Proceedings of the 6th International Conference on
Computation of Shell and Spatial Structures
IASS-IACM 2008: “Spanning Nano to Mega”
28-31 May 2008, Cornell University, Ithaca, NY, USA
John F. ABEL and J. Robert COOKE (eds.)

Nonlinear Dynamic Analysis of Space Frame Structures

Chung-Yue WANG*, Ren-Zuo WANG**

* Department of Civil Engineering, National Central University
Chungli, 32054, Taiwan, R.O.C
cywang@cycu.edu.tw

* National Center for Research on Earthquake Engineering
Taipei 106, Taiwan, R.O.C.
rzwang@ncree.org.tw

Abstract

In this paper, a novel formulation for the nonlinear motion analysis of reticulated space frame structures is developed by applying a new concept of computational mechanics, named the vector form intrinsic finite element (VFIFE or V-5) method. The V-5 method models the analyzed domain to be composed by finite particles and the Newton’s second law and Euler’s equation of motion are applied to describe each particle’s motion. By tracing the motions of all the mass particles in the space, it can simulate the large geometrical and material nonlinear changes during the motion of structure without using geometrical stiffness matrix and iterations. The analysis procedure is vastly simple, accurate, and versatile. The formulation of VFIFE type space frame element includes a new description of the kinematics that can handle large rotation and large deformation, and includes a set of deformation coordinates for each time increment used to describe the shape functions and internal nodal forces. A convected material frame and an explicit time integration scheme for the solution procedures are also adopted. Numerical examples are presented to demonstrate capabilities and accuracy of the V-5 method on the nonlinear dynamic stability analysis of space frame structures

1. Introduction

Nonlinear analysis methods developed since last century are used to study the behavior of structures with material and geometrical nonlinearities. Gallagher and Padlog (1963) first introduce the geometrical stiffness matrix into the nonlinear analysis of structure by considering the nonlinear strain terms in the formulation. Argyris et al. [1] have tried to modify the definition of bending moment to derive a modified geometrical stiffness matrix to satisfy the equilibrium requirement at each deformed state. Yang and Kuo [2] proposed a method to decompose the displacement of structural element into rigid body displacement and natural deformation displacement in each incremental step of the computation and this kind of decomposition can lead the geometrical stiffness matrix pass the rigid body motion test. It is well known that the core idea of the nonlinear analysis of structure is how to clearly identify the rigid body component and the deformation component in the motion. Recently, a novel computation method called as the vector form intrinsic finite element (VFIFE, simply called V-5) method was proposed by Ting et al. [3, 4] and Shi et al. [5]. The VFIFE method has been successfully applied to the nonlinear motion analysis of 2D frame (Wu et al. [6]) and the dynamic stability analysis of space truss structure (Wang et al. [7, 8, 9]). Due to some special characteristics of the VFIFE method, it is very easy to be applied to study the highly nonlinear dynamic behavior of a structure system from continuous to discontinuous states. In this paper, the theory of space frame element in VFIFE (Wang [10]) is briefly introduced.

2. Fundaments of the 3D Fame in VEFIFE

A novel computational method so called the Vector Form Intrinsic Finite Element is developed by Ting et al. (2004 a, b) to handle engineering problems with the following characters: (1) containing multiple deformable bodies and mutual interactions, (2) material non-linearity and discontinuity, (3) large deformation and arbitrary rigid body motions of deformable body. Since the conventional FEM based on variational method requires the total virtual work to be zero but does not require the balance of forces at nodes. These unbalanced residual forces will do some non-zero work under virtual rigid body motion and cause the inaccuracy and un-convergence of the calculation results. The computation procedure and some concepts of this VFIFE method are similar to the FEM. But the major difference is that the VFIFE does not apply the variational principle on the stress expressed equilibrium equations in its formulation. Instead, VFIFE maintains the intrinsic nature of the finite element method and makes strong form of equilibrium at nodes, the connections of members.

