Structural Parameter Estimation of Two Bridges from Site Data using an Eigen Value Realization Algorithm

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ABSTRACT: The Eigen Value Realization Algorithm is used to identify the structural frequencies from the ambient data recorded from site. These frequencies are compared with those estimated from other classical methods and also from the finite element model. It is shown that the different methods of data extraction give very comparable and consistent results with each other and also with the numerical model.

1 INTRODUCTION

The structural parameter extraction from response data to assess the condition of structures is a well known method. Seminal work has been done in this area, a good review of which has been given by Doebling, et al (1996). Of the various methods proposed, in this work the Eigen Value Realization Algorithm (ERA) pioneered by Juang and Pappa (1985) has been taken for implementation. The Natural Excitation Technique (NExT) was proposed by James, et al (1993) which viewed the cross correlation of the ambient vibration with a reference signal as analogous to the free vibration of a system.

Using ERA to assess the system parameters has been done by Conte, et al. (2004) using 64 channels of simultaneous acceleration data acquisition. Clayton (2002) used 21 accelerometers over a continuous 4 day recording to assess the structural parameters.

Two bridges were instrumented and the acceleration data gathered was processed to extract the system parameters. The typical length of the acceleration data is a 10 second data length sampled at 200 Hz. The bridges were sparsely instrumented with only eight accelerometers. The sensors were placed at four locations on the bottom chord. The locations chosen for placement were the centre of span and one point at quarter span on both the girders. Here the sensors were placed to measure the vertical and transverse accelerations.

It is shown by the analysis which follows, that even the sparsely instrumented bridge, with very limited spatial resolution of sensors, still exhibits the system properties within the structural significant zone. Within the significant frequency period it is shown that the eigen value realization algorithm, in one case out-performs the classical peak picking method, whereas in the other case, the classical peak picking method gives better results as compared with the numerical model.

The Eigen Value realization algorithm, however, predicts the modes corresponding to higher frequencies as well. It is also a good algorithm for realizing closely placed modes and natural frequencies. This study shows that even using sparsely distributed sensors over very short data lengths can give very consistent data using this algorithm.

2 FORMULATION

2.1 Impulse Response of Linear Time Invariant systems:

The multi-step predictor model is represented by Juang and Phan (2001) as:

$$y[k] = -\sum_{i=1}^{p} \alpha_i y[k-i] + \sum_{i=0}^{p} \beta_i u[k-i] \tag{1}$$

$$y[k+j] = -\sum_{i=1}^{p} \alpha_i^j y[k-i] + \sum_{i=0}^{q} \beta_i^j u[k-i] + \sum_{i=1}^{j-1} \beta_o^j u[k+j-i] \tag{2}$$

Where:

$$\left.\begin{aligned} \alpha_n^j &= \alpha_{n+1}^{j-1} - \alpha_1^{j-1}\alpha_n; 1 \le n < p \\ \alpha_p^j &= -\alpha_1^{j-1}\alpha_p \\ \beta_n^j &= \beta_{n+1}^{j-1} - \alpha_1^{j-1}\beta_n; 1 \le n < p \\ \beta_p^j &= \alpha_1^{j-1}\beta_p \\ \beta_0^j &= \beta_1^{j-1} - \alpha_1^{j-1}\beta_0 \end{aligned}\right\} \tag{3}$$

Giving an input of 1 at time instance k, and noting the output, it can be easily verified that β_0^j is the impulse response of the system.

Equating the impulse response to the impulse response of the state space model as shown by Juang and Phan (2001), as both the formulations represent the same linear time invariant system:

$$\beta_0 = D; \beta_0^1 = CB; .....; \beta_0^{s-1} = CA^{s-2}B; \tag{4}$$

These are the Markov parameters of the system.

