MOTION AND CONTROL ANALYSIS OF STRUCTURE UNDER EARTHQUAKE EXCITATION

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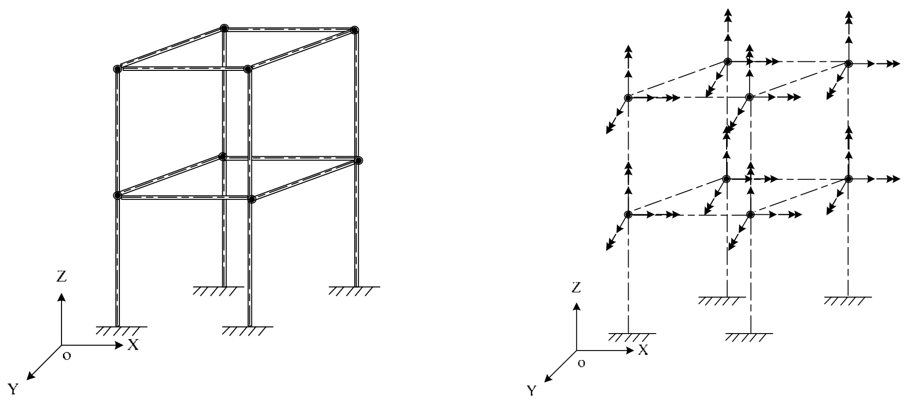
Abstract: In this paper, the explicit vector form intrinsic finite element (VFIFE) method is used to conduct the motion and control analysis of structure under earthquake excitation. This VFIFE method can do the motion analysis of structure with large deformation and rotation from continuous states to discontinuous states. Three dimensional frame elements of the VFIFE method are used to model the progressive failure of structure. Two dimensional frame elements of VFIFE are used to conduct the passive control analysis of structure under earthquake excitation.

1. INTRODUCTION

To prevent the immeasurable losses of human lives and social properties due to earthquakes, the seismic resistance evaluation and retrofitting of civil infrastructures becomes an important issue of many countries in the world. Besides experimental and theoretical study, the numerical simulation is another way to assist engineers to understand the nonlinear dynamic failure behavior of structure under the earthquake excitation. Large deformation, progressive failure and collapse are the most critical deformation states causing structure damage and the threatening to lives. However, the mechanical behaviors inherited in these failure modes are extremely complicated and some physical quantities are difficult to be measured in the test. Therefore, the numerical techniques for the motion and control analyses of a given structure system under earthquake loading are essential in the evaluation of the capability of seismic resistance.

Nonlinear analysis methods developed since last century are used to study the behavior of structures with material and geometrical nonlinearities. Gallagher and Padlog (1963) first introduce the geometrical stiffness matrix into the nonlinear analysis of structure by considering the nonlinear strain terms in the formation. The rigid body motion of the structure will change the directions of internal forces and causes fictitious strains (Jagannathan et al., 1975a, b). Later Kohnke (1978) proposed the concept of fictitious force to remedy this phenomenon. Yang and Chiou (1987b) developed a rigid body test method to evaluate the geometrical stiffness matrix. Leu and Yang (1990)；Leu and Yang(1991) stated that the structure has to satisfy the equilibrium equation during the rigid body motion to get rid of the error in the numerical analysis due to the fictitious forces.

Argyris et al. (1978) and Elias (1986) have tried to modify the definition of bending moment to derive a modified geometrical stiffness matrix to satisfy the equilibrium requirement at each deformed state. Many researchers used the iterative method to solve the system equilibrium equations of incremental form (Mallett and Marcal (1968); Rajasekaran and Murray (1973); Leu and Yang (1990, 1991)). Leu and Yang (1990, 1991) proposed a predictor-corrector mechanism in the nonlinear analysis to accurately predict the elastic buckling behavior of space truss structure. Yang and Kuo (1994) proposed a method to decompose the displacement of structural element into rigid body displacement and natural deformation displacement in each incremental step of the computation and this kind of decomposition can lead the geometrical stiffness matrix pass the rigid body motion test. It is clear seen that the core idea of the nonlinear analysis of structure is how to clearly identify the rigid body component and the deformation component in the motion.



