developed by Shinozuka and Jan (1972) was utilized. The duration of the excitation time history is 1000s with the sampling frequency of 256 Hz. Various damage scenarios with different damage severities and locations are created. Random white measurement noises are also introduced into the structural responses to investigate the effect of measurement noise on damage detection quality.

4.1 Damage detection results with measurement noise free

To demonstrate the effectiveness of the proposed method, 8 damage scenarios with combination of various damage severities and locations are considered and investigated. The preset damage severities μ and identified damage severities $\hat{\mu}$ of every story are presented in Table 2. Scenarios 1 to 5 simulate single damage with different damage severities at different locations. Scenarios 6 to 8 simulate multi damages at different stories with same or different damage severities. Seen from Table 2, the identified damage severities are very close to the actual damage severities even when damage severity is only 2%. The damage locations and damage severities can be accurately detected when the measurement noise is not taken into account.

Table 2 Numerically identified damage severities for Scenarios 1-8 (MDOF system) (%)

Scenario	Story 1			Story 2			Story 3		
	μ	$\hat{\mu}$	$\hat{\mu}^N$	μ	$\hat{\mu}$	$\hat{\mu}^N$	μ	$\hat{\mu}$	$\hat{\mu}^N$
1	-2	-2.01	-1.96	0	-0.01	0.00	0	-0.01	0.00
2	-5	-5.02	-4.95	0	-0.02	0.00	0	-0.02	0.00
3	-20	-20.09	-20.05	0	-0.12	-0.08	0	-0.12	-0.07
4	0	-0.10	0.00	-5	-5.12	-4.86	0	-0.13	0.00
5	0	0.00	0.00	0	0.00	0.00	-10	-9.86	-9.78
6	-10	-10.01	-10.06	-10	-10.01	-10.16	0	-0.01	-0.04
7	0	0.00	0.00	-2	-1.66	-1.63	-10	-9.83	-9.81
8	-5	-5.83	-5.70	-10	-10.84	-10.72	-20	-20.75	-20.65

4.2 Damage detection results with measurement noise

To investigate the effect of measurement noise on damage detection quality, a white noise with the noise intensity of 15% is added to contaminate displacement responses for the eight damage scenarios. The identified results with measurement noise, $\hat{\mu}^N$, are also presented in Table 2. It can be seen that even the measurement noise intensity is as high as 15%, the identified damage severities are very close to the identified ones without considering the measurement noise and those preset values. This indicates that the proposed method is insensitive to measurement noise. The robustness and reliability

of the proposed method are numerically demonstrated. Further information can be found in Zhang et al. (2008).

5 EXPERIMENTAL INVESTIGATION

5.1 Experimental arrangement

To access the feasibility and reliability of the statistical moment-based structural damage detection method, shaking table tests of eight damage scenarios were conducted on three shear building models under white noise or color noise excitations in this study. The height of the first floor of the building models was 280 mm, while the height of the upper two floors was 300 mm using the centerline dimensions. The size of steel beams was 394×50× 30 mm and that of columns was 900×50× 3 mm. The beams and columns were properly connected by steel blocks and screws to form rigid joints. The building model was then fixed on a steel base plate of 15 mm thickness by four angle bars. All the columns and beams were made of high strength steel of 435 MPa yield stress and 200 GPa modulus of elasticity. An additional mass of about 10 kg was placed on each beam of Model 1. The lumped mass values calculated by summing up the weight of the corresponding beam and connection accessories were 14.32 kg, 14.32 kg and 14.21 kg for the first, second and third floor of Model 1, respectively and 5.41 kg, 5.41 kg and 5.23 kg for Model 2 and 3 correspondingly. To simulate inherent energy dissipation capacity of a real structure, a liquid damper was installed at the first story (Fig. 1).

Fig. 1 Shear building models

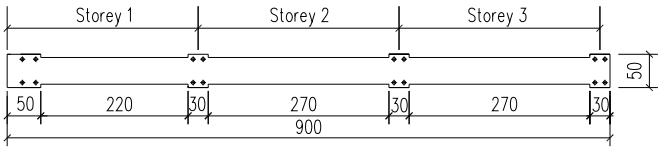

Fig. 2 Locations of saw cuts (unit: mm)

To simulate different damage locations of the structure, both sides of the two columns per model were accordingly cut at the stories which were designated to have damage. Two kinds of saw cut depth were introduced, 2.5 mm and 5 mm, to simulate different damage severities. The location of saw cuts at different stories was presented in Fig. 2. To assess the reliability of the proposed method, eight damage scenarios were designed. The theoretical values of the damage severity μ for a given damage scenario were computed using the material and geometric properties and presented in Table 3. Although the so-obtained theoretical values of damage severity are only approximations of the real ones of building models, they can be taken as references for comparison with the identified damage severity.

The unidirectional shaking table facility at the Hong Kong Polytechnic University used in this study was manufactured by the MTS Corporation, with dimensions of 3 m by 3 m in plane (MTS 469DU). Three B&K 4370 accelerometers (made in Denmark) and three laser displacement sensors with the range of 100 mm and the resolution of ±0.01 mm per testing model were used to measure acceleration and displacement responses of each floor. One more B&K 4370 accelerometer and one more laser displacement sensor were installed on the base plate of the building to measure ground excitations and absolute displacements of the shaking table. The signals from the accelerometers and lasers were then transferred to B&K 2635 signal conditioners. The 32 channel data acquisition equipments were used to acquire data during the

shaking table tests. The sampling rate was set at 500 sample/sec. White noise and color noise time histories with the frequency ranging from 0.5 to 40 Hz and the duration of 1000s were numerically simulated and then inputted into the control system of the shaking table to generate ground excitations.