2.2 Eigen Value Realization Algorithm (ERA)

On the basis of these Markov parameters, a block Hankel matrix is built up of size pm x γr:

$$H(0) = \begin{bmatrix} Y_1 & Y_2 & ... & Y_\gamma \\ Y_2 & Y_3 & ... & Y_{\gamma+1} \\ : & : & : & : \\ Y_p & Y_{p+1} & ... & Y_{\gamma+p-1} \end{bmatrix} = \begin{bmatrix} CB & CAB & ... & CA^{\gamma-1}B \\ CAB & CA^2B & ... & CA^\gamma B \\ : & : & : & : \\ CA^{p-1}B & CA^pB & ... & CA^{\gamma+p-2}B \end{bmatrix} = P_p Q_\gamma \tag{5}$$

A singular value decomposition of this matrix is done as follows:

$$\left.\begin{aligned} H(0) &= R\Sigma S^T; \\ \Sigma &= \begin{bmatrix} \Sigma_n & 0 \\ 0 & 0 \end{bmatrix}; \end{aligned}\right\} \tag{6}$$

Taking R_n as the matrix with the first n rows of R and S_n as the matrix with the first n columns of S:

$$\left.\begin{aligned} &H(0) = R_n \Sigma_n S_n^T; R_n^T R_n = S_n^T S_n = I_n; \\ &H(0) = [R_n \Sigma_n^{1/2}][\Sigma_n^{1/2} S_n^T] \cong P_p Q_\gamma \end{aligned}\right\} \tag{7}$$

A possible solution is for both P^T and Q^T to equal Σ_n. This is also an optimal solution. From the form of (23) it can then be concluded that in this sense, the matrix B corresponds to the first r columns of Q and the matrix C to the first m rows of P.

Doing a singular value decomposition of H(1) in the same pattern:

$$H(1) = [R_n \Sigma_n^{1/2}] A [\Sigma_n^{1/2} S_n^T] \cong P_p A Q_\gamma \tag{8}$$

The least square convergence of A works out to be:

$$A = \Sigma_n^{-1/2} R_n^T H(1) S_n \Sigma_n^{-1/2} \tag{9}$$

$$D = Y(0) \tag{10}$$

With this, all the system matrices can therefore be computed from the available data in an optimal sense.

2.3 Ambient Vibrations using Natural Excitation Technique (NExT)

For ambient vibrations, the force vector is unknown. In this case, since the forcing function cannot be represented one of the recorded data sets is considered as a reference channel. All other data are then computed with respect to the reference channel. This technique is called the NExT technique or the natural excitation technique. This is based upon that under certain assumptions, the cross correlation functions between the response measurements and a single reference measurement can be treated as free responses. The assumption is that the excitation is stationary, broadband and uncorrelated.

The formulation has been done by James, et al (1993) by taking the basic equation of motion and post multiplying by a reference channel $Q_i(s)$ and taking the expected values on both sides:

$$MR(\ddot{Q}\ddot{Q}_i)(t,s) + CR(\dot{Q}\ddot{Q}_i)(t,s) + KR(Q\ddot{Q}_i)(t,s) = R(P\ddot{Q}_i)(t) \tag{11}$$

Under the assumption that the acceleration, displacement and velocity are weakly stationary and that the excitation is uncorrelated with the reference:

$$MR(\ddot{Q}\ddot{Q}_i)(\tau) + CR(\dot{Q}\ddot{Q}_i)(\tau) + KR(Q\ddot{Q}_i)(\tau) = 0 \tag{12}$$

So the cross-correlations can be considered as free response data as they satisfy the homogenous equations of motion. So in the eigenvalue realization algorithm using ambient data, one of the sensors is considered as the reference channel, and the cross-correlations of the other channels with this channel are considered as the data.