(a) (b)

Figure 1 (a) A space frame structure, (b) Discrete particles modeling of a space frame structure system by the VIFE method.

A new computational method so called the Vector Form Intrinsic Finite Element (VFIFE) is developed by Ting et al. (2004 a, b) to handle engineering problems with the following characters: (1) containing multiple deformable bodies and mutual interactions, (2) material non-linearity and discontinuity, (3) large deformation and arbitrary rigid body motions of deformable body. Since the conventional FEM based on variational method requires the total virtual work to be zero but does not require the balance of forces at nodes. These unbalanced residual forces will do some non-zero work under virtual rigid body motion and cause the inaccuracy and un-convergence of the calculation results. The computation procedure and some concepts of this VFIFE method are similar to the FEM. But the major difference is that the VFIFE does not apply the variational principle on the stress expressed equilibrium equations in its formulation. Instead, VFIFE maintains the intrinsic nature of the finite element method and makes strong form of equilibrium at nodes, the connections of members. All the forces balanced at each node are obtained from the principle of virtual work and the associated deformations are satisfied with the compatibility condition of deformation. Instead of initial boundary value problems as treated by conventional finite element approaches, VFIFE treats the motion of structures or solids as an initial value problem only. In other words, the continuous bodies are represented by a set of particles as shown in Fig. 1. It is also different from those of the explicit finite element approaches which use the lumped mass techniques. The lumped mass technique is nothing but a solution procedure of explicit finite element under the mathematical framework. However, the mass points or particles in VFIFE have physical significance which is interpretable.

The particulate nature of the VFIFE method allows it does not have to solve the matrix equations and can avoid dealing the mathematical difficulties in solving matrix equations with ill conditions.

Recently, VFIFE method has been successfully applied to the nonlinear motion analysis of 2D frame (Wu et al. 2006) and the dynamic stability analysis of space truss structure (Wang et al. 2005, 2006). Wang et al. (2006b) have briefly introduced the formulation of the 3D frame element and applied to the collapse analysis of frame structure under earthquake analysis. Due to the characters of the VFIFE method, it is very easy to be applied to study the highly nonlinear dynamic behavior of a structure system from continuous to discontinuous states.

In this paper, the applications of frame element in VFIFE on the passive control of structure are presented.

2. FUNDAMENTALS OF THE FRAME ELEMENT IN VFIFE

The basic modeling assumptions for the VFIFE method for 3D frame structures are essentially the same as those in classical structural analysis. A frame is constructed by means of prismatic members and joints. Members are subjected to forces and moments as shown in Fig. 1(b). Joints have work equivalent masses and mass moment of inertias, and can be modeled as discrete rigid bodies. Motions of the joints can be described by the principles of virtual work or equations of motion for particles. Members have no mass, and are thus in static equilibrium. Therefore, the structural configuration of a space frame structures can be described by the positions of a set of discrete rigid bodies as shown in Fig. 1(b).

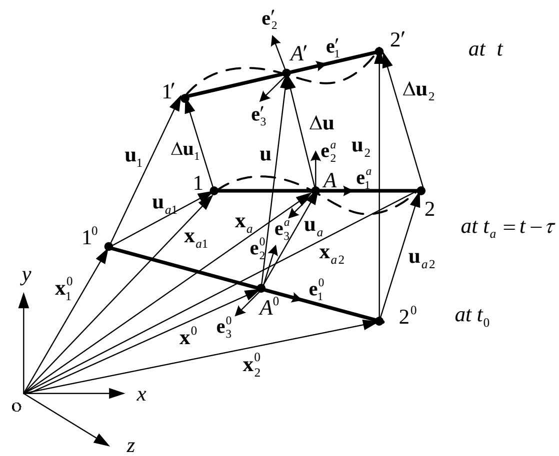


Figure 2 Nodal displacements and displacement increments of a frame element in the spatial-temporal space.

