The 3rd International Conference on Structural Health Monitoring of Intelligent Infrastructure
Vancouver, British Columbia, Canada
November 13-16, 2007

VIRTUAL RELIABILITY ASSESSMENT OF STRUCTURES WITH DAMAGE, SARA – PART II

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Abstract

A complex interdisciplinary approach to the reliability assessment of reinforced concrete structures is presented. The main feature of the presented methodology is utilization of virtual testing (non-linear computer simulation) for realistic modelling of structural behaviour and resistance.

Nonlinearities in material, damage, degradation, non-linear response of supports and other effects are employed in FEM modelling of the particular structure, including uncertainties and randomness involved in all these phenomena. Solution of the stochastic response requires repeated analyses of the structural model with random realizations of input parameters. Since the nonlinear structural analysis used for the realistic prediction of structural resistance is computationally rather demanding, a suitable statistical technique (Latin hypercube sampling) is utilized to get sufficiently accurate results keeping the number of simulations acceptably small.

The particular methodologies for non-linear analysis and stochastic treatment have been worked out and verified in practice. Their integration into an innovative tool for reliability analysis of engineering