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STRUCTURAL DAMAGE IDENTIFICATION USING MODE SHAPES

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Abstract

In this paper, a new global algorithm for damage assessment of structure by using finite element methods and modal data. Damage is considered as a change in the structural stiffness parameters. To remove drawback of incomplete measurement, unmeasured parts of mode shapes of a structure are characterized as a function of the structural parameter and the measured parts of mode shapes. Elemental damage equations, which relate to partially measured mode shapes of damaged structures to changes of structural parameters are developed using incomplete measured mode shapes through a condensation method. These equations are solved to find change of structural parameters utilizing an optimization method. Monte Carlo simulation is applied to noise polluted modal displacements to investigate the sensitivity of this method to measurement errors. The algorithm is verified in numerical simulation environments using a planer truss. Results show the good ability of this method to detect any damage of structures in presences of noise in acquired data.

INTRODUCTION

The subject of structural damage detection has been receiving a growing amount of attention from researchers in the civil, the mechanical and the aerospace fields of engineering. A large number of structural failures have been reported over the past decades, causing considerable loss of life and property. Therefore, early detection, monitoring, analysis, and repair of a damaged structure, are vital for the safe performance of the structure. Dynamic damage detection methods are most common of system identification methods. Detailed literature review has been provided by Doebling et al [1] and Stubbs et al [2]. References cited in these reviews proposed many different methods for identifying and localizing damage from vibration response measurements. The majority of the cited references rely on the finite element modeling process and/or linear modal properties for damage diagnosis.

Dynamic damage detection methods are more sensitive and accurate in comparison to static damage detection methods and are developed by many researchers. These methods use eigenvalues and eigenvectors of a structure to assess its damage. Lim and Kashanagi [3] developed the best achievable eigenvector concept. In this method a measured mode shapes compared with the best achievable eigenvector based on a candidate set of assumed damage cases to detect damage in structures. Kabe [4] developed a method for stiffness matrix adjustment using modal test data and preserved structural connectivity information of a mass-spring system. Lim [5] proposed a systematic method that provides precise identification of damage location and extends when exact measured modes at every finite element DOF are used. Also a procedure was presented to perform damage detection with inaccurate and incomplete measured modes. Kim and Stubbs [6] proposed the damage index method of a structure which is computed by using mode shape before and after the damage of structure.

A mode-based damage identification method is proposed by Ren et al [7] to predict location and severity of damage based on the work done by Araújo dos Santos [8]. In this work, it was demonstrated, when multiplying the damaged eigenvalue equations with the damaged or undamaged modes, provides more equations than the strain energy-based method which will guarantees the damage localization. Hemez and Farhat [9] applied their element-by-element sensitivity update method to a 10-bay truss data. They examined some specific issues to surround the location of damage in this structure, including the selection of the type and the number of finite elements, the modeling of the cantilever boundary condition, the selection of the modes used in the update, and the limitations of the sensitivity-based technique.

As a drawback of FEM-update techniques, the requirement of reducing FEM degrees of freedom or extending the measured modal parameters may result in the loss of physical interpretability and errors due to the stiffness diffusion that smears the damage-induced localized changes in stiffness matrix into the entire stiffness matrix. To overcome this drawback Bakhtiari-Nejad et al [10] proposed a damage detection method using incomplete measured mode shape. They expressed mode shape as a function of stiffness by assuming one of the modal displacements to be equal to one and derived a set of equations which related modal displacement of damaged structures to changes of structural parameters. Using an optimization criterion, the derived equation was solved to obtain changes of structural parameters. The proposed method applied to a frame and a truss structure using noisy simulated data.

In the present work, the unmeasured part of an eigenvector of a structure is expressed as a function of measured part of the mode shape, frequency, stiffness and mass matrix of the structure. Based on the work of Ren et al (2001) an element level damage equation was characterized using mode shapes of intact structure and partially measured mode shapes of damaged structure. To overcome the problem of undetermined equations, an optimization criterion is used to solve equations for estimating the structural parameters. Noise in the measurement is simulated by adding a proportional random error to the exact data obtained form finite element models of damage structures.