The basic modeling assumptions of the 3D frame structure in VFIFE are:

1) Each joint and discrete rigid particle has three translational degrees of freedom and three rotational degrees of freedom.
2) The mass of each particle is not changed during the motion. However, the matrix of the mass moment of inertias is changed with the directions of the principal axes of the frame element.
3) The connection between frame elements can be rigid or hinge.
4) The equivalent internal forces and external forces applied on each particle are the summation

of the internal force and external forces of the nodes of the elements connecting with the particle.

5) Each frame element satisfies the static equilibrium condition, while its internal forces are characterized by the nodes connected to the corresponding particles. The motion of each rigid body particle satisfies the Newton's second law and the Euler's equation.
6) The frame element is straight and prismatic and is made of linear elastic material.

To effectively separate the rigid body displacements from the total displacement of a deformable body, VFIFE constructs a convected material frame to describe the deformation and internal forces of the frame component moving in the spatial-temporal space. The configuration of the frame element at time t_a is chosen as the material frame for the analysis. The displacement vector of the two nodes on the frame element at time t_a is defined as ($\mathbf{u}_{a1}$, $\mathbf{u}_{a2}$) and the time increment used in the analysis is defined as $\tau = t - t_a$. As shown in Fig. 2, the displacement vector of an arbitrary point A at time t_a is defined as $\mathbf{u}_a$. The directional vectors of the principal axis of the cross section at time t_0, t_a and t are denoted by ($\mathbf{e}_1^0$, $\mathbf{e}_2^0$, $\mathbf{e}_3^0$), ($\mathbf{e}_1^a$, $\mathbf{e}_2^a$, $\mathbf{e}_3^a$) and ($\mathbf{e}_1'$, $\mathbf{e}_2'$, $\mathbf{e}_3'$), respectively. The procedure of the construction of these principal directions is clearly explained in the paper for space truss element by the first author (Wang et al., 2006a). The detail of the calculation of internal forces applied on each mass particle can be seen in the paper by Wang et al. (2006b).

Since the VFIFE method uses the motion and relative displacements of particles to identify the internal forces among them. This feature allows users to apply the displacement control type excitation to the structure. In the seismic analysis, variations of the displacements and rotations of the element nodes connected to ground can be assigned according to the history of ground motion. In the following sections, the methods of applying VFIFE on the passive control analysis are explained.

3. MODELLING OF THE PASSIVE CONTROL DEVICES BY THE VFIFE METHOD

The types of devices used in passive control of structures can be characterized by their material property as viscous damper, rubber bearing and by their internal mechanism as Tuned Mass Damper (TMD), Friction Pendulum Bearing (FPB). The modeling of some passive control devices are described as following:

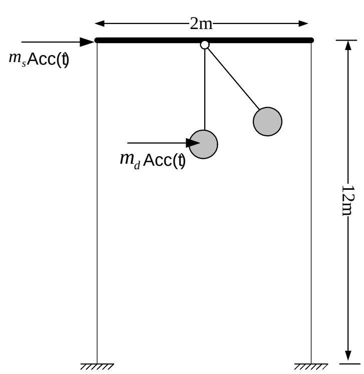


Fig. 3 A frame structure equipped with TMD.

3.1 Tuned Mass Damper (TMD)

Since the VFIFE can perform the motion analysis with large rigid body motion, the motion of tuned mass damper can be modeled by a truss member with a lumped mass at one end as shown in Fig. 3. The length of the pendulum, L, can be calculated according to the design frequency ω_d by the following equation.

$$L = g / \omega_d \tag{1}$$

Where g is the gravity acceleration.

