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STRUCTURAL DAMAGE IDENTIFICATION OF PLATE STRUCTURES BASED ON FREQUENCY RESPONSE FUNCTION AND NATURAL FREQUENCIES

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Abstract

In this paper, a structural damage identification method (SDIM) is developed for plate-like structures. This Method is derived using dynamic equation of undamaged/damaged plate, in which local change in flexural rigidity is characterized utilizing a damage distribution function. The SDIM requires to modal data in the intact state and frequency response of damage state where most of vibration based damage identification techniques requires to modal data in both states. Change of mode shapes of damaged plate are approximated as a linear combination of mode shapes of intact plate and are considered in dynamic equation of damaged plate. Constant Coefficients of linear combination have been evaluated using perturbed equation of motion and the damage distribution function. Two strategies for making the inverse problem damage identification are introduced in connection with damage the present SDIM: (1) by using sensitivity of natural frequencies and (2) by using FRF-data, a sufficient number of equations can be derived to detect magnitude and location of damage. The feasibility of presented method is validated through some numerically simulated damage identification test taking into account random noise in FRF-data.

INTRODUCTION

Occurred damage within a structures leads to change of structural vibration characterizes and can be used in turn to detect, locate and quantify existence structural damage. The finite element model (FEM) update techniques have been proposed in the literature [1]. The existing experimental-data-based SDIMs can be classified into several groups depending on the type of experimental data used to detect, locate, and/or quantify structural damage. They include changes in modal data [2–5], frequency response functions (FRFs) [6, 7], strain energy [10], transfer function parameters [11], flexibility matrix [12], residual forces [13]. As a drawback of FEM-update techniques, the requirement of reducing FEM degrees of freedom or extending the measured modal parameters may result in the loss of physical interpretability and the errors due to the stiffness diffusion that smears the damage-induced localized changes in stiffness matrix into the entire stiffness matrix.

The modal-data-based structural damage identification method (SDIM) has some shortcomings. First, the modal data can be contaminated by measurement errors as well as modal extraction errors because they are indirectly measured test data. Second, the completeness of modal data cannot be met in most practical cases because they often require a large number of sensors. On the other hand, using measured FRFs may have certain advantages over using modal data. First, the FRFs are less noise contaminated because they are directly measured from structures. Second, the FRFs can provide much more information about damage in a desired frequency range than modal data are extracted from a very limited number of FRF data around resonance [14]. Thus, the use of FRFs seems to be very promising for structural damage identification.

Identifying ways to minimize the experimental measurement errors, structure model errors, and the damage identification analysis errors has been an important issue in most structural damage identification researches. Some researchers have investigated the damage-induced changes in natural frequencies, mode shapes, and curvature mode shapes with varying the location and severity of a damage. However, very few attentions have been given to the effects of the change of mode shapes, damage-induced coupling of vibration modes and the higher vibration modes omitted in the analysis on the accuracy of predicted vibration characteristics of the damaged beam, from a damage identification viewpoint.

The purposes of the present paper are: to develop an FRF-based SDIM, to investigate effects of the mode shape changes on the accuracy of the predicted vibration characteristics of damaged plates, and finally to verify the feasibility of the present SDIM through some numerically simulated damage identification tests.

THEORY

Dynamic Equation of Motion of Intact Plate

The dynamic equation of motion for a plate is expressed as follows:

$$\frac{\partial^4 w}{\partial^4 x} + 2\frac{\partial^4 w}{\partial^2 x \partial^2 y} + \frac{\partial^4 w}{\partial^4 y} + m\ddot{w} = \frac{f(x,y,t)}{D} \quad (1)$$

where *w(x, y, t)* is the flexural deflection, *f(x, y, t)* is the external force applied normal to the surface of plate, *D* is the flexural rigidity for the intact plate, and *m* is the mass density per area. In Eq. (1), the dot (.) indicates the derivative with respect to time t. Assume that a harmonic point force is applied at a specified point (x_F, y_F) as[9,15]:

$$f(x,y,t) = F_{\circ}\delta(x - x_F)\delta(y - y_F)e^{i\omega t} \quad (2)$$

where $F_{\circ}$ is the amplitude of harmonic point force and ω is the excitation (circular) frequency. The forced vibration response of an intact plate can be assumed, by superposing *M* natural modes as follows:

$$w(x,y,t) = \sum_{m=1}^{M} W_m(x,y) q_m(t) \quad (3)$$

where q_m are the modal coordinates and W_m are the natural modes satisfying the eigenvalue problem of the intact plate:

$$D\nabla^4 W_m = m\Omega_m^2 W_m \qquad m = 1,\ldots,M \quad (4)$$

and the orthogonally property:

