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RELEVANCE VECTOR MACHINE REGRESSION APPLIED TO STRUCTURAL HEALTH MONITORING

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Abstract

A recently-developed Bayesian learning methodology, the Relevance Vector Machine (RVM), can identify a regression model based on given training data with the advantages of sparseness (i.e. utilizing only a small number of relevant basis expansion terms by automatically pruning others) and automatic regularization of ill-conditioned problems. In training of the RVM, the probability distribution over the unknown regression model parameters is updated based on given training data by using Bayes' Theorem with a special prior probability distribution called the Automatic-Relevance-Determination prior. RVM uses the same kernel basis expansion as in the popular machine learning methodology, the Support Vector Machine (SVM). However, SVM gives only a single estimate while RVM gives probabilistic predictions and it usually does so with a drastically smaller number of basis terms than occurs in the corresponding SVM for the same regression problem. A preliminary investigation of the applicability of RVM regression to structural health monitoring is made. RVM is used to learn a probabilistic relationship between the structural feature vectors, such as changes in mode shapes and natural frequencies, and corresponding damage index vectors describing the damage state of the structure. A two-step approach to SHM is investigated, in which one trained RVM is used to detect the locations of any damage from dynamic data and another trained RVM is then used to estimate the damage severity at each location. The mathematical procedure for RVM and the results for illustrative examples are presented. These preliminary results suggest that RVM is worthy of further investigation as a data processing tool for structural health monitoring.

INTRODUCTION

There is a well-recognized need for an efficient and reliable structural health monitoring (SHM) methodology to detect and assess structural damage from dynamic data for both maintenance and safety of structures. Despite significant advances in sensor hardware, there is still no well-established methodology that meets this need.

Many studies of SHM are based on the updating of a physically-based structural model using a measured structural response (time histories or features extracted from them, e.g. [13,14,4]). Other SHM studies have used a pattern recognition approach that updates a nonlinear regression model such as an Artificial Neural Network (ANN) (e.g. [6,15]). This study presents a regression approach to SHM by utilizing a Bayesian learning methodology called the

Relevance Vector Machine (RVM) that updates the probability distribution over the unknown model parameters based on given data and an Automatic-Relevance-Determination (ARD) prior [11]. It uses the same kernel basis expansion as in the popular learning methodology, the Support Vector Machine (SVM). One advantage of a regression approach to SHM is that it does not require the computational effort needed to initially develop and then update a detailed structural model of the system. RVM also has merit as a regression method compared with SVM or ANN in that it is able to provide probabilistic predictions using a smaller number of relevant basis functions. This sparseness is produced by using the ARD prior along with Bayesian model class selection [7,1] to select the most probable (plausible) model class based on the data by maximizing its evidence (also called its marginal likelihood). This model class selection procedure automatically produces optimal regularization of regression problems, making them well-conditioned.

RVM has already been investigated for damage classification rather than damage regression as considered here [8], but if the number of categories increases, classification (or pattern recognition) approaches might be computationally inefficient. Also, most previous work on RVM applied to regression problems has used a scalar output, as in the original theory [11]. In this study, RVM is applied using vector outputs to examine its effectiveness; this is an on-going research topic in machine learning in order to make RVM a good tool for general regression problems [10].

The regression application of RVM to dynamic data from a structure for SHM consists of three phases:

1) Phase I (feature extraction phase): Determine which features from the dynamic data to use for a training dataset. The key point in this stage is to examine which features extracted from the data are more sensitive to structural damage but less sensitive to modeling and/or measurement errors.
2) Phase II (training phase): In this stage, the mathematical form and unknown parameters defining a regression model are identified by using a training dataset.
3) Phase III (operating phase): Based on the most plausible regression model determined in the previous phase, prediction for damage assessment is performed on operating data (features extracted in real-time from sensor signals).

TRAINING AND TESTING DATASETS

There has been a lot of research conducted on what to use as damage-informative features for SHM: modal properties such as natural frequencies and mode shapes or the changes in these quantities [4,13,14], the ratios between changes of the measured eigenvalues [2], Ritz vectors, or the changes in them [3,9,6], damage signature defined as the ratio of the change of eigenvectors to the change of eigenvalues [15], mode shapes scaled by the inverse square of their natural frequencies [8], and so on. The goal has been to search for features that are sensitive to structural damage but insensitive to modeling and/or measurement errors.

