The 3rd International Conference on Structural Health Monitoring of Intelligent Infrastructure
Vancouver, British Columbia, Canada
November 13-16, 2007

INCORPORATING STRUCTURAL HEALTH MONITORING DATA TO RELIABILITY-BASED STRUCTURAL CONDITION ASSESSMENT

Bhasker Dubey
Concordia University, Canada

Ashutosh Bagchi
Concordia University, Canada

Sabah S. Alkass
Concordia University

Abstract

The present infrastructure around the world is deteriorating rapidly because of extensive usage, ageing and negligence through the decades. Although many structures are being monitored currently, structural condition assessment based on SHM data and its use in the infrastructure management system have remained very difficult. This paper presents a reliability-based assessment of a structure using SHM data and a proposal to use the reliability indices in infrastructure management system. Usually the deterioration of structural components and the load distribution are modeled based on historical statistical data for different types of structures. However, every structural system is unique and behaves slightly differently from another structure of the same type and configuration. The SHM system can help in assessment of behavioral changes for an individual structure and its components. A deterioration model can be developed based on this information. The Crowchild Bridge in Calgary, Canada has been used as an example for demonstrating this method. A finite element (FE) model for the bridge has been constructed and validated using the available data form field tests, and a large volume of simulated data has been generated using the FE model for a number of possible combinations of known loads. The deterioration effect has been generated by reducing stiffness or modifying the section.

INTRODUCTION

Due to overuse and unsatisfactory inspection, monitoring, and maintenance, the condition of present infrastructures is deteriorating rapidly. This rapid change has caused a great concern among owners, researchers, and decision makers. Upgrading and replacing existing structures using traditional materials and methodologies is no longer affordable. People are searching for newer technologies and rehabilitation schemes such as Structural Health Monitoring (SHM) and Fibre Reinforced Polymers (FRPs). SHM refers the broad concept of assessing the ongoing, in-service performance of structures using a variety of measurement techniques [1]. Over the past two decades, SHM is emerging as a tool for the maintenance and management of existing civil infrastructure. The advancement in technology makes this task more important and interesting. But data interpretation techniques in SHM often lead to inverse problems. The amount of data available in SHM techniques makes this task more complicated. So, the development of appropriate data interpretation techniques is very important. The actual purpose behind collecting and storing data will only be served when the cause and effect relationship behind the data is known. In this study, a small but important effort in this direction has been made.

The structures which incorporate SHM systems with various types of sensors for monitoring the in-service performance often referred to as smart structures (although smart structure terminology is more applicable to structures with both sensing and active control capabilities). These structures have emerged as a potential solution in diagnosing infrastructure deterioration before it becomes critical. The Crowchild Bridge in Calgary is one of the fine examples of these types of structures. In this study a Finite Element Model of the Crowchild bridge has been developed and validated with the field test data. To simulate the degradation in the bridge deck, the stiffness of the deck elements has been reduced over the time and the computed deflection at a number of specified locations has been used for establishing a degradation function. The aim of this work is to develop a methodology to calculate the degradation rate of a bridge when the deformation history for a set of specified points is known. In the field, the deformation can be measured (directly or indirectly). Once the degradation rate is determined and updated using SHM data, the reliability index for a structure can be calculated. The concepts of structural reliability are discussed in the next section.

BACKGROUND

The ultimate aim of SHM techniques is to get an idea about the performance and behavior of a structure in the present environment. On the basis of this information many decisions like maintenance strategies, remaining service life of the structure, allowable load and many more, can be taken. To have an idea about a structure's performance and its condition, usually the following two methods are used: (1) based on visual inspection the condition rating is assigned, and (2) the structural reliability is calculated on the basis of past experience and historical data of loads on a structure and the strength of materials. According to Thoft-Christensen and Baker [2], structural reliability should be considered as having two meanings: a general, and a mathematical one.