3 PROBLEM DEFINITION AND RESULTS

3.1 Bridge descriptions

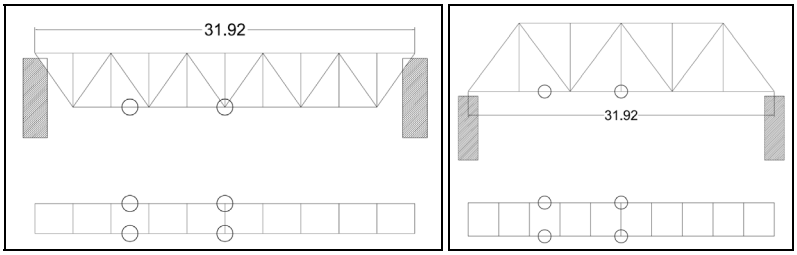

Figure1. Sensor Placements on Underslung and Through girders

Both the bridges under study are Open Web Steel Girders having 31.92 meters span from bearing centers. For both of these girders eight accelerometers were placed on the bottom chord member. These were oriented as four in the vertical direction and four in the horizontal direction perpendicular to the bridge axis. Along the girder they were placed at centre of span and at quarter span on both the girders. The sensor placed at the centre of span in the vertical direction on the east girder was considered as being the reference sensor.

Since the spatial resolution of the accelerometers was sparse, the mode shapes are not worked out. For both the bridges under consideration, the model order considered is 30. Using this model order, the data sets generated from the results of the eigen value realization algorithm are compared with classical frequency domain data extraction systems and the numerical model.

3.2 Results for Under Slung Girder

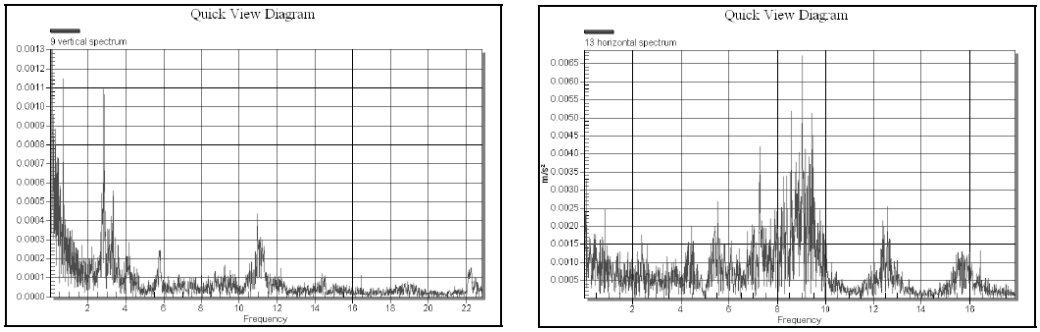

Figure 2. Power Spectral Density Function of the Acceleration Data

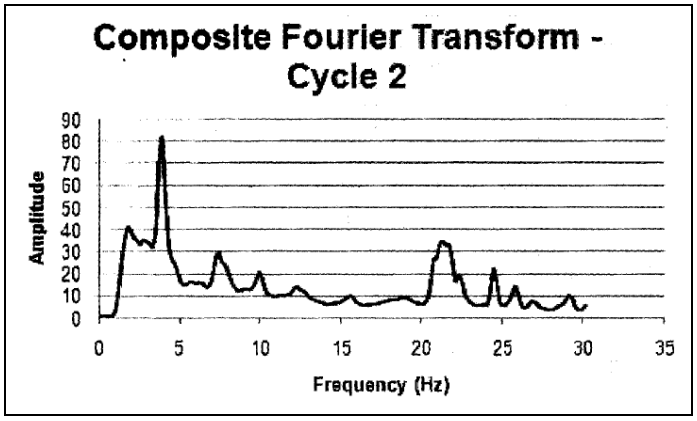

Figure 3. Statistical Data from Fourier Transform for Peak Picking

Table 1. Frequency representation using different techniques

Mode	Frequency from ERA (Hz)	Frequency from Peak Picking (Hz)	Frequency from Numerical model (Hz)
Mode 1	3.83	3.91	4.20
Mode 2	7.11	7.42	7.35
Mode 3	9.96	9.96	9.73
Mode 4	13.38	-	14.98
Mode 5	13.91	-	15.0