THEORY

Mode Shape Equation

For an undamaged structure the modal characteristics of structure are described by the eigenvalue equation:

$$(K - \omega_i^2 M)\phi_i = 0 \qquad (1)$$

where $K(n \times n)$ and $M(n \times n)$ are stiffness and mass matrices of structure respectively; ω_i and ϕ_i are the ith eigenvalue and mode shape of structure respectively and n is the number of degrees of freedom. Degrees of freedom of a structure can be portioned into two categories; measured and unmeasured parts. Therefore the stiffness and mass matrices of a structure can be rewritten as:

$$K = \begin{bmatrix} K_{aa} & K_{ab} \\ K_{ba} & K_{bb} \end{bmatrix} \quad , \quad M = \begin{bmatrix} M_{aa} & M_{ab} \\ M_{ba} & M_{bb} \end{bmatrix} \qquad (2)$$

where subscripts a and b indicate the degrees of freedom associated to the measured and unmeasured location of structure respectively. By using Eq. (2), Eq. (1) can be rewrite as:

$$\left(\begin{bmatrix} K_{aa} & K_{ab} \\ K_{ba} & K_{bb} \end{bmatrix} - \omega_i^2 \begin{bmatrix} M_{aa} & M_{ab} \\ M_{ba} & M_{bb} \end{bmatrix}\right)\begin{pmatrix} \phi_{ia} \\ \phi_{ib} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (3)$$

Eq. (3) can be expanded as:

$$(K_{aa} - \omega_i^2 M_{aa})\phi_{ia} + (K_{ab} - \omega_i^2 M_{ab})\phi_{ib} = 0 \quad (4\text{-a})$$

$$(K_{ba} - \omega_i^2 M_{ba})\phi_{ia} + (K_{bb} - \omega_i^2 M_{bb})\phi_{ib} = 0 \quad (4\text{-b})$$

Using Eq. (4-b) the unmeasured parts of mode shapes can be calculated as:

$$\phi_{ib} = (K_{bb} - \omega_i^2 M_{bb})^{-1}(K_{ba} - \omega_i^2 M_{ba})\phi_{ia} \quad (5)$$

Eq. (5) expressed the unmeasured parts of mode shapes of structures as a function of stiffness matrix (structural parameters), mass matrix and natural frequencies.

Element Damage Equations

Using the eigenvalue problem of the intact structure as given by Eq. (1), and substituting the eigenvalue problem of the damaged, the l1th mode shapes can be written as:

$$[(K + \delta K) - \omega_{ld}^2 M]\phi_{ld} = 0 \quad (6)$$

Pre-multiplying Eq. (6) by ϕ_i^T , transposing and rearranging it yields:

$$\phi_{ld}^T (K + \delta K)\phi_i = \omega_{ld}^2 \phi_{ld}^T M \phi_i \quad (7)$$

Using the eigenvalue problem of an intact structure as given by Eq. (1) and substituting $M\phi_i$ by $\frac{1}{\omega_i^2} K\phi_i$ in the right hand side of Eq. (7) results:

$$\phi_{ld}^T (K + \delta K)\phi_i = \frac{\omega_{ld}^2}{\omega_i^2} \phi_{ld}^T K \phi_i \quad (8)$$

expanding the left hand side of Eq. (8) and rearranging, it yields:

$$\phi_{ld}^T \delta K \phi_i = \left(\frac{\omega_{ld}^2}{\omega_i^2} - 1\right) \phi_{ld}^T K \phi_i \quad (9)$$

Eq. (9) expresses the relation between the measured modal parameter of a damaged structure and change in the stiffness matrix of a structure. This equation requires a complete measured mode shape of a structure which is time consuming and expensive for most of the structures. Also in structures which have translational and rotational degrees of freedom, measurement of rotational degree of freedom needs expensive equipment. By using Eq. (3), Eq. (8) can be rewritten as:

$$\begin{bmatrix}\phi_{lda}^T & \phi_{ldb}^T\end{bmatrix}\begin{bmatrix}\delta K_{aa} & \delta K_{ab}\\ \delta K_{ba} & \delta K_{bb}\end{bmatrix}\begin{bmatrix}\phi_{ia}\\ \phi_{ib}\end{bmatrix} = (\frac{\omega_{ld}^2}{\omega_i^2}-1)\begin{bmatrix}\phi_{lda}^T & \phi_{ldb}^T\end{bmatrix}\begin{bmatrix}K_{aa} & K_{ab}\\ K_{ba} & K_{bb}\end{bmatrix}\begin{bmatrix}\phi_{ia}\\ \phi_{ib}\end{bmatrix} \quad (10)$$

