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MULTILEVEL STRUCTURAL HEALTH MONITORING USING AN INVERSE WAVE PROPAGATION METHOD

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Abstract

Health monitoring and evaluation of civil and aeronautical structures is becoming even more important to guarantee structural performance, safety and economic reliability. Modern designs and materials pose new challenges and more sophisticated methods are required to satisfy operational conditions and management demands. Recent sensor technology achievements open up the possibility for thousands of different types of sensors for innovative monitoring techniques. While most damage identification methods are based on the modal analysis, this paper presents a different approach based on the analysis of the wave propagation behavior with an adapted minimum rank perturbation theory.

Two solution schemes for the inverse problem with regularization are described; the first is based on dynamic programming, while the second on the wavelet convolution; both with a general finite element model. Results show that different levels of structural monitoring are possible (global or detailed). From the same data, a fast monitoring analysis can be done to identify damage and then, a detailed study of the damaged sub-region is possible for a full evaluation. The wavelet convolution solution scheme allows the use of time limited spatially distributed data setting off an opportunity for whole field data sensors for structural monitoring and damage evaluation.

INTRODUCTION

Modern civil and aeronautic structures are getting more complex, new concepts and materials are increasingly being used, and environmental or operational conditions are more demanding. Moreover, structures are important components of integrated systems (i.e. highway bridges in integrated transport systems), where monitoring is necessary, and in some cases, in real time[1]. At the same time, sensor technology has grown and expanded very many possibilities on measuring, evaluation and control of engineering systems; multiple sensors of different types are now affordable and we can envisage the use of thousands of embedded sensors in smart structures[2,3]. Under these operational conditions, health monitoring of structures pose different challenges and alternatives for innovative theory and experimental schemes[4].

Structural evaluation to identify damage, deterioration and/or operational anomalies in modern complex civil is essential to determine their safety, operational reliability and residual life[1]. Traditionally, most of the damage detection programs are based on visual inspections, which are costly and difficult due to the inaccessibility to most of the parts of the structure. Furthermore, internal damage can not be detected and no quantitative value of the damage and the residual strength are given. Recent health monitoring systems have included different nondestructive testing techniques, but most of them are highly localized and no global evaluation of the structural condition is possible[5]. Until now, it is being recognized that the vibration and modal analysis techniques are the only ones with the potential to serve for global description[1,6]. In these cases, health monitoring is done through changes in the vibration signature characterized by the natural frequency, damping ratios, and mode shapes. In general, an undamaged model (typically a Finite Element Model) is used as a reference to compare the dynamic responses of the model with the experimental measurements from the real structure. Many algorithms have been developed for this purpose and generally are classified under four different categories: optimal-matrix updates, sensitivity methods, eigenstructure assignment techniques, and minimum-rank perturbation methods[7].

In reaction to the limitations of the modal analysis based algorithms[8,9], an inverse force identification method, called Sub-Domain Inverse method (SDIM), was adapted for the identification of damage. The SDIM is based on the analysis of the wave propagation within the structures[10,11], thus it requires few experimental tests, and it can solve a large number of unknowns. The sub-structural analysis is also possible without loss of generality and furthermore, it is size tunable to macro or micro complex structures. Appropriate sensors distribution defining sub-regions, permits unambiguous identification of the damaged sub-regions and from the same data, different data mining strategies may be used for global monitoring and to zoom into the structure.

STRUCTURAL CHANGES AS RESPONSE VECTORS

Within a structural context, damage appears as a change of stiffness and/or mass. Also, a stress wave propagation phenomenon in a structure is affected by these structural changes and hence, it could become very good carrier of this information. It is in this context, that we define change of stiffness and/or mass as damage and use wave propagation responses in order to detect it.

Using the finite element method notation, the changed condition is represented as a perturbation of the stiffness and mass matrices form the undamaged state as $[\mathbf{K_D}] = [\mathbf{K}_0] - [\mathbf{\Delta K}]$, $[\mathbf{M_D}] = [\mathbf{M}_0] - [\mathbf{\Delta M}]$, then the governing equations for the damaged condition become as:

$$[M_0]\{\ddot{u}\} + [C_0]\{\dot{u}\} + [K_0]\{u\} = \{P\} - [\Delta K]\{u\} + [\Delta M]\{\ddot{u}\} = \{P\} + \{D\} \qquad (1)$$

The vector $\{\mathbf{D}\}$ clearly has the information about the damage and we therefore call it the Damage Response Vector (DRV). If somehow, we can determine $\{D\}$ then it is a direct (but subtle) process to extract the desired damage information. As an illustration, consider the case with only a stiffness change: $[\Delta K][u] = [D]$.