1) In the most general sense, the reliability of a structure is its reliability to fulfill its design purpose for some specified reference period.
2) In a narrow sense it is *probability* that a structure will not attain each specified limit state (ultimate or serviceability) during a specified *reference period.*

Here we are more concerned about the narrow sense. To understand the reliability in terms of probability, a simple example is taken from [2]. If S is the total load effect (total demand), and R is the resistance (or capacity), the limit state function and M(R,S) and the probability of failure P_f can be defined as in Equations 1 and 2, respectively.

$$M(R,S) = R - S \quad (1)$$

$$P_f = P(M \le 0) = \int_{-\infty}^{+\infty} F_R(x) f_S(x) dx \quad (2)$$

where $M = R\text{-}S$, and F_R is the probability distribution function of R and f_S the probability density function of S. In this case, distribution of R and S are both assumed to be stationary with time. Similarly the reliability R, defined as

$$R = 1 - P_f \quad (3)$$

If r is the fixed value of random variables R, the probability of failure becomes,

$$P_f = P(r - s \le 0) = 1 - F_S(r) \quad (4)$$

To quantify the structural reliability, an index, called the reliability index (β), is used and it is given by

$$\beta = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_Q^2}} \quad (5)$$

where β is the inverse of the coefficient of variation of the function $g(R,Q) = R\text{-}Q$ when R and Q are uncorrelated. For normally distributed random variables R and Q, it can be shown that the reliability index is related to the probability of failure by

$$\beta = \Phi^{-1}(P_f) \text{ or } P_f = \Phi(-\beta) \quad (6)$$

where Φ is standard normal cumulative density function (CDF). In this study it has been considered that R and Q are uncorrelated, normally distributed, random variables.

Generally, the resistance (R) and the load effect (S) depend on a number of time variant or time independent random variables x. As a result, the probability distributions $f_s(x)$ and $F_R(x)$ cannot be defined explicitly. Moreover, structural resistance may depend on the resistance of structural components, so the probability of failure may be expressed as the multidimensional integral [3]

$$p_f = \int_D f_x(x)dx \quad (7)$$

where $f_x(x)$ is the joint probability density of the basic random variables; D is the failure domain defined by $G_i(x) \leq 0$; and $G_i(x)$ the limit state function of the i^{th} component. The integral in Equation 2 cannot be solved analytically in all but very special cases. In practice, to evaluate this integral, approximate methods such as fist- or second-order reliability methods or Monte Carlo simulation techniques are usually employed [3]. An introduction to structure reliability and risk assessment can be found in [4].

DETERIORATION MODEL

In this paper, it is assumed that the live load effect does not depend on time and remains the same during the service life of the structure. But the capacity, or resistance, of the bridge certainly depends on time, and it degrades over time. The time dependent structural resistance of an element can be expressed as a combination of the initial resistance, R_0 and a time-dependent resistance degradation function $g(t)$ as proposed in [5] (Equation 8).

$$r(t) = R_0.g(t) \quad (8)$$

Enright and Frnagopol [4] proposed a resistance degradation model for RC bridge beams subjected to reinforcement corrosion. But a resistance degradation function for the use in a predictive analysis has not yet been developed with a sufficient degree of accuracy [4]. In this study, an attempt is made to develop a more precise model using the SHM data. Using M-FEM [6], a finite element program for structural analysis, model updating and vibration-based damage identification, and the SHM data, a resistance degradation model has been proposed for a steel free deck bridge, namely the Crowchild Bridge in Canada. Degradation is a general function of stiffness reduction and here, it is assumed that resistance degradation is directly proportional to reduction in stiffness, which is indicated in the change in deformation pattern. Thus, $g(t)$ in Equation 8 can be replaced by $k(t)$ as shown in Equation 9, where $k(t)$ represents the change in stiffness over time. The stiffness degradation function, $k(t)$ can be obtained using the displacement envelope, $d(t)$ at the location(s) of interest with respect to time due to given sets of live loads.