Transpose of Eq. (10) can be expanded and rearranged as:

$$\phi_{ia}^T\delta K_{aa}\phi_{lda} + \phi_{ib}^T\delta K_{ba}\phi_{lda} + \phi_{ia}^T\delta K_{ab}\phi_{ldb} + \phi_{ib}^T\delta K_{bb}\phi_{ldb} - (\frac{\omega_{ld}^2}{\omega_i^2}-1)(\phi_{ia}^T K_{ab}\phi_{ldb} + \phi_{ib}^T K_{bb}\phi_{ldb})$$

$$= (\frac{\omega_{ld}^2}{\omega_i^2}-1)(\phi_{ia}^T K_{aa}\phi_{lda} + \phi_{ib}^T K_{ba}\phi_{lda}) = R_{li} \qquad l = 1,\ldots nm \; i = 1,\ldots nu \qquad (11)$$

where R_{li} is the vector of residual. Using Eq. (5), the unmeasured portion of a mode shapes of damaged structure can be computed based on the measured part, therefore Eq. (11) can be rewritten as:

$$\phi_{ia}^T\delta K_{aa}\phi_{lda} + \phi_{ib}^T\delta K_{ba}\phi_{lda} - \left[\phi_{ia}^T\delta K_{ab} + \phi_{ib}^T\delta K_{bb} - (\frac{\omega_{ld}^2}{\omega_i^2}-1)(\phi_{ia}^T K_{ab} + \phi_{ib}^T K_{bb})\right]\times$$
$$(K_{bb} + \delta K_{bb} - \omega_{ld}^2 M_{bb})^{-1}(K_{ba} + \delta K_{ba} - \omega_{ld}^2 M_{ba})\phi_{lda} = R_{li} \qquad (12)$$

Using nu mode shapes of intact structure and nm mode shapes of damaged structures, $nm \times nu$ equation will be derived. The solution of the element damage equations for the unknowns allows locating and quantifying damage. These two types of damage equations expressed by Eq. (5) and Eq. (12) can be used either independently or combined. The advantage of combining two equations is that more equations are available for damage detection. To investigate the efficiency of the proposed method to handle element damage equation, in this work only these type of equations have been used in the damage detection process.

OPTIMIZATION FUNCTION

Using Fox formulation [11], change in the mode shape of a structure can be expressed as a linear combination of mode shapes of intact structures as:

$$\delta\phi_l \cong \sum_{k=1}^{n}\alpha_{lk}\phi_l \;, \quad \alpha_{lk} = \frac{\phi_k^T\delta K\phi_l}{(\omega_l^2-\omega_k^2)},\; k \neq l \;, \quad \alpha_{ll} = 0 \qquad (13)$$

Then the eigenvalue problem of the damaged structure can be described by:

$$[K + \delta K - (\omega_l^2 + \delta\omega_l^2)M]\phi_{ld} = 0 \qquad (14)$$

Imposing Eq. (12) in to Eq. (14) yields:

$$E_l = [K + \delta K - (\omega_l^2 + \delta\omega_l^2)M](\phi_l + \delta\phi_l) \qquad (15)$$

where, E_l is a vector of errors due to approximation of Eq. (12). There is some error due to approximated expression used for computing mode shape change. Eq. (15) can be simplified as:

$$E_l = (K - \omega_l^2 M)\delta\phi_l + (\delta K - \delta\omega_l^2 M)(\phi_l + \delta\phi_l) \quad (16)$$