3.2 Viscous Damper

In the VFIFE method, as shown in Fig. 4, the control force f_d provided by the viscous damper is determined by the relative velocity between the two joint points of the damper by the following equation

$$f_d = C\left|\frac{\Delta L}{\Delta t}\right|^{\alpha} \operatorname{sgn}\left(\frac{\Delta L}{\Delta t}\right) \tag{2}$$

Where the $\Delta L / \Delta t = v_D$ is the relative velocity of the two ends of the damper, C is the viscous coefficient of the damper, α is the nonlinearity parameter of the damper and the Δt is the time step used in the numerical analysis.

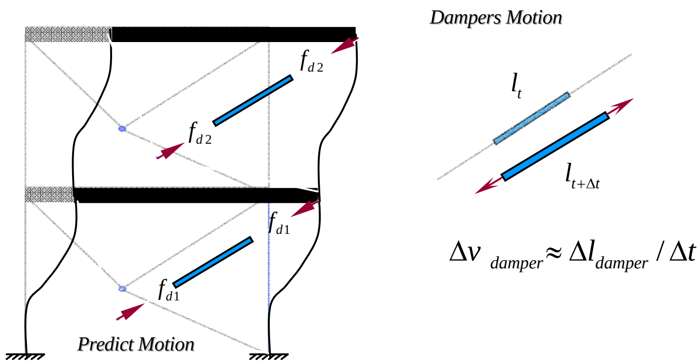


Fig. 4 Calculation of the control force provided by the viscous damper.

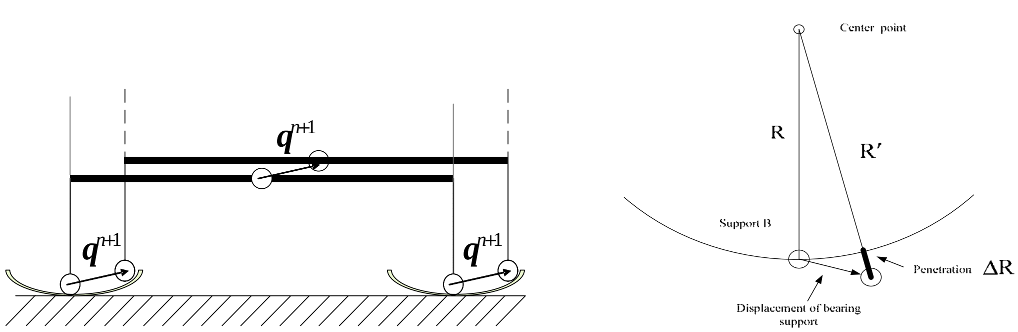


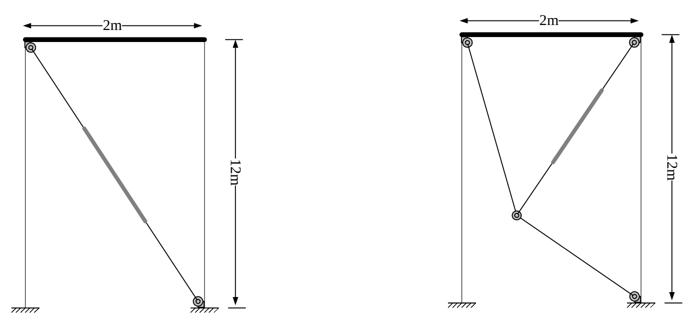
Fig. 5. Modeling of the Friction Pendulum Bearing

3.3 Friction Pendulum Bearing (FPB)

The friction pendulum bearing belongs to the sliding base isolation type device. As shown in Fig. 5, the contact analysis procedure is used to calculate the contact force between the sliding surface and the support of the bearing by the penalty method.