In this study, a two-step approach is performed to detect and assess structural damage [15]. The first step is to identify any damage locations. Damage signatures defined as the ratios of the change in mode shape components to the change of a reference eigenvalue (e.g. fundamental frequency squared) are utilized as inputs to the RVM and a damage location index vector as output:

$$DS_j \text{ (Damage signature from } j^{th} \text{ mode)} = \frac{\Delta\phi_j}{\Delta\omega_1^2} \quad \text{and} \quad L = \left\{L_1, L_2, ..., L_{N_L}\right\}^T \qquad (1)$$

where N_L is the number of possible damage locations and L_i has value of 1 if damage exists at the i^{th} location and 0 otherwise.

In the second step, utilizing the identified damage locations from the first step, damage severity is estimated by using the changes in mode shape components and natural frequencies from the undamaged (baseline) structure as input and a damage severity index vector as output:

$$E = \left\{E_1, E_2, ..., E_{N_E}\right\}^T \quad (2)$$

where N_E is the number of damage locations identified from the first step and E_i has values between zero and one representing the fractional stiffness reduction at corresponding structural elements.

Systematic methodologies for pattern recognition or regression, such as ANN, SVM and RVM, are classified as supervised learning methods since the damage states for a training dataset are assumed to be a known *priori*. This condition can not be satisfied with real data since it is not possible to induce various damage states in an existing structure. Supervised learning, however, enables the SHM results to provide more information regarding damage severity and location. In this study, we implement a supervised learning approach by using a finite-element (FE) model of the structure to generate the training dataset by introducing various damage patterns in the FE model. However, the FE model used for training is not the same as the one representing the actual structure in the testing phase; realistic modeling error is reflected in the difference between the actual structural system and the FE model used to generate a training dataset. Also, a certain amount of noise is added to the simulated acceleration time histories to encourage robustness against measurement errors during the operating phase.

RVM PROCEDURE

In this section, the procedure for training and testing of RVM is presented. The original RVM algorithm was presented only for a scalar output [11]. In this study, we use the expanded RVM methodology that is applicable for vector outputs [10].

RVM Training

Let $\underline{f}(\underline{x}|\underline{\theta})$ denote the chosen regression function relating the feature vector $\underline{x} \in P^L$ to the most probable value of the output vector $\underline{y} \in P^M$ (damage location or severity index vector) when the model parameter vector $\underline{\theta}$ is specified. This function is embedded in a probability model by introducing an uncertain prediction error to account for the fact that no model gives prefect predictions, so:

$$\underline{y} = \underline{f}\left(\underline{x} \mid \underline{\theta}\right) + \underline{\varepsilon} \quad (3)$$

where $\underline{\varepsilon}$ is modeled as a Gaussian vector with zero mean and covariance matrix $\mathbf{\Omega} = \mathrm{diag}(\sigma_1^2, \ldots, \sigma_M^2)$. This choice of the probability model for the prediction error is motivated by Jaynes' Principle of Maximum Entropy [5]. It gives a Gaussian (Normal) predictive probability model (PDF) for the output vector $\underline{y}$:

$$p\left(\underline{y} \mid \underline{x}, \underline{\theta}, \underline{\sigma}^2\right) = \mathcal{N}\left(\underline{y} \mid \underline{f}\left(\underline{x} \mid \underline{\theta}\right), \mathbf{\Omega}\right) \quad (4)$$

where $\underline{\sigma}^2 = [\sigma_1^2, \ldots, \sigma_M^2]^T$.