$$\int_A mW_m W_n dxdy = \delta_{mn} \qquad m,n = 1,...,M \tag{5-1}$$

$$\int_A DW_m \nabla^4 W_n dxdy = \Omega_m^2 \delta_{mn} \qquad m,n = 1,...,M \tag{5-2}$$

where Ω_m are the natural frequencies of the intact plate and δ is the Kronecker symbol. Substituting Eq. (3) into Eq. (1) and applying Eqs. (4) and (5) yields the modal coordinate equations as:

$$\ddot{q}_m(t) + \Omega_m^2 q_m(t) = f_m(t) \quad m,n = 1,...,M \tag{6}$$

where f_m are the modal forces defined by :

$$f_m(t) = W_m(x_F, y_F) F_{\circ} e^{i\omega t} \quad m,n = 1,...,M \tag{7}$$

Solution of Eq.(6) results modal coordinate $q_m(t)$ as:

$$q_m(t) = \frac{W_m(x_F)}{\Omega_m^2 - \omega_m^2} F_{\circ} e^{i\omega t} \tag{8}$$

DAMAGE DISTRIBUTION FUNCTION

In the damage state stiffness reduction has been expressed as a local piece-wise uniform thickness reduction as shown in Fig. 1.

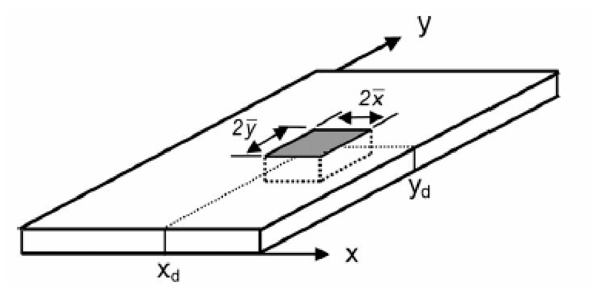

Fig. 1. A piece-wise uniform damage distribution

The damage has the constant magnitude $-1 < \delta D < 0$ over a small finite segment of area $4\overline{xy}$, with its center at (x_d, y_d).The local damage can be represented by:

$$\delta D(x,y) = \delta d\left[H(x_d - \bar{x}) - H(x_d + \bar{x})\right]\left[H(y_d - \bar{y}) - H(y_d + \bar{y})\right] \tag{9}$$

where, $H(x-a)$ is the Heaviside's unit function defined as:

$$H(x-a) = \begin{cases} 1 & x > a \\ 0 & x < a \end{cases} \tag{10}$$

MODE SHAPES CHANGES

The dynamic equation of motion for the damaged plate by considering the introduced piece-wise uniform damage distribution function is expressed as follows:

$$D(\frac{\partial^4 w_d}{\partial^4 x}+2\frac{\partial^4 w_d}{\partial^2 x\partial^2 y}+\frac{\partial^4 w_d}{\partial^4 y})+\delta D(x,y)(\frac{\partial^4 w_d}{\partial^4 x}+2\frac{\partial^4 w_d}{\partial^2 x\partial^2 y}+\frac{\partial^4 w_d}{\partial^4 y})$$
$$+m\ddot{w}_d = f(x,y,t) \qquad (11)$$

Assuming that mode shape changes of the structure to be a linear combination of the mode shapes of the intact structures would result in the following[14]:

$$W_{md} = W_m + \delta W_m = W_m + \sum_{n=1}^{M}\alpha_{mn}W_n \qquad (12)$$

Substituting Eq. (12) in the eigenvalue equation of the damaged plate yields:

$$D\sum_{n=1}^{M}\alpha_{mn}(\frac{\partial^4 W_n}{\partial^4 x}+2\frac{\partial^4 W_n}{\partial^2 x\partial^2 y}+\frac{\partial^4 W_n}{\partial^4 y})+$$

$$\delta D(x,y)(\frac{\partial^4 W_m}{\partial^4 x}+2\frac{\partial^4 W_m}{\partial^2 x\partial^2 y}+\frac{\partial^4 W_m}{\partial^4 y})-m\Omega_m^2 W_m - m\Omega_m^2\sum_{n=1}^{M}\alpha_{mn}W_n = 0 \qquad (13)$$

Pre-multiplying Eq. (13) by W_k for $m=k$, integrating over the area of the plate and considering the orthogonally property of the mode shapes results:

$$\alpha_{mn} = \frac{1}{\Omega_m^2-\Omega_n^2}\int_A \delta D(x,y)[\frac{\partial^4 \delta W_m}{\partial x^4}+2\frac{\partial^4 \delta W_m}{\partial x^2\partial y^2}+\frac{\partial^4 \delta W_m}{\partial y^4}]W_j dA \quad i\neq j \quad (14)$$

Also derivation of Eq. (5-1) for $m=n$ yields:

$$\alpha_{mm}\int_A m w_m^2 dA = 0 \qquad (15)$$

and therefore α_{mm} vanished.