The shaking table tests were conducted according to the procedure as follows: (1) conduct shaking table tests on undamaged shear building models under white noise and color noise excitations and record data; (2) identify the lateral stiffness values of the undamaged models using the fourth-order statistical moment vector of story drifts and the first two damping ratios identified by HT method in conjunction with RDT technique; (3) conduct shaking table tests on the damaged shear building models with eight damage scenarios step by step and record data; (4) identify the lateral stiffness values of the damaged models using the constrained optimization method; (5) for every damage scenario, detect damage locations and damage severity values by comparing the identified lateral stiffness of the damaged model with that of the corresponding undamaged model; and (6) compare the damage detection results with the theoretical ones to assess the feasibility and reliability of the proposed method. During the shaking table tests, the background noise of the laboratory was measured and recorded by the acceleration sensors. The measurement noise intensities of recorded acceleration time histories ranged from 8% to 10% which were larger than 3% considered as a high level of noise that already makes it difficult to identify damage location of a structure by other detection methods (Worden *et al*. 2005).

5.2 Data analysis and discussion

The damage detection results of the eight damage scenarios in comparison with the theoretical ones are presented in Table 3. Scenario 1 and 5 are detected using only white noise excitation, while Scenario 3, 7 and 8 are detected using only color noise excitation. Scenario 2, 4 and 6 are investigated by both white noise tests and color noise tests.

Table 3 Identified damage severity (DS) values in comparison with theoretical ones

Scenario	Damage Location	Total Saw Cut Depth (mm)	Type of Excitation	Damage severity (%)					
				Storey 1		Storey 2		Storey 3	
				μ	$\hat{\mu}$	μ	$\hat{\mu}$	μ	$\hat{\mu}$
1	Story 1	5.0	WN	-5.36	-5.15	0	-0.92	0	0
2	Story 1	10.0	WN	-11.26	-11.16	0	-0.27	0	0
			CN		-12.60		-0.61		0
3	Story 2	5.0	CN	0	0	-7.45	-9.83	0	0
4	Story 2	10.0	WN	0	0	-15.33	-16.32	0	-1.05
			CN		0		-17.26		-1.61
5	Story 1 & 3	10.0 5.0	WN	-11.26	-13.42	0	0	-7.45	-10.33
6	Story 2 & 3	10.0 5.0	WN	0	0	-15.33	-19.90	-7.45	-6.44
			CN		0		-18.72		-7.53
7	Story 1 & 3	10.0 5.0	CN	-11.26	-11.72	-7.45	-7.95	0	-2.21
8	Story 1,2 & 3	10.0, 5.0 & 10.0	CN	-11.26	-10.03	-7.45	-9.58	-15.33	-20.50

It can be seen from Table 3 that there are only small differences between the identified damage severities and the corresponding theoretical ones. Even for the light damage at different

locations, such as the damage of 5.36% at the first floor in Scenario 1, the damage of 7.45% at the second floor in Scenarios 3, 7 and 8 and the damage of 7.45% at the third floor in Scenarios 5, satisfactory identification results are obtained. For Scenario 2, 4 and 6, the identified results either by the white noise tests or by the color noise tests not only match the theoretical values well but also have small relative difference with each other. The reliability of the statistical moment-based damage detection method is, therefore, demonstrated by both white noise tests and color noise tests.

6 CONCLUDING REMARKS

A new structural damage detection method has been proposed in this study based on the statistical moments of dynamic responses of building structures. The principle of the new method is put forward in terms of a single-degree-of-freedom (SDOF) system and then extended to multi-degree-of-freedom (MDOF) systems with resort to the least-squares method. Numerical examples show that the method could be used to accurately identify both damage location and damage severity of shear buildings. A significant advantage of the proposed damage detection method lies in that it is not sensitive to measurement noise. Even when the measurement noise intensity is as high as 15%, the proposed method still presents high reliability on detecting damage locations and damage severity.

The proposed method has been experimentally examined in this study by white noise and color noise shaking table tests. Three shear building models without any structural damage were designed and manufactured for the shaking table tests and then eight damage scenarios with different damage locations and preset damage severities were inflicted on the three building models step by step during the experiment. The identified results of the eight damage scenarios by using the statistical moment-based damage detection method were compared with those theoretical values. Different damage locations of either single damage scenarios or multi-damage scenarios could be satisfactorily detected. It was found that the experimentally identified damage severity values match those theoretical values well even for the light damage scenarios. The feasibility and reliability of the statistical moment-based damage detection method was demonstrated by both the white noise shaking table tests and the color noise ones. Nevertheless, the statistical moment-based damage detection method was only demonstrated by simple shear buildings. Further investigation on more complicated structures is deserved.

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References

Alvandi A. and Cremona C. 2006. Assessment of vibration-based damage identification techniques, *Journal of Sound and Vibration*, 292:179-202.

Doebling S.W., Farrar C.R. and Prime M.B. 1998. A summary review of vibration-based damage identification methods, *The Shock and Vibration Digest*, 30:91-105.

Shinozuka M. and Jan C.M. 1972. Digital Simulation of Random Processes and Its Applications, *Journal of Sound and Vibration*, 25:111-128.

Worden K., Farrar C.R., Manson G. and Park G. 2005. Fundamental axioms of structural health monitoring, *Structural Health Monitoring 2005: Advancements and Challenges for Implementation*, Chang F.K. (editor), DEStech Publications, Lancaster, Pennsylvania, 26-41.

Zhang, J., Xu, Y.L., Xia, Y. and Li, J. 2008. A new statistical moment-based structural damage detection method. *Structural Engineering and Mechanics-An International Journal* 30(4): 445-466