$$r(t) = R_0.k(t) \quad (9)$$

THE CROWCHILD BRIDGE

The Crowchild Bridge located in Calgary, Alberta is a two-lane traffic overpass with three continuous spans. The details of the bridge superstructure are shown in Figure 1. The superstructure is said to be the first continuous-span steel-free bridge deck in the world [7]. It is composed of five longitudinal steel girders (900 mm deep), a polypropylene fiber reinforced concrete slab deck, and prefabricated glass fiber reinforced concrete barriers. The five longitudinal girders are spaced at 2 m. Four evenly spaced cross-frames in each span and steel girder diaphragms at the supports hold the main girders in place. The main girders are also connected by evenly spaced steel straps placed across the top of the girders to provide lateral restraint to them. The girders and straps are connected to the deck slab by stainless steel stubs. The deck is 9030 mm wide and does not contain any internal steel reinforcement. The slab thickness is 275 mm along the girders and 185 mm elsewhere.

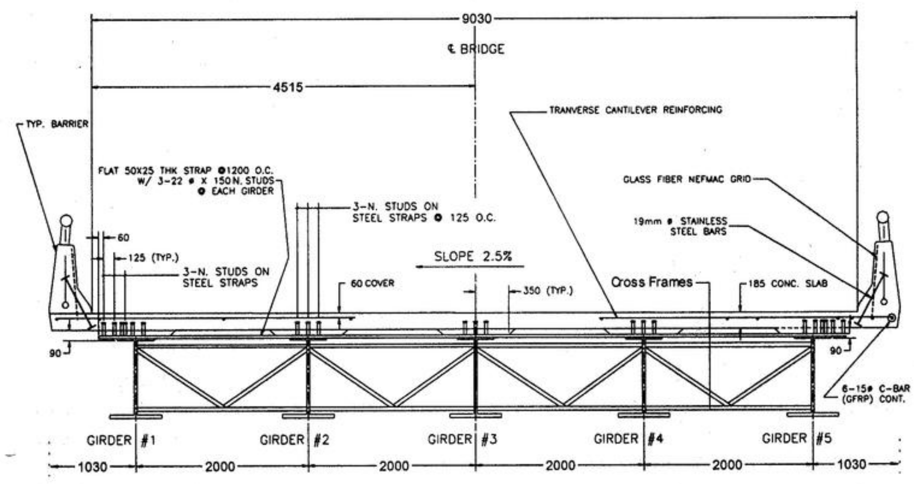


Figure 1. Cross Section of Crowchild Bridge (Excerpted from [9])

A monitoring program for the bridge has been developed by ISIS Canada. Static and ambient vibrations tests have been conducted on the bridge by the University of British Columbia in 1997 [8], and the University of Alberta in 1998-99 [9]. This bridge consists of three continuous spans named as north span, interior span, and south span which have lengths of 29.830m, 32.818m and 30.230m, respectively. The cross section of the bridge is shown in Figure 1. During a static load test in 1997, two 80,000 lbs trucks were placed at six different positions as shown in Figure 2, while a single truck was placed in three different positions, not shown here (details are available in [1]). The north span of the bridge was surveyed at five equally spaced points on each girder and deformation for each position was measured in mm. Distance between the points was 5000 mm. Points *a* and *e* were at 4915 mm from north abutment and Pier no. 1, respectively as shown in Figure 2.

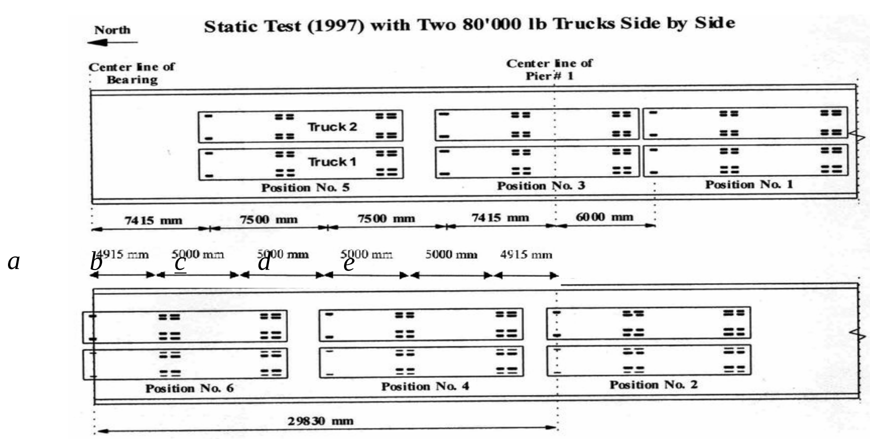


Figure 2. Positions of trucks and equally spaced points (excerpted from [10])