DYNAMIC EQUATION OF MOTION OF DAMAGED PLATE

By evaluating the mode shapes change using Eq. (12), the eigenvalue equation of the damaged plate is expressed as follows:

$$D\nabla W_m + D\nabla\delta W_m + \nabla(\delta D W_m)+\nabla(\delta D\delta W_m)-(\Omega_m^2+\delta\Omega_m^2)m(W_m+\delta W_m)=0 \quad (16)$$

Multiplying Eq.(11) by W_m , integrating over the area of the plate and considering the orthogonally property yields:

$$\delta\Omega_m^2 = \int_A \delta D(x,y)[\frac{\partial^4 W_m}{\partial x^4}+2\frac{\partial^4 W_m}{\partial x^2\partial y^2}+\frac{\partial^4 W_m}{\partial y^4}]W_m dA+$$

$$\int_A \delta D(x,y)\left[\frac{\partial^4 \delta W_i}{\partial x^4} + 2\frac{\partial^4 \delta W_i}{\partial x^2 \partial y^2} + \frac{\partial^4 \delta W_i}{\partial y^4}\right] W_i \, dA \tag{17}$$

And similarly Frequency Response function of the damaged plate when change of mode shape has been taken into account can be expressed as:

$$w(x,y,t) = \sum_m^M W_m \left(\frac{W_m(x_F, y_F)}{\Omega_m^2 - \omega_n^2} + \sum_n^M \frac{\lambda_{mn} W_n(x_F, y_F)}{(\Omega_n^2 - \omega_n^2)(\Omega_n^2 - \omega_n^2)} \right) F_\circ e^{i\omega t} \tag{18}$$

$$+ \sum_m^M \sum_n^M \alpha_{ml} W_l \left(\frac{W_m(x_F, y_F)}{\Omega_m^2 - \omega_n^2} + \sum_n^M \frac{\lambda_{mn} W_n(x_F, y_F)}{(\Omega_n^2 - \omega_n^2)(\Omega_n^2 - \omega_n^2)} \right) F_\circ e^{i\omega t}$$

$$\lambda_{mn} = \int_A W_n \delta D(x,y) \left(\frac{\partial^4 W_m}{\partial x^4} + 2\frac{\partial^4 W_m}{\partial x^2 \partial y^2} + \frac{\partial^4 W_m}{\partial y^4} \right) dxdy \tag{19}$$

Using the introduced definition for damage distribution function integral of equations (17) and (18) can be divided to N sub integral, where N is the number of considered damage zones. Therefore change of the natural frequencies and Frequency Response Function can be expressed as nonlinear function of $\delta D(x, y)$. Solving this nonlinear function for $\delta D(x, y)$, using an optimization criterion will yield the location and severity of damage.

NUMERICAL RESULTS

To investigate ability of the presented method a square simply support isotopic uniform plate has been considered. As shown in Fig. 2 the plate has been divided into 5 zones along *x* and *y* axes which generates 25 unknown parameters. To avoid adverse effect of symmetry of the plate on results, the plate has been divided into unequal zones. Two cases of damage have been considered; in the first case, the flexural rigidity of the zone 19 has been reduced by 20 percent and in case two the flexural rigidity of the zone 7 reduced by 20 percent.

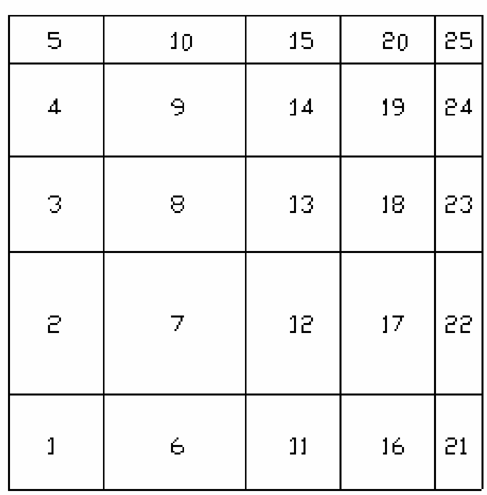

Fig. 2. Divided zones of Example plate

To investigate effects of neglecting mode shape change on the accuracy of the frequency changes evaluation, these changes are evaluated by Eq.(17) in to cases: first by considering mode shapes change (second part of Eq.(17)) second by omitting this term. Impact of excluding (including) of the mode shapes changes for the simulated damage cases are compared in Fig.3 and 4.