Let $D_N = \{(\underline{x}_i, \underline{y}_i): i = 1, \ldots, N\}$ denote a training dataset generated from a FE model where $\underline{x}_i \in P^L$ is the i^{th} example of the feature vector and $\underline{y}_i \in P^M$ is the corresponding output. As in SVM, the regression function is expressed in terms of a kernel basis expansion where the i^{th} kernel function is centered at data point $\underline{x}_i$ (we use Gaussian radial basis functions in the examples later):

$$f_m\left(\underline{x} \mid \underline{\theta}\right) = \theta_{m0} + \sum_{i=1}^{N} \theta_{mi} k(\underline{x}, \underline{x}_i) = \underline{\tau}(\underline{x})^T \underline{\theta}_m, \quad m = 1, ..., M \quad (5)$$

where $\underline{\theta}_m = [\theta_{m0}, \theta_{m1},\ldots,\theta_{mN}]^T \in P^{N+1}$, $\underline{\theta}=[\underline{\theta}_1^T,\ldots,\underline{\theta}_M^T]^T \in P^{M(N+1)}$ and $\underline{\tau}(\underline{x}) = [1, k(\underline{x}, \underline{x}_1), \ldots, k(\underline{x}, \underline{x}_N)]^T \in P^{N+1}$. Using Bayes' Theorem to incorporate the information from the data D_N leads to the posterior PDF for the model parameter vector $\underline{\theta}$:

$$p\left(\underline{\theta} \mid D_N, \underline{\alpha}, \underline{\sigma}^2\right) = \frac{p\left(D_N \mid \underline{\theta}, \underline{\sigma}^2\right) p\left(\underline{\theta} \mid \underline{\alpha}\right)}{p\left(D_N \mid \underline{\alpha}, \underline{\sigma}^2\right)} \tag{6}$$

where $\underline{\alpha} = [\alpha_0,\ldots, \alpha_N]^T$ contains hyperparameters that control the prior for $\underline{\theta}$.
The components of each $\underline{y}_i$ are independent (since $\mathbf{\Omega}$ is diagonal) so the likelihood function can be expressed as a product of Gaussians for each output component:

$$p\left(D_N \mid \underline{\theta}, \underline{\sigma}^2\right) = \prod_{m=1}^{M} N\left(\underline{\upsilon}_m \mid \mathbf{\Phi}\underline{\theta}_m, \sigma_m^2 \mathbf{I}\right) \tag{7}$$

where $\underline{\upsilon}_m = [(\underline{y}_1)_m,\ldots, (\underline{y}_N)_m]^T$ and $\mathbf{\Phi} = [\underline{\tau}(\underline{x}_1), \ldots, \underline{\tau}(\underline{x}_N)]^T \in P^{Nx(N+1)}$. The ARD prior is:

$$p\left(\underline{\theta} \mid \underline{\alpha}\right) = \prod_{m=1}^{M} N\left(\underline{\theta}_m \mid \underline{0}, \mathbf{A}^{-1}\right) \tag{8}$$

where matrix $\mathbf{A}=\mathrm{diag}(\alpha_0,\ldots, \alpha_N)$. Using (6), the posterior PDF for $\underline{\theta}$ is:

$$p\left(\underline{\theta} \mid D_N, \underline{\alpha}, \underline{\sigma}^2\right) \propto \prod_{m=1}^{M} N\left(\underline{\theta}_m \mid \hat{\underline{\theta}}_m, \mathbf{\Sigma}_m\right) \tag{9}$$

where $\hat{\underline{\theta}}_m = \sigma_m^{-2}\mathbf{\Sigma}_m\mathbf{\Phi}^T\underline{\upsilon}_m$ and $\mathbf{\Sigma}_m = (\sigma_m^{-2}\mathbf{\Phi}^T\mathbf{\Phi}+\mathbf{A})^{-1}$ give the most probable value of $\underline{\theta}_m$ and its covariance matrix, respectively.