3.3 Results for Through Girder

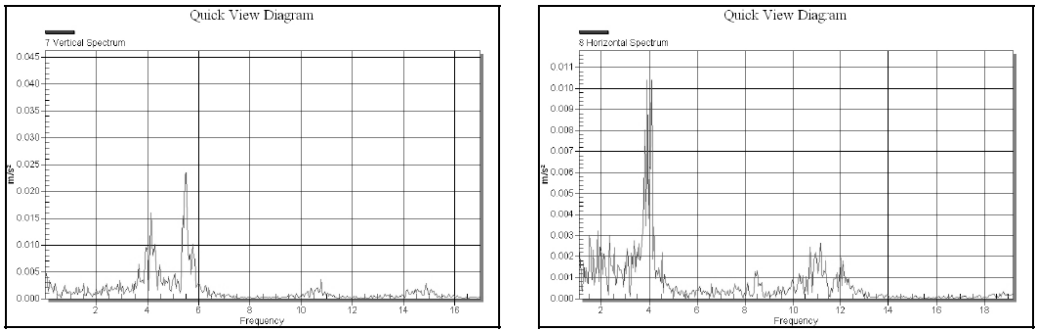

Figure 4. Power Spectral Density Function of the Acceleration Data

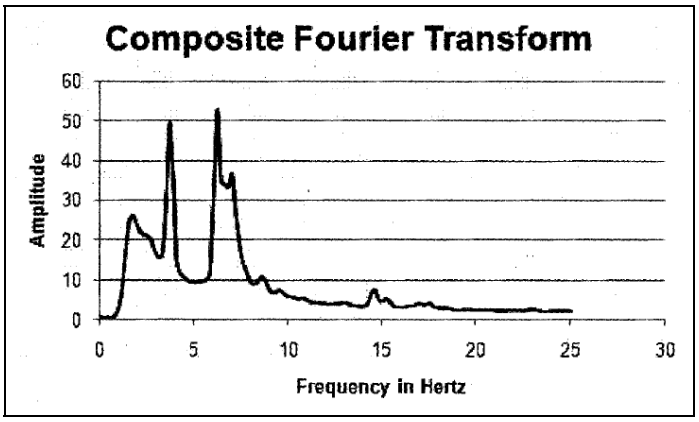

Figure 5. Statistical Data from Fourier Transform for Peak Picking

Table 2. Frequency representation using different techniques

Mode	Frequency from ERA (Hz)	Frequency from Peak Picking (Hz)	Frequency from Numerical model (Hz)
Mode 1	3.34	-	3.43
Mode 2	3.87	3.71	4.05
Mode 3	6.18	6.25	5.92
Mode 4	7.28	7.04	7.52
Mode 5	9.34	-	8.78
Mode 6	9.95	-	9.92
Mode 7	13.52	-	15.38
Mode 8	13.76	-	17.83

4 CONCLUSIONS

From the above analysis it is seen that even with few sensors, the structural frequencies less than 15 Hertz are very well represented. The predominant structural frequencies are in this range for structures of this nature. Therefore it can be concluded that the Eigen Value Realization Algorithm is a robust method for structures of this kind. It is also seen that for the frequency range under consideration, the numerical model also predicts the eigen values within an acceptable tolerance of the site measured values.

The differences between the ERA predicted values and the finite element frequencies are within 10% for frequencies less than 15 Hz, which is the expected significant structural frequency. The mode shapes have not been derived because the sparsely instrumented sensor locations would not give the mode shape with any degree of confidence. The computed frequencies can be kept for future references to assess the change in the structural condition.

A similar exercise has been done to ascertain the damping ratios with reasonably good results, considering that the damping ratios are generally known to have significant estimator variation

depending upon the assessment algorithm chosen. These have not been included here due to space constraints.

Time domain algorithms typically need longer data sets. It has been proposed that in further studies of the bridges, roving accelerometers be used to give a better spatial mapping of the system with limited sensors and also the data lengths be increased so that the algorithm would perform better in estimation of the mode shapes. From this exercise the optimal data length would be estimated for robust estimation of all the structural parameters.

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