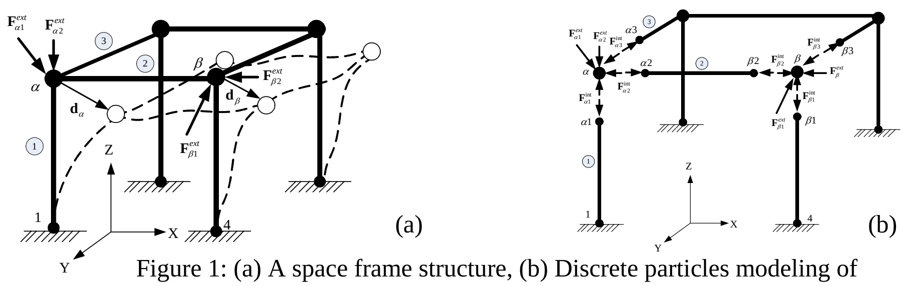


Figure 1: (a) A space frame structure, (b) Discrete particles modeling of space frame structure system by the VFIFE method.

In other words, the continuous bodies are represented by a set of mass points through lumped mass technique as shown in Fig. 1. Each mass point satisfies the law of mechanics, i.e. the conservation of linear and angular momentums. Similar to other well-developed VFIFE elements, a convected material frame and explicit time integration for the solution procedures are also adopted in the formulation of 3D frame element. The description of kinematics to discrete rigid body and deformation displacements, and a set of deformation coordinate for each time increment to describe deformation and internal nodal forces can be found in the thesis of Wang [2005a]. The formulation of space frame element in V-5 is an extension of the theories for space truss element. The correspondence between these two types of element can be identified from the work done by the authors (Wang et al. [7]). The basic modeling assumptions for the VFIFE method for 3D frame structures are essentially the same as those in classical structural analysis. A frame is constructed by means of prismatic members and joints. Members are subjected to forces and moments. The corresponding general internal forces $\hat{\mathbf{f}}^* = (\hat{f}_{2x}, \hat{m}_{1y}, \hat{m}_{1z}, \hat{m}_{2x}, \hat{m}_{2y}, \hat{m}_{2z})^T$ of the frame element in the deformation coordinate system can be derived by the principle of virtual work. From the static equilibrium equations, all the internal forces at the two nodes of the frame element can be calculated. After calculating all the internal forces of element nodes, one can sum over all internal forces $-\mathbf{F}_\beta^{\text{int}}$ and external forces $\mathbf{F}_\beta^{ext}$ applied on a rigid body particle β and obtain the following equation of motion without damping effect:

$$\mathbf{M}_\beta \ddot{\mathbf{d}}_\beta = \mathbf{F}_\beta^{ext} - \mathbf{F}_\beta^{\text{int}} \tag{1}$$

Where $\mathbf{M}_\beta$ is the general mass matrix and $\ddot{\mathbf{d}}_\beta$ is the general displacement vector of the particle β. In the present analysis, the explicit time integration technique is used to solve Eq. (1). Since the VFIFE method uses the motion and relative displacements of particles to identify the internal forces among them. This feature allows users to do the displacement control type excitation.

3. Numerical Examples

Example 1: Buckling of Space Frame Structure

Figure 2 shows a reticulated space frame structure composed of 12 members subjected to a vertical load P at its highest node. Each member has a cross sectional area $A = 0.9in^2$, principal mass moment of inertials $I_2 = 0.2in^4$, $I_3 = 0.02in^4$, $J = 0.0331in^4$, Young's modulus $E = 4.398x10^5\ psi$ and shear modulus

$G = 1.59x10^5$ *psi* . This large deformation, post buckling problem has been studied by many researchers (Papadrakakis [11], Meek and Tan [12], Hsiao and Horng [13]). We use the function of displacement control of V-5 to calculate the load-displacement relation of this frame structure and compare it with the results obtained by previous researchers. It is seen in Fig. 3(b) that the V-5 method can accurately analyze the buckling of space frame with large deformation.