In the next step, Bayesian model class selection [1] is used to select the most probable hyperparameters $\hat{\underline{\alpha}}$ and variances $\hat{\underline{\sigma}}^2$, based on data D_N. If we take a uniform prior probability distribution over all possible model classes defined by $\underline{\alpha}$ and $\underline{\sigma}^2$ then by applying Bayes' Theorem at the model class level, the most probable model class is the one that maximizes the log evidence [1], which is given by [12]:

$$\begin{aligned} L\left(\underline{\alpha}, \underline{\sigma}^2\right) &= \ln p\left(D_N \mid \underline{\alpha}, \underline{\sigma}^2\right) = \ln \int p\left(D_N \mid \underline{\theta}, \underline{\sigma}^2\right) p(\underline{\theta} \mid \underline{\alpha}) d\underline{\theta} \\ &= -\frac{1}{2}\sum_{m=1}^{M}\left[N \ln 2\pi + \ln |\mathbf{C}_m| + \underline{\upsilon}_m^T \mathbf{C}_m^{-1} \underline{\upsilon}_m\right] \\ &= L\left(\underline{\alpha}_{-i}, \underline{\sigma}^2\right) + \sum_{m=1}^{M}\left\{\log \alpha_i - \log\left(\alpha_i + s_{mi}\right) + \frac{q_{mi}^2}{\alpha_i + s_{mi}}\right\} \end{aligned} \tag{10}$$

where $L(\underline{\alpha}_{-i},\underline{\sigma}^2)$ is the evidence with $\underline{\tau}_i$ excluded, $\mathbf{C}_m=\sigma_m^2\mathbf{I}+\mathbf{\Phi}\mathbf{A}^{-1}\mathbf{\Phi}^T$, $s_{mi}=(\alpha_i S_{mi})/(\alpha_i-S_{mi})$ and $q_{mi}=(\alpha_i Q_{mi})/(\alpha_i-S_{mi})$ and S_{mi} and Q_{mi} are calculated as follows:

$$S_{mi} = \sigma_m^{-2}\underline{\tau}_i^T\underline{\tau}_i - \sigma_m^{-4}\underline{\tau}_i^T\mathbf{\Phi}\mathbf{\Sigma}_m\mathbf{\Phi}^T\underline{\tau}_i \tag{11}$$

$$Q_{mi} = \sigma_m^{-2}\underline{\tau}_i^T\underline{\upsilon}_i - \sigma_m^{-4}\underline{\tau}_i^T\mathbf{\Phi}\mathbf{\Sigma}_m\mathbf{\Phi}^T\underline{\upsilon}_i \tag{12}$$

The maximum of the log evidence is determined by finding the stationary points of $L(\underline{\alpha},\underline{\sigma}^2)$ with respect to each α_i and σ_m^2. For example, to find $\hat{\alpha}_i$:

$$\frac{\partial L\left(\underline{\alpha}, \underline{\sigma}^2\right)}{\partial \alpha_i} = \sum_{m=1}^{M}\left\{\frac{1}{\alpha_i} - \frac{1}{\alpha_i + s_{mi}} - \frac{q_{mi}^2}{\left(\alpha_i + s_{mi}\right)^2}\right\} = 0 \tag{13}$$

and similarly, an expression can be found for $\hat{\sigma}_m^2$. Since s_{mi} and q_{mi} depend on all of the α_i's (since $\mathbf{\Sigma}_m$ does), an iterative algorithm [12] is applied to solve these equations for each $\hat{\alpha}_i$ and $\hat{\sigma}_m^2$. In practice, many of the α_i's approach infinity during this iterative process and so the corresponding θ_{mi} for each m=1,…, M get set to zero (they have zero mean and zero variance), thereby pruning the corresponding kernel $k(\underline{x},\underline{x}_i)$ from the regression function. In the end, only the kernels corresponding to a small number of the $\underline{x}_i$ are retained (these are called the relevance vectors).

RVM Testing

In the operating phase, we make predictions for the output (damage index vector) corresponding to a new feature vector based on the robust posterior predictive probability distribution as follows. Let $\underline{\tilde{x}}$ denote a feature vector extracted from dynamic data from the structure. The robust predictive probability for the corresponding damage index vector $\underline{\tilde{y}}$ based on the most probable model class is given by the Theorem of Total Probability:

$$p\left(\underline{\tilde{y}} \mid \underline{\tilde{x}}, D_N, \underline{\hat{\alpha}}, \underline{\hat{\sigma}}^2\right) = \int p\left(\underline{\tilde{y}} \mid \underline{\tilde{x}}, \underline{\theta}, \underline{\hat{\sigma}}^2\right) p\left(\underline{\theta} \mid D_N, \underline{\hat{\alpha}}, \underline{\hat{\sigma}}^2\right) d\underline{\theta}$$