Finite Element Model

Using M-FEM [6] these conditions have been simulated and the maximum deformation in each section has been recorded for each respective position. Results show that maximum deformation in each section occurs when the test vehicles are placed in position 6 and the values are in agreement with the static load test performed on this bridge in 1997. This position has been chosen for simulation of stiffness degradation, as it corresponds to the maximum deformation. To simulate the deterioration in bridge deck condition, the stiffness of the deck has been reduced gradually, and the maximum deformation has been computed at each section of the bridge as indicated in Figure 2.

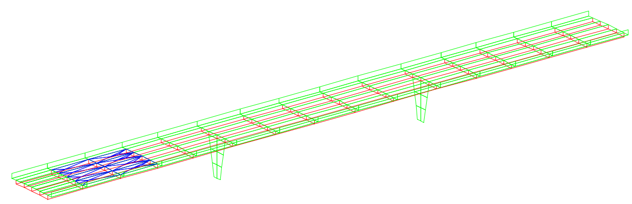
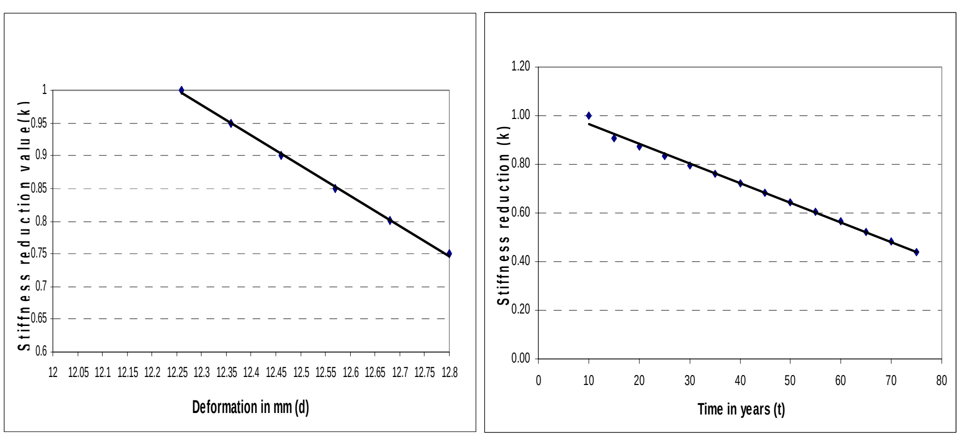


Figure 3. A Finite Element Model of the Crowchild Bridge

Figure 3 shows the FEM model, developed using SHM, of the bridge when a truck is placed in position 6. Using SHM data the maximum deformation can be plotted with respect to time, and hence stiffness reduction can be plotted with respect to time. In this study, by assuming that deformation is directly proportional to stiffness reduction, a relation between stiffness reduction and time has been obtained as shown in Figure 4.



(a) (b)

Figure 4. (a) Change in stiffness with respect to time; (b) Change in Stiffness with respect to maximum deformation.

Using Equation 9 the value of time varying resistance can be calculated as

$$r(t) = R_0.(-0.008t + 1.0443) \qquad (10)$$

Estes [10] has developed a limit state equation for the bridge deck. This limit state equation has been modified using equation (10) as

$$g(t) = R_0.(-0.008t + 1.0443) - M_{DL} - M_U \qquad (11)$$

where M_{DL} = dead load moment; M_U = live load moment. Due to space limitation, the other details of this equation is not discussed here.