Summing these errors over the number of measured modes nm , provides the error produced by all nm equations as:

$$g' = \sum_{l=1}^{nm} \|E_l\|^2 = \sum_{l=1}^{nm} \left\| [K - \omega_l^2 M]\delta\phi_l + [\delta K - \delta\omega_l^2 M](\phi_l + \delta\phi_l) \right\|^2 \quad (17)$$

If the stiffness matrix of a damaged structure is close to the stiffness matrix of an intact structure, produced error by Eq. (17) must be minimized; therefore, the introduced objective functions can be expressed as ($\delta K = A\delta P A^T$, P is the structural stiffness parameters, *A* is connectivity matrix of the structure [10]):

$$g = \sum_{l=1}^{nm} \left\| [K - \omega_l^2 M]\delta\phi_l + [\delta K - \delta\omega_l^2 M](\phi_l + \delta\phi_l) \right\|^2 + \delta P^T \delta P \quad (18)$$

where $\delta P^T \delta P$ is the norm of vector of parameters change, which will be described later. Since change in the structural stiffness parameter is always negative, an inequality constraint is introduced as:

$$\delta P < 0 \quad (19)$$

The optimization problem can now be stated as:

$$Min\, g(\{\delta P\}) \quad (20)$$

subjected to the nonlinear equality constraints given in Eq. (12) and inequality constraints of Eq. (19). This problem can be solved by the MATLAB optimization toolbox using the FMINCON routine. This routine implements the Sequential Quadratic Programming (SQP) to minimize the nonlinear cost function subjected to linear and nonlinear equality and inequality constraints. SQP converts a nonlinear minimization to a linear minimization using a Hessian matrix of cost function and gradient of nonlinear constraints. Since, this problem must be solved iteratively and, like any iterative algorithm, the estimators need initial values for unknown parameters to start the iteration. The choice of initial value controls the convergence of the algorithm and dictates, to some extent, the computational effort required to achieve a solution. In these, paper origin ($\delta P = 0$), is considered as an initial trial for the optimization problem. It may increase the number of required iterations to solve optimization, but did not influence uniqueness of results. Since, inequality constrained of Eq.(19) bound the search domain of optimization criteria and therefore the results is unique. Examination of other random initial trial provides that the initial trial do not influence results of this study.

NUMERICAL VERIFICATION

A one-storey-one-bay frame as shown in Figure 7 is considered to verify the damage identification method described in this paper. The FEM analysis is carried out to simulate the experimental data by using two-node beam elements. The number of nodes and elements are 16 and 15 respectively. The unknown parameters are flexural rigidity of elements, EI, where I is the moment of inertia of the cross-sectional of beam elements.

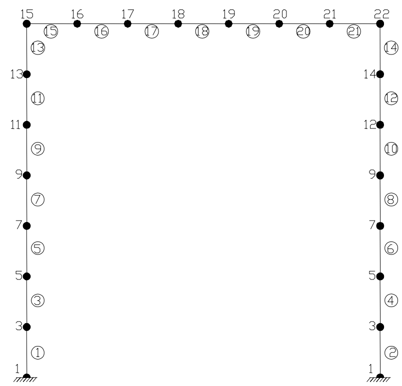

FIGURE 7:. PLANER FRAME STRUCTURE

Here four damage cases are considered to investigate capabilities of the present method in detection of occurred damage of flexural structure. In the first damage case, the stiffness of element 10 was reduced by 20 percent. In the damage case number two the stiffness of elements 12 and 20 were reduced by 20 and 40 percent respectively. In the damage case three the stiffness of elements 3, 9 and 18 were reduced by 30, 20 and 40 percent respectively. In the damage case four, the stiffness of elements 7, 12 and 20 were reduced by 20, 30 and 20 percent respectively.

First, the partially measured mode shapes of damaged structures and the fifteen first mode shapes of intact structures has been considered in the damage detection process and has been assumed only translational degrees of freedom can be measured. Nodes number 6,10,15,11 and 19 are selected as measurement locations to measure the translational displacements.