$$= N\left(\underline{\tilde{y}} \mid \underline{f}(\underline{\tilde{x}} \mid \underline{\hat{\theta}}), \tilde{\mathbf{\Omega}}\right) \tag{14}$$

where $\underline{f}(\underline{\tilde{x}} \mid \underline{\hat{\theta}}) = \hat{\mathbf{\Theta}}^T \underline{\tau}(\underline{\tilde{x}})$, $\hat{\mathbf{\Theta}} = [\,\underline{\hat{\theta}}_1, ..., \underline{\hat{\theta}}_M\,] \in P^{(N+1)xM}$, $\tilde{\mathbf{\Omega}} = \text{diag}(\tilde{\sigma}_1^2, ..., \tilde{\sigma}_M^2) \in P^{MxM}$, $\tilde{\sigma}_m^2 = \hat{\sigma}_m^2 + \underline{\tau}(\underline{\tilde{x}})^T \hat{\mathbf{\Sigma}}_m \underline{\tau}(\underline{\tilde{x}})$ and $\hat{\mathbf{\Sigma}}_m = (\hat{\sigma}_m^{-2} \mathbf{\Phi}^T \mathbf{\Phi} + \hat{\mathbf{A}})^{-1}$.

ILLUSTRATIVE EXAMPLE: SHM FOR FIVE-STOREY SHEAR BUILDING

A five-storEy shear building has previously been used to study the applicability of ANN for estimating the damage locations and severity [15]. A similar study is performed here using the vector output RVM. However, for more realism, modeling errors are introduced in this study when generating a training dataset by using 95% of the floor mass and 90% of the interstory stiffness when constructing the FE model from which mode shapes and natural frequencies are calculated for various damage states. (The previous study [15] did not consider modeling error when generating a training dataset).

Training consists of two steps as explained before. In the first step, damage signatures and indices (1) are used as inputs and outputs, respectively. After training a RVM to find the damage locations, another RVM is trained to estimate damage severities using changes in the modal parameters (here fundamental frequency and mode shape as inputs and damage severity indices (2) as outputs).

When training the RVM to find the damaged stories, the total number of possible damage cases is 32 (including an undamaged case) because multiple stories may be damaged. For each of these damage cases, a 50% stiffness reduction is imposed for each damaged story to generate the feature vectors from the training FE structural model. The RVM trained to find damage locations is then tested by using the mode shapes and natural frequencies from simulated dynamic data from the FE model that represents the actual shear building. To provide a severe test, only the fundamental frequency and mode shape are extracted from noisy data obtained by adding 5% root-mean-square discrete white-noise to simulated acceleration time histories to reflect the measurement noise. The datasets used in this test phase are as follows:

(1) Single damage in each story with 20% stiffness reduction (see results in Table 1a)
(2) Single damage in each story with 80% stiffness reduction (see results in Table 1b)
(3) Damage in two stories with 20% stiffness reduction in each (see results in Table 2)
(4) Damage in the 2nd and 3rd stories with selected stiffness reductions r_2% and r_3%, respectively (see results in Table 3)

In these tables, RVM output values near one indicate the location of damage. Note that testing datasets are not included in the training dataset. From Tables 1-3, it can be concluded that the RVM works well for identifying damage locations with a performance comparable with the previous study using ANN (see the results in [15]).

After locating the damaged structural members, the severity of the damage is estimated. Two different cases are considered when training and testing for this purpose:

(1) The fundamental frequency and mode shape are used together and are generated from FE model with a single damage at the 2nd story. These features are generated by imposing 10%, 20%, 30%, 40%, 50%, 60% and 70% stiffness reduction to the corresponding story. For the test case, the stiffness at the 2nd story is reduced by 20%.