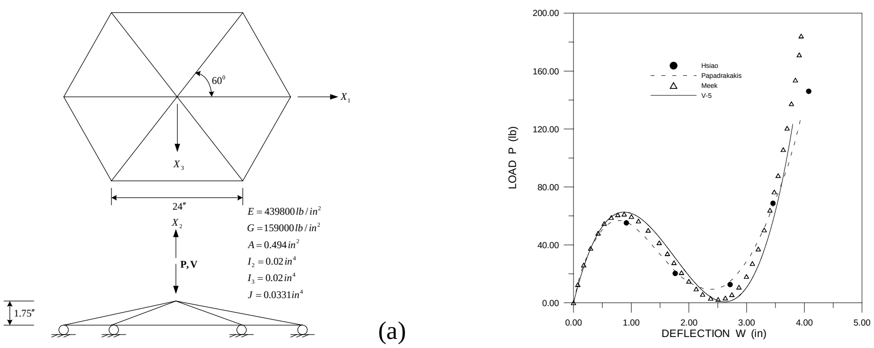
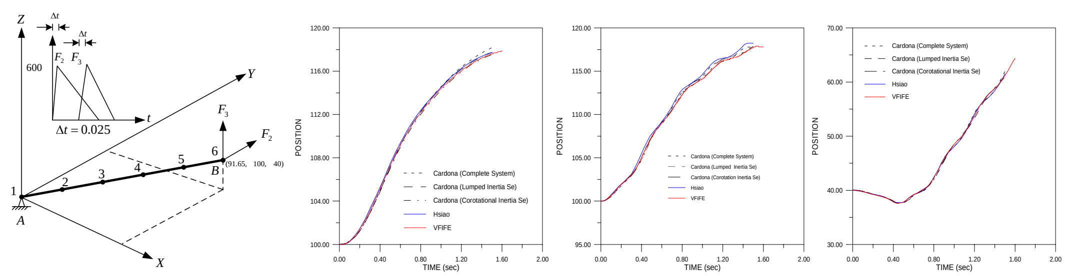


Figure 2: Bucking analysis of reticulated space frame structure composed of 12 members.
(a) top view and side view, (b) vertical load- displacement relation of roof tip.



(a) (b) (c) (d)

Figure 3: Deformation and rotation of an articulated rod, (a) loading functions, (b) history of the Y coordinate of the tip of rod with $E = 2.1x10^9$ *psi* , (c) history of the Y coordinate of the tip of a rod with $E = 6.3x10^6$ *psi* , (d)) history of the Z coordinate of rod tip with $E = 6.3x10^6$ *psi* .

Example 2: Articulated-free rod

An articulated- free rod subjected to an impulse force F_2 in Y direction and an impulse force F_3 in the Z direction as shown in Fig. 3 was studied. This problem has been investigated by many researchers (Géradin and Cardona [14], Hsiao et al. [15], Crisfield [16]) as a benchmark problem for frame structure having large rotation and large deformation. The rod has length 141.42 inch, cross sectional area 9 in^2 , mass density $\rho = 2.54\times10^{-4} lb - s^2 in^{-4}$, and Poisson's ratio $\nu = 0.3$. Rods with different Young's modulus were selected to study their difference in motions with large deformation and rotation. Five frame elements were used to model the rod. Figure 3(b) shows the history of the Y coordinate of the tip of a rod with Young's modulus $E = 2.1x10^9$ *psi* and Fig. 3(c) shows the history of the Y coordinate of the tip of a rod with Young's modulus $E = 6.3x10^6$ *psi* . These two figures reveal the effect from the rigidity of rod on the deformation of rod. Figure 3(d) shows the history of the Z coordinate of the tip of a rod with Young's modulus $E = 6.3x10^6$ *psi* . From Fig.

3, it also find that the V-5 method can accurately analyze the motion of frame structure with large rotation and deformation based on a new concept of computational mechanics.