In the numerical examples, noise is simulated by adding a series of pseudorandom numbers on the theoretically calculated frequencies and mode shapes. In this study, 1 percent proportional uniform noise applied to model displacement and natural frequencies has been considered noise free. Next, in order to investigate the effect of input error on the parameter estimates, the kth component of the noisy measured eigenvector for the kth ϕ_{lk}^{m} can be computed of the lth simulated noise free eigenvector ϕ_{lk}^{0} as:

$$\phi_{lk}^{m} = \phi_{lk}^{0}(1+\zeta_{l}^{k}) \tag{23}$$

where ζ_{l}^{k} is a random number. The results of Monte Carlo's analysis for truss and frame models are given in Figures 6 to 9.

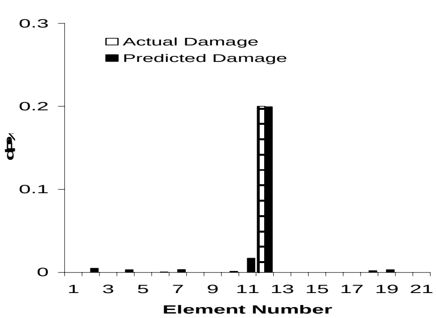

FIGURE 16. PREDICTED DAMAGE OF FRAME SCENARIO 1 (NOISY DATA)

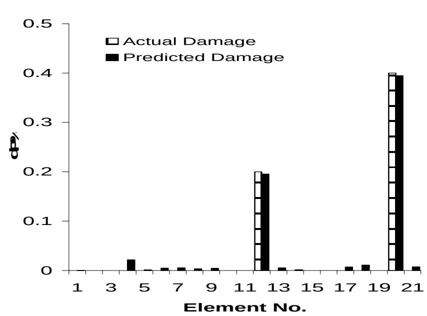

FIGURE 17. PREDICTED DAMAGE OF FRAME SCENARIO 2 (NOISY DATA)

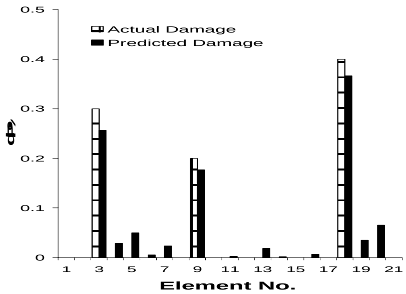

FIGURE 18. PREDICTED DAMAGE OF FRAME SCENARIO 3 (NOISY DATA)

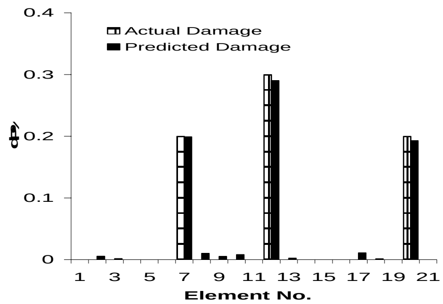

FIGURE 19. PREDICTED DAMAGE OF FRAME SCENARIO 4 (NOISY FREE DATA)

As results show, this method is capable of detection of the magnitude and location of damaged elements with noisy data. The damaged elements are identified by an acceptable accuracy where as an additional slight damage is shown on intact element due to noise presence in mode shapes measurements. Maximum error of parameter identification of the frame example is less than 5 percent at damaged elements and less than 10 percent at intact elements. Without using element damage equations to obtain the same results at least two first mode shapes of frame examples are required [11]. Since amplitude of mode shape decreases at higher mode shape, measurements of higher mode shape are more noise contaminated which adversely affects the results of damage detection. Efficiency of the proposed method can become more significant by increasing the number of unknowns in large structures.

CONCLUSION

This paper presents an approach for damage detection in structures utilizing incomplete measured mode shapes and natural frequencies. The unmeasured part of mode shapes of structures is characterized as a function of structural stiffness parameters and measured modal displacements. More equations have been obtained using element damage equations which need complete mode shapes. This drawback is solved by presenting mode shape equations and dividing structural degrees of freedom to measured and unmeasured parts. An optimal criterion is used to solve these sets of equations to obtain changes of structural parameters. Results of a planer frame represent the ability of this method to evaluate the severity and location of damage using exact and noise polluted data. Results show that this method is above to detect structural damage using a few modal data and measurement efforts.

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