(2) Similarly to single-damaged case, 10% to 70% stiffness reductions are imposed on the 2nd and 3rd stories. The same features as in (1) are obtained from the FE model for training. For the test case, damage is imposed as a 50% and 30% stiffness reduction at the 2nd and 3rd stories, respectively.

Note that in the second step, the damage locations are already known. The estimated damage severities are very close to the exact values: 19.42% for the first case and 50.51% and 32.19% for the 2nd and 3rd stories for the second case. The performance is comparable to that of the study using ANN for the same example (see the results in [15]).

We conclude that the proposed vector output RVM successfully estimates both the damage locations and the damage severity for this simple illustrative example.

CONCLUSIONS

Parallel to the development of sensor technology, there is a need for efficient and reliable data processing tools for structural health monitoring. In this study, a regression method based on the vector output RVM is introduced to determine the damage locations and severity from changes in modal parameters. RVM automatically selects the most probable model class to provide the best predictions for damage assessments by maximizing the evidence for the model class based on the regularizing ARD prior. Once the RVM is trained, it is efficient for on-line SHM based on extracting the selected feature vector from dynamic data.

From the illustrative example, we conclude that the proposed vector output RVMs show promise for estimating both the damage locations and the damage severities from changes in the structure's dynamic characteristics, such as its modal parameters, when the SHM is performed in two steps: first identify damage locations and then estimate damage severities. We are currently investigating a one-step process for SHM where a single RVM is trained to estimate damage locations and severities simultaneously. Another phase of future work is to study the applicability of the RVM SHM method to experimental data, such as the Phase II IASC-ASCE Structural Health Monitoring Experimental Benchmark Data that was studied in [4].

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Table 1: Identify single damage of (a) 20% and (b) 80% stiffness reduction in each storey.

Story	(a) Damage Case 1					(b) Damage Case 2				
	1	2	3	4	5	1	2	3	4	5
1	**1.07**	-0.10	-0.19	-0.02	0.00	**1.06**	-0.43	-0.01	0.01	0.00
2	0.27	**0.75**	0.17	-0.04	0.00	0.10	**0.57**	0.17	0.00	0.00
3	0.12	0.11	**0.88**	0.15	0.00	0.03	-0.09	**1.22**	-0.02	0.00
4	-0.28	-0.32	0.42	**0.98**	0.00	-0.06	-0.16	0.14	**0.87**	0.00
5	0.42	0.11	0.42	0.31	**0.40**	0.18	0.10	0.14	0.27	**0.40**

Table 2: Identify damage of 20% stiffness reduction in both x_1/x_2 stories.

Damage Case 3	Storey				
	1	2	3	4	5
1/2	**0.81**	**0.96**	0.11	-0.05	-0.02
1/3	**0.97**	0.33	**1.33**	-0.32	0.58
1/4	**1.12**	0.33	0.20	**1.21**	0.30
1/5	**1.07**	0.29	0.23	-0.36	**1.30**
2/3	-0.12	**0.94**	**0.99**	-0.22	0.16
2/4	-0.20	**1.03**	0.01	**0.79**	0.25
2/5	-0.24	**0.86**	0.03	-0.23	**1.12**
3/4	-0.06	0.03	**1.00**	**0.64**	0.52
3/5	-0.05	0.02	**0.83**	-0.07	**0.72**
4/5	-0.01	-0.01	0.08	**0.79**	**0.52**

Table 3: Identify damage in the 2nd/3rd stories of r_2/r_3% stiffness reduction.

Damage Case 4	Storey				
	1	2	3	4	5
50/10	-0.02	**0.97**	**0.25**	-0.18	0.01
50/20	-0.05	**1.01**	**0.42**	-0.18	0.05
50/30	-0.05	**1.03**	**0.64**	-0.26	0.15
50/40	-0.07	**1.01**	**0.74**	-0.15	-0.02
50/50	-0.07	**0.97**	**1.02**	-0.08	0.04
50/60	-0.02	**0.85**	**1.25**	-0.08	0.03
50/70	0.24	**0.88**	**1.36**	-0.13	-0.18
50/80	-0.28	**0.42**	**0.98**	-0.03	0.22
50/90	-0.14	**0.13**	**0.50**	-0.10	0.21