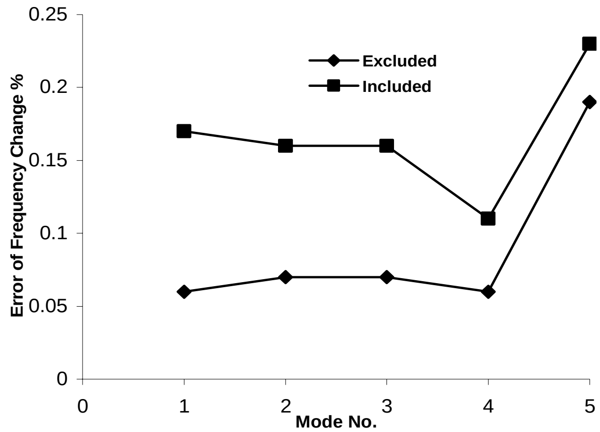

Fig. 3. Evaluation of Frequency Change (Case 1)

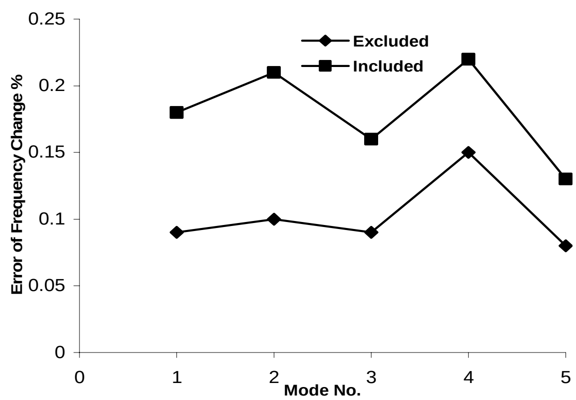

Fig. 4. Evaluation of Frequency Change (Case 2)

Using the presented equation to evaluate change of the natural frequencies and Frequency Response Function (five first natural frequency and Frequency Response Function measured at two point), damages of the simulated cases have been predicted by simulated noisy data. Results of detected damage are shown in Fig.5 and Fig.6.

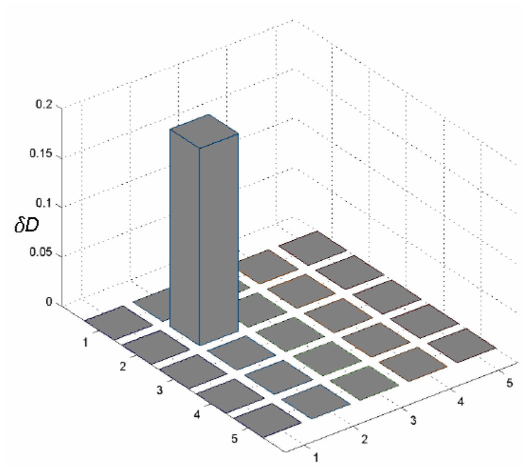

Fig. 5. Predicted Damage of Case 1

As the results show in this SDIM damage can be predicted using noise polluted data. By changing the position of excitation force and the measurement location higher number of equations can be derived. Therefore damage cases can be detected more accurate. This method is applicable to other type of structure such as beam or composite plate using analytically evaluated mode shape of these structures.

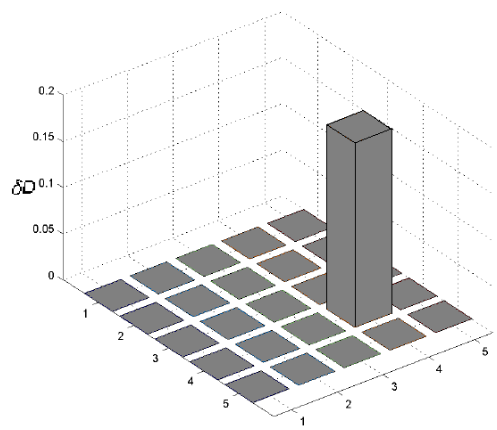

Fig. 6. Predicted Damage of Case 2

CONCLUSIONS

In this paper, an FRF-based SDIM is derived from dynamic equation of motion for damaged plate. The appealing features of the present SDIM are as follows:

a) The modal data of damaged structure are not required in the analysis.
b) A large number of equations can be readily derived by varying the excitation frequency as well as the response measurement point. The feasibility of the present SDIM is verified through some numerically simulated damage identification tests. It is shown that presented method is able to predict damage using noisy modal data.

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