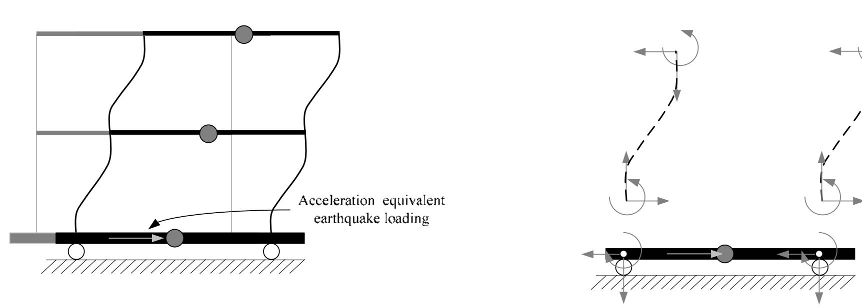
4. NUMERICAL EXAMPLE

Energy dissipation or damping system has been widely used in construction to dissipate much of the earthquake-induced energy in elements. Toggle-brace-dampers with various configurations have been used for the delivery of forces from the energy dissipation devices to the structural frame. Constantinou et al. (2001) investigated the "toggle-brace-damper" system and verified its ability to amplify the axial displacements of damper and the efficiency of energy dissipation through both cyclic loading tests and shaking table tests with a single degree of freedom steel model. Huang et al. (2005) conducted shaking table tests to investigate the seismic responses of a three-story steel model with and without linear viscous toggle-brace-dampers. In his research, a procedure for determining the relationship between the displacement magnification factor and the geometry of the toggle-brace mechanism is established to facilitate practical applications. For the structure with complex configuration, the design and testing of ability of damper system are difficult and costly. The numerical simulation can be an effective tool to assist the engineering analysis.



(a) (b)

Fig. 6. Frame structure with brace damper subject to earthquake excitation: (a) diagonal brace-damper system; and (b) upper toggle brace damper system.



(a) (b)

Fig. 7 Shaking table type loading based on the ground linear and angular accelerations

As shown in Fig. 6, the behaviors of frame structure with brace damper subjected to earthquake loading were investigated by the VFIFE method. The mass density of the column is $\rho = 0.01kg/m^3$, Young's modulus $E = 288000\ kN/m^2$, mass moment of inertial $I = 1\ m^4$, cross sectional area $A = 1\ m^2$. Rigid beam of mass density $\rho = 50kg/m^3$ is used to simulate the floor. Both linear and nonlinear dampers were used to do the simulation. The parameters of the linear damper are $\alpha = 1.0$, $C = 191kN/(m/\sec)$ and are $\alpha = 0.5$, $C = 1500(N-(\sec/mm)^{\alpha})$ for the nonlinear damper. The length of the damper is *8.062 m* with cross sectional area A = 1 m^2 and has mass density $\rho = 0.01kg/m^3$. The length of the two brace is 4.123 m and 8.062 m, respectively. These braces have the same mass density $\rho = 0.01kg/m^3$ and the cross sectional area $A = 1m^2$.

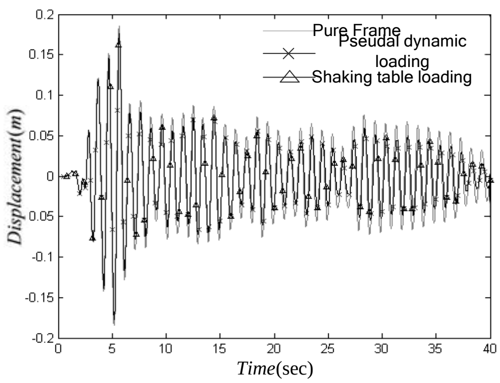


Fig. 7 Comparison of analytical responses of linear diagonal brace damper frame and pure frame subjected to pseudo dynamic and shaking table type loadings.