4. Conclusions

A vastly simple numerical procedure is developed in this paper for motion analyses of the nonlinear response and stability of reticulated space frame structures subjected to large geometrical changes and complicated excitations. Due to the nature of discrete independent particle point, it is not required to set essential boundary conditions of the system. It is very easy to prescribe the displacement and forcing conditions on each particle during the procedure of analysis. Through the numerical analyses of a few benchmark problems of features as large rotation and dynamic instability, the newly proposed method demonstrates its accuracy and superior capability on the nonlinear motion analysis of space frame structure. As well, the vector form nature of the V-5 method allows it to be linked with parallel computation techniques to study the large scale problems that have complicated geometrical variations and loading histories.

References

[1] Argyris JH, Dunne PC and Scharpf DW. On large displacement-small strain analysis of structures with rotational degree of freedom. *Computer Methods in Applied Mechanics Engineering*, 1978; 14: 401-451.

[2] Yang YB, and Kuo SR. *Theory & Analysis of Nonlinear Framed Structures*. Prentice-Hall. 1994.

[3] Ting EC, Shih C, and Wang YK. Fundamentals of a vector form intrinsic finite element: Part I. basic procedure and a plane frame element. *Journal of Mechanics*, 2004; 20: 2. 113-122.

[4] Ting EC, Shih C, and Wang YK. Fundamentals of a vector form intrinsic finite element: Part II. plane solid elements. *Journal of Mechanics*, 2004; 20: 2. 123-132.

[5] Shih C, Wang YK, and Ting EC. Fundamentals of a vector form intrinsic finite element: Part III. Convected material frame and examples. *Journal of Mechanics*, 2004; 20: 2, 133-143.

[6] Wu TY, Wang RZ, and Wang CY. Large deflection analysis of flexible planar frames, *Journal of the Chinese Institute of Engineers*, 2006; 29: 4. 593-606.

[7] Wang CY, Wang RZ, Chuang CC, and Wu TY. Nonlinear Dynamic Analysis of Reticulated Space Truss Structure. *Journal of Mechanics*, 2006; 22: 3. 235-248.

[8] Wang RZ. *Vector Form Motion Analysis of Structure*, Ph. D. Thesis, Department of Civil Engineering, National Central University, Taiwan (in Chinese). 2005.

[9] Wang RZ, Chuang CC, Wu TY, and Wang CY. Vector form analysis of space truss structure in large elastic-plastic deformation. *Journal of the Chinese Institute of Civil & Hydraulic Engineering*, 2005; 17: 4. 633-646.

[10] Wang RZ, Kang LC, Wu SY, and Wang CY. Numerical Simulation of the Whipping Problem of High Pressure Pipe. *Chinese Society of Structural Engineering*, 2005; 20: 4. 120-143 (in Chinese).

[11] Papadrakakis, M. Post-buckling analysis of spatial structures by vector interation methods. *Computers and Structures* 1981; 14; 5-6:393-402.

[12] Meek JL, Tan HS. Geometrically nonlinear analysis of space frames by an incremental iterative technique. *Computer Methods in Applied Mechanics and Engineering* 1984; 47:261-282.

[13] .Hsiao KM, Horng HJ. A corotational procedure that handles large rotations of spatial beam structures. *Computers and Structures* 1987; 27; 6: 769-781.

[14] Géradin M, Cardona A. A superelement formulation for mechanism analysis. *Computer Methods in Applied Mechanics and Engineering* 1992; 100:1-29.

[15] Hsiao KM, Lin JY, Lin WY. A consistent co-rotational finite element formulation for geometrically nonlinear dynamic analysis of 3-D beams. *Computer Methods in Applied Mechanics and Engineering* 1999; 169: 1-18.

[16] Crisfield MA. Dynamic analysis of 3D beams with joints in presence of large rotations. Computer Methods in Applied Mechanics and Engineering 2001; 190: 4195-4230.