Where the time and space information of $\{u(t)\}$ and $\{D(t)\}$ were arranged as the rectangular arrays $[u]$ and $[D]$, respectively. This set of equations to determine $[\Delta K]$ is ill-conditioned; however, we can solve it using the same approach as used in the Minimum Rank Perturbation Theory[12]. This gives the solution representation as:

$$[\Delta K] = [Du^T]\left[[Du^T]^T[uu^T]\right]^{-1}[Du^T]^T \qquad (2)$$

The inner matrix inversion is accomplished using singular value decomposition (SDV) and taking only a sub-set of the singular values. All the required information such as damage location, extent of stiffness reduction, and so on, is contained in the matrix $[\Delta K]$.

The key issue in the damage detection scheme is the ability to identify the vector $\{D\}$. The components of this vector are associated with the nodes' degrees of freedom (DOF) of the FEM model and therefore, constitute a large set of unknowns. Note that it appears in Equation (1) as a force vector similar to $\{P\}$; indeed, one interpretation of $\{D\}$ is as a collection of externally applied loads acting on the undamaged structure to give a response similar to the damaged structure. Our problem therefore reduces to one of determining a large set of unknowns applied forces – it is precisely for this situation that the Sub-Domain Inverse Method was developed.

FORCE IDENTIFICATION INVERSE PROBLEM

A typical forward problem in engineering entails determining the system response when both the system and the inputs are known. Inverse Problems are situations where some aspects of the system are unknown (material properties, boundary conditions, behavior of a non linear joint, for example) while other aspects are known and we attempt to use measurements to determine the unknowns. A common characteristic of inverse problems is those are notoriously ill-conditioned and require a level of sophistication far beyond the corresponding forward problems. Most of the engineering problems are of the inverse type, such as the force identification problems.
Consider a structural system represented by the following dynamic equation:

$$[M]\{\ddot{u}(t)\}+[C]\{\dot{u}(t)\}+[K]\{u(t)\}=\{P(t)\} \quad (3)$$

Identification of vector $\{P(t)\}$ is possible from experimental measurements from time-space limited sensors such as accelerometers, or space-time limited sensors such as Moiré imaging. Thus, two different approaches are investigated: the first is based on dynamic programming and the second on the wavelet convolution.

The probability of achieving a good inverse solution is enhanced if the number of unknowns can be reduced. This would be possible if we could just isolate the sub-region or sub-domain of interest (the aircraft wing form the fuselage, the loose joint from beams, and the center span from the rest of a bridge). Unfortunately, this introduces a set of unknowns associated with the boundaries. The idea of the SDIM is shown in Figure 1; the sub-domain has, in addition to the primary unknowns of interest, a set of additional unknown tractions that are equivalent to the remainder of the structure.

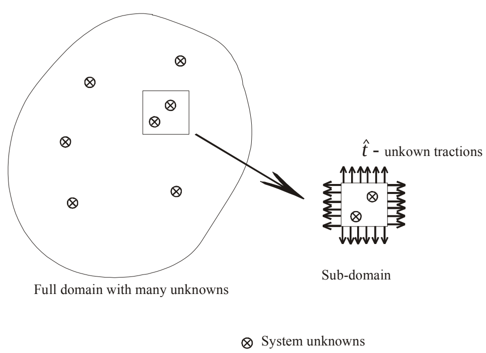


Figure 1. Sub-domain with the system unknowns plus the unknown boundary tractions.

The significant advantage of analyzing a sub-domain with unknown tractions on the boundaries, over analyzing the complete domain is the opportunity to remove non linearities from the analysis. For example, a sloshing fuel tank in an aircraft or a sunken foundation pile would simple not be included in the sub-domain. It may also be that the totality of unknowns in the complete system is itself unknown again justifying a sub-domain. A further advantage is that it gives us a formal way of scaling our analysis; that is, global analysis as well as detailed analysis is possible within the same framework.

Dynamic Programming for Force Identification
Force identification from Equation (3) is solved using an algorithm based on Bellman's dynamic programming in conjunction with a general finite element program to supplement with additional information the measured data. Solution is calculated from an error function with regularization terms to give robustness to the solution originated by the ill-conditioning of the inverse problem. In general, the error function is:

$$\boldsymbol{E}(\mathrm{u},\mathrm{g}) = \sum_{n=1}^{N}\left[\{\mathrm{d}-\mathrm{Qu}\}_n^T[\mathrm{W}]\{\mathrm{d}-\mathrm{Qu}\}_n + \lambda\{\mathrm{g}\}_n^T[\mathrm{H}]\{\mathrm{g}\}_n\right] \quad (4)$$

Where, $[\mathrm{Q}]$ is the matrix that relates the measured data $\{\mathrm{d}\}_n$ with the degrees of freedom of the system $\{\mathrm{u}\}_n$; $\{g\}_n$ is the vector of the sub-set of forces to be identified; $[\mathrm{H}]$ is the regularization matrix, and λ is the regularization parameter according to Tikhonov theory. A two step solution is initiated with a backward iterative scheme; the second step calculates the forces using a forward iteration process. Detailed description of the equations and how they are solved are included in References 10 and 11. Key feature of this solution scheme is that complete time information is required for the measured data.