COMPUTATIONAL PROCEDURE

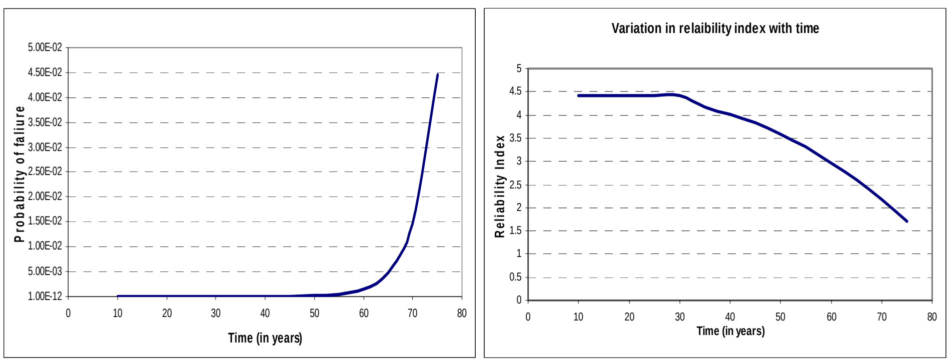
A computer program in C++ has been developed to compute the probability of failure and reliability index using equation (12). This program uses Monte Carlo Simulation to calculate P_f for a given limit state equation, and then calculates the reliability index using Equation 6. It is known that failure occurs when $g(.)<0$; therefore an estimate of the P_f can be found by

$$\overline{P_f} = N_f / N \qquad (12)$$

where N_f is the number of simulation cycles in which $g(.)<0$, and N is total number of simulation cycles. To check the accuracy of P_f the variance and covariance of estimated P_f have been calculated. The variance of the estimated P_f can be computed by assuming each simulation cycle to constitute a Bernoulli trial [11].

RESULTS OF RELIABILITY ANALYSIS

Figure 5(a) shows that for the first 50 years there the value of P_f is very low and it doesn't change too much. However, if no maintenance is performed the P_f tends to change rapidly after 50 years which means a higher rate of degradation. Figure 5(b) shows the variation of reliability index (β) with respect to time. At the time of consideration the value of β is found to be 4.5, and it starts decreasing after 30 years. At year 50 it goes down to 3.5 which corresponds to a failure probability of $P_f = 1.7\times10^{-4}$.



(a) (b)

Figure 5. (a) Time Vs Probability of failure; (b) Time Vs Reliability Index

DECISION ANALYSIS

In United States, the target reliability index for non-redundant bridges is 3.5. This target reliability index relates to the cumulative probability of failure of the bridge remaining serviceable over a 50-70 year lifetime without requiring any major rehabilitation [12]. The accuracy of reliability index calculation depends on the accuracy of the input data and highly accurate input data can be obtained using SHM. In this study this value has been assumed sensitive enough to take rehabilitation decision.

Probability Updating

Once value of reliability index is known the probability of maintenance can be updated using this information. Let's define the probability of performing first rehabilitation at nth relative year P_1^n. Assuming there are three probabilities of performing first rehabilitation and one based on historical information these probabilities are given in Table 1. SHM provides precise information about the structure's condition. Hence the calculated reliability index will also give a more accurate assessment of structure behavior. The reliability of SHM predictions/outputs are shown in Table 2, which are generally based on previous experience. This means that if output is Z_0 and the true state of nature is θ_0 then probability of first rehabilitation is given by $P(Z_0/\theta_0)$.

Table 1. Probabilities of performing first rehabilitation at respective years

Year	Probability
0	P_0^1
3	P_3^1
6	P_6^1

Table 2. Probabilities of SHM information shows the true state.