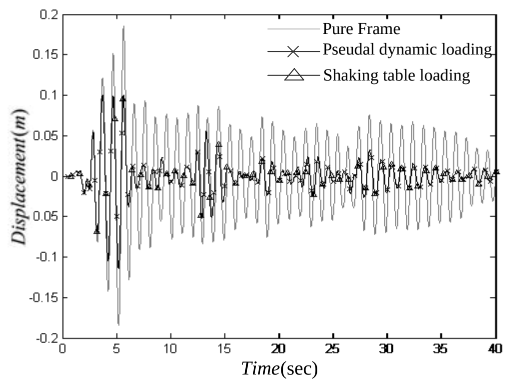


Fig. 8 Comparison of analytical responses of linear upper toggle-brace damper frame and pure frame subjected to pseudo dynamic and shaking table type loadings.

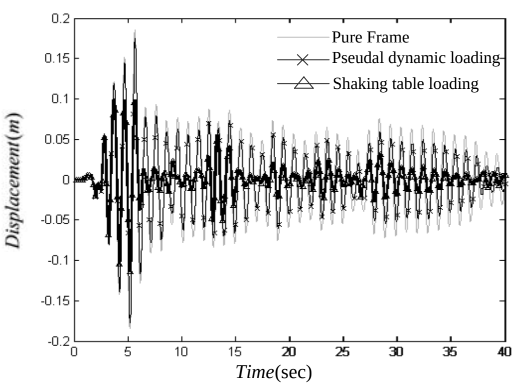


Fig. 9 Comparison of analytical responses of nonlinear diagonal brace damper frame and pure frame subjected to pseudo dynamic and shaking table type loadings.

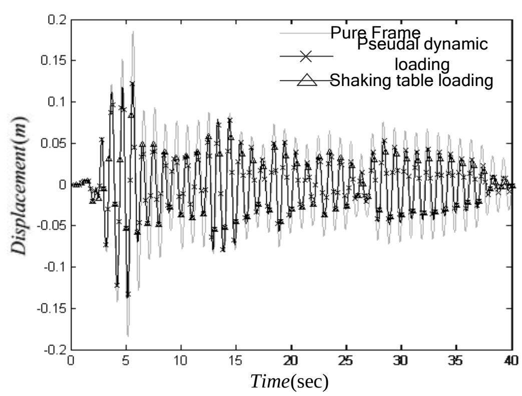


Fig. 10 Comparison of analytical responses of nonlinear upper toggle-brace damper frame and pure frame subjected to pseudo dynamic and shaking table type loadings.

Two ways of loading deployment were used for the structure to subject El Centro earthquake. One is the pseudo dynamic loading conventionally used in the structural dynamic analysis and the other one is the shaking table type excitation. Since the VFIFE is very flexible to apply load and displacement control on the structure, a rigid plate is added into the process of simulation. An acceleration equivalent force is calculated by multiplying the mass of the rigid plate by the ground acceleration of the structure. The equation of motion of the rigid plate can be solved to ensure the acceleration of the plate is equivalent to the desired ground acceleration. The dynamic displacements of the points on the plate connected to the structure are used into the VFIFE analysis as the displacement boundary conditions (see Fig. 7). This technique can be used to simulate the dynamic behaviors of structure subjected ground motion with rotational degrees of freedom.

From the comparisons shown in Figs. 8-10, one can realize that the toggle-brace-damper can amplify the damper displacement and dissipate energy effectively compared with the diagonal brace damper. Besides, the shaking type loading excitation is accurate and rational. This newly proposed VFIFE method is quite effective for engineers to conduct motion and control analyses of structures under various types of dynamic excitations.

5. CONCLUSIONS

A novel numerical method called the Vector Form Intrinsic Finite Element (VFIFE) method for the motion analysis of frame structure is presented. Numerical examples demonstrate that the VFIFE method can effectively conduct passive control analysis of structure under earthquake excitation. Due to some special features of VFIFE, it can conduct simulations of the hybrid control of structure with progressive failure and collapse. It is believed that the further development of VFIFE method can provide engineers as an effective and friendly tool to analyze very complicated engineering problems.

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