Force Identification Through the Wavelet Deconvolution.
The basic idea of the wavelet deconvolution is to include information from space-time limited sensors. In this case, the force is expanded in terms of the wavelet function $\phi_m(t)$ and the displacements at each node are expanded in terms of the wavelet functions $\psi_m(x,t)$, which are the responses at the node due to a force $\phi_m(t)$. Then,

$$P(t) = \sum_{m=1}^{M_g}\tilde{P}_m\phi_m(t) \qquad \text{and} \qquad u(x,t) = \sum_{m=1}^{M_g}\tilde{P}_m\psi_m(x,t) \quad (5)$$

To solve the inverse problem, the following function error is used

$$\boldsymbol{E}(\overline{\mathrm{P}},\mathrm{d}) = \boldsymbol{A} + \boldsymbol{B}(\lambda_t,\lambda_s) = \{\mathrm{d}-\overline{\mathrm{Q\Psi P}}\}^T[\mathrm{W}]\{\mathrm{d}-\overline{\mathrm{Q\Psi}P}\} + \{\overline{\mathrm{P}}\}^T[\lambda_t[\mathrm{H_t}]+\lambda_s[\mathrm{H_s}]]\{\overline{\mathrm{P}}\} \quad (6)$$

Where $\boldsymbol{A}$ and $\boldsymbol{B}$ are positive operators; $\boldsymbol{A}$ measures the error between the experimental data with the simulated data; and $\boldsymbol{B}$ is a stabilization term for the time and space regularization. Minimizing $\boldsymbol{E}$ with respect to $\overline{\mathrm{P}}$ yields:

$$[\overline{\mathrm{G}}]\{\overline{\mathrm{P}}\} = \{\hat{\mathrm{u}}\} \quad (7)$$

where,

$$[\overline{\mathrm{G}}] \equiv [\overline{\mathrm{Q\Psi}}]^T[\mathrm{W}][\overline{\mathrm{Q\Psi}}] + \lambda_t[\mathrm{H_t}] + \lambda_s[\mathrm{H_s}], \qquad \{\hat{\mathrm{u}}\} \equiv [\overline{\mathrm{Q\Psi}}]^T[\mathrm{W}]\{\overline{\mathrm{d}}\} \quad (8)$$

A simple triangular form for the wavelet function $\phi(t)$ behaves as a filter for high frequency undesired components and to keep the information within the frequency range of interest. Also, the inverse deconvolution is not needed while it is obtained directly from the solution[13]. When the number of unknowns is very large, a direct matrix inversion method for Equation (7) turns out to be time consuming; then the Quasi-Toeplitz structure of matrix $[\overline{G}]$ can be used to adapt fast (Levinson[14]) o super-fast[15] algorithms, which are of $O(n^2)$ or $O(n\log^2 n)$ respectively.

MULTILEVEL HEALTH MONITORING EXAMPLE

For demonstration of the SDIM potential for monitoring and damage detection, two different scenarios are simulated. The first, is a bidimensional truss structure fully instrumented, while the second is a plate structure partially instrumented in which all the nodal displacements are measured, but none of the angular rotations.

Bidimensional Truss with Complete Sensor Instrumentation
Consider a two dimensional truss structure divided into 6 sub-regions, instrumented with bi-directional sensors at all nodes and with highly located damage in one sub-region (Figure 2).

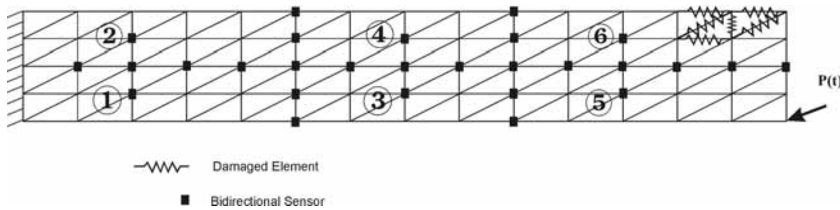
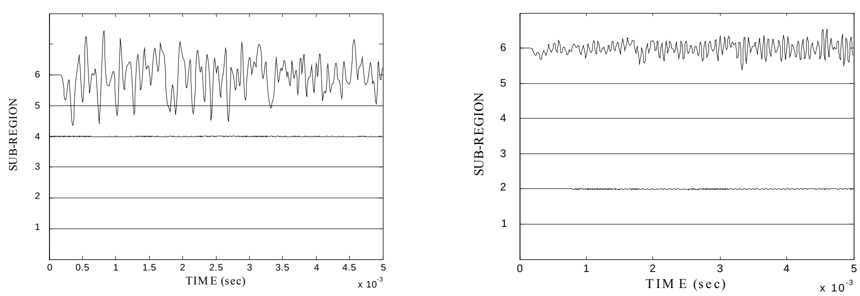


Figure 2. Fully instrumented truss.

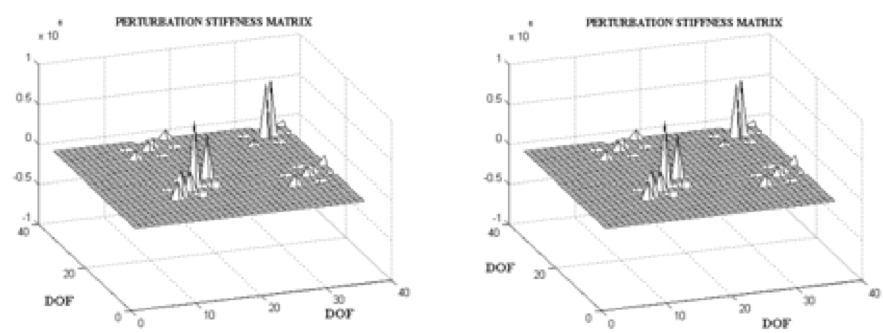
Assume that monitoring is run under controlled conditions, that is, the external excitation force and the undamaged structural conditions are known. In this case, the proposed monitoring strategy is based on the following:

a. All sensor data are recorded (70 bi-directional sensors).
b. Sub-region limiting sensors and one internal sensor in each sub-region are only considered for the first step, that is 28 bi-directional sensors (indicated in Figure 2), which corresponds to the 40% of the total recorded data.
c. DRV's are calculated for the limiting sub-region degrees of freedom and for the internal nodes associated to the internal sensors (56 DOF out of 140).
d. All nodes directly related to damage sub-regions will show none zero DRV's. Particular attention is given to the corresponding DRV's of the 6 internal nodes (see Figures 3a and 3b).
e. Once damaged sub-region is identified (sub-region 6), a second analysis step is done for damage evaluation. Thus, considering all the data from the 18 sensors in that sub-region, the DRV´s are calculated for all the 36 DOF of the corresponding nodes.
f. Once the DRV's are obtained, the sub-region perturbation stiffness matrix $[\Delta K]$ is calculated using Equation (2). In this case, matrix inversion using the SVD was calculated with 20 singular values. Figures 4(a) and 4(b) show a comparison of the calculated and exact perturbation matrices $[\Delta K]$.



(a) "x" direction (b) "y" direction

Figure 3. DRV´s for both directions for the internal nodes in each sub-region.



(a) Calculated (b) Exact

Figure 4. Perturbation stiffness matrices for the truss sub-region 6.

For very large structures, this process can be divided into several steps to gradually zoom from the whole structure to the damaged sub-regions. In this first example, the structure was divided into six sub-regions and damage was immediately identified from the DRV´s of the monitoring internal sensors in each sub-region.

Partially Instrumented Plate

Most practical cases are not fully instrumented due to sensor limitations. This is the case of a structure that can have complete displacement sensors in the either “x”, “y” and/or “z” directions, but no sensors for angular rotations. Having no information from the nodal angular rotations is equivalent to assume zero torsional DRV’s for those nodes. This limitation affects the performance of the SDIM because when none zero DRV’s are neglected, the algorithm compensates their effect on other DRV’s, leading to misleading results. In some cases, when the neglected DRV’s are small, damage analysis can be appropriate to some extent.

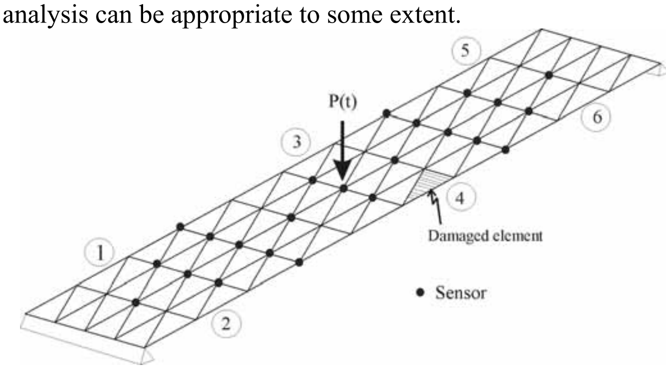


Figure 5. Partially instrumented plate structure.

In our second example, damage is simulated in a plate structure (Figure 5) with 3 DOF per node, z displacement and “x” and “y” rotations. The structure is fully instrumented with displacement sensors at all nodes and experimental conditions are under known excitation load and initial undamaged conditions.

Following a similar procedure as previous example, an initial step identifies the damaged sub-region considered monitoring of the DRV´s of the six internal monitoring nodes. Figure 6 shows the DRV´s in the “z” direction at the monitoring nodes, indicating that the damaged sub-region is number 4.

The second step to quantify damage uses the displacement sensor information of the complete plate to calculate the “z” DRV´s for all the nodes (45 forces). Figures 7(a) and 7(b) show the computed and exact diagonal terms of the perturbation stiffness matrices and, although the damaged element can be easily identified for having the largest magnitudes (DOF 26, 31 and 32), the neighboring elements show a small stiffness reduction as if they were damaged (DOF 21, 27, 33, 36 and 37). This effect, within a 20% error, is originated by assuming zero torsional forces. At the same time, the damage magnitude with respect to the exact perturbation matrix is within a 50%, as if damage were distributed over the neighboring elements.

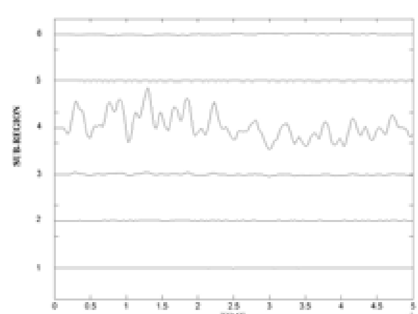
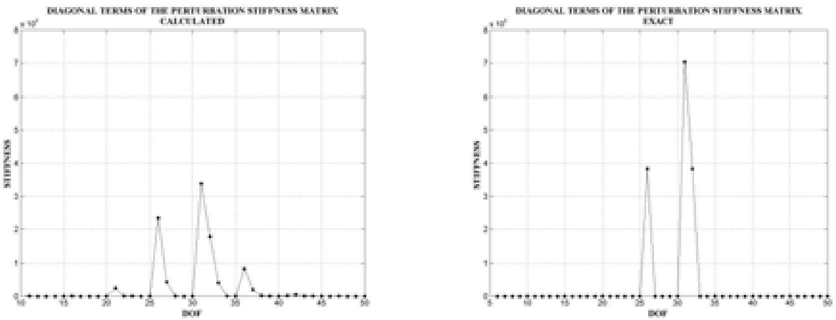


Figure 6. Damage Response Vectors (DRV’s) for the monitoring internal nodes.



(a) Calculated (b) Exact

Figure 7. Diagonal terms of the perturbation stiffness matrices for the plate sub-region 4.

DISCUSSION AND CONCLUSIONS

The capacity of the SDIM to handle very many sensors and unknowns makes it a very flexible and advantageous scheme for structural health monitoring and damage detection. Structural partitioning and sub-domain can be adapted as necessary and, as long as there is enough information from the sensors, analysis is possible in many forms from a fast general evaluation to a complex detailed analysis. Unknowns or non linearities can be removed from the analysis and substituted by boundary unknown tractions of the sub-domain that can be easily identified from the SDIM.

The damage response vectors, once they are identified by the SDIM contain sufficient information for the damage evaluation regardless the severity or location, assuming that damage manifests itself as a change of stiffness or mass. Multilevel analysis for damage evaluation can be done and the monitoring strategy depends on the size and type of structure, as well as the instrumentation available. Main feature of this scheme is that at each step the damage analysis is simpler than considering the whole structure and measured data. When the structure is fully instrumented, damage evaluation is straight forward, but sensor limitations lead to inaccurate results as it can be seen from the plate analysis example.

Although in this study only time data is used from discrete distributed sensors, it is possible to extend the SDIM to handle spatial distributed data from imaging sensors. Preliminary results with adapted techniques demonstrate that the use of a wavelet deconvolution can be used with Moiré images taken at a discrete number of times[13]. Modification of the SDIM is possible but it requires the adaptation of fast and super-fast Toeplitz-like solvers and further research is needed. On the other hand, it enables the use or many other sensors of different type with spatial information. It is believed that the use of spatial distributed data will enhance results for sensor limited cases.

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