Outcome Z_X	True State (θ_n)		
	θ_0	θ_3	θ_6
Z_0	$P(Z_0/\theta_0)$	$P(Z_0/\theta_3)$	$P(Z_0/\theta_6)$
Z_3	$P(Z_3/\theta_0)$	$P(Z_3/\theta_3)$	$P(Z_3/\theta_6)$
Z_6	$P(Z_6/\theta_0)$	$P(Z_6/\theta_3)$	$P(Z_6/\theta_6)$
	$\sum$ = 1.0	$\sum$ =1.0	$\sum$ =1.0

where, θ_n is the true state of nature (which is not known) corresponding to the rehabilitation at the n^{th} year, Z_n is the SHM output indicating that the first rehabilitation is required at the n^{th} year, and $P(Z_n/\theta_n)$ is the probability of performing the first maintenance at the n^{th} year if true state is $\tilde{\theta_n}$. As shown in Table 2, SHM data gives three outcomes in this sort of situation and there are three true states of natures. In the Table 2 $P(Z_0/\theta_0)$ indicates the probability of outcome to perform rehabilitation at the 0^{th} year when true state of natures also happens to be θ_0. On the basis of these new probabilities the pre-posterior probabilities of first rehabilitation can be calculated using the Bayesian approach [13] and total probability theorem. The updated probability will be given by:

$$P_{1,u}^n = P(\theta_n/Z_x) = \frac{P(Z_x/\theta_n)P_1^n}{\sum P(Z_x/\theta_n)P_1^n} \qquad (13)$$

$$P(Z_x) = \sum P(Z_x/\theta_n)P_1^n \qquad (14)$$

where x = 0, 3, 6 and n = 0, 3, 6.

Table 3. Updated probabilities for each outcome

Sample outcome Z_X	Updated probability			
	$P_{1,u}^{0}$	$P_{1,u}^{3}$	$P_{1,u}^{6}$	
Z_0	$P(\theta_0 / Z_0)$	$P(\theta_3 / Z_0)$	$P(\theta_6 / Z_0)$	$\sum = 1.0$
Z_3	$P(\theta_0 / Z_3)$	$P(\theta_3 / Z_3)$	$P(\theta_6 / Z_3)$	$\sum = 1.0$
Z_6	$P(\theta_0 / Z_6)$	$P(\theta_3 / Z_6)$	$P(\theta_6 / Z_6)$	$\sum = 1.0$

The Value of Information (VI)

To decide whether SHM information should be used or not, the value of SHM information needs to be calculated. VI is calculated as follows:

$$VI = E(T) - E(R) \quad (15)$$

where *E(T)* is the expected cost of rehabilitation after updating the probabilities excluding the cost of SHM information, *E(R)* is the expected cost of rehabilitation calculated without considering the updated probabilities. *E(T)* is calculated using updating probabilities for the first rehabilitation. To calculate the expected rehabilitation cost and the cost associated with SHM a pre-posterior analysis needs to be done which involves a decision tree approach. This work is not in the scope of this present paper. If the value of information, *VI* exceeds the cost associated with the SHM system, C_{SHM} the SHM information will be regarded to be beneficial.

TOTAL EXPECTED COST (TEC) CALCULATION

It is assumed in this study that VI comes out to be greater than C_{SHM} , so TCE will be calculated as shown in equation (16):

$$TEC = E(T) + C_{SHM} \quad (16)$$

The cost of SHM information will depend on several factors such as the type of SHM (periodic monitoring or continuous monitoring), the degree of sophistication of SHM instruments, the type of bridge, *etc*.

CONCLUSION

In this study an attempt to calculate the expected rehabilitation cost using SHM information has been made. A degradation model for the Crowchild bridge deck has been developed using a deformation and stiffness deterioration relationship. A Finite Element model of the Crowchild Bridge has been used to simulate the bridge behavior. It is assumed that for this bridge degradation model is linearly proportional to time. In that case, for the first 40 years, the bridge deck doesn’t show any major degradation, but after this time period there is a drastic change in bridge deck degradation. After 50 years, the reliability index crosses down to a value of 3.5. Updated probabilities have been used to calculate the total expected rehabilitation cost (*TEC*). The value of information approach has been used to decide whether to use or to not use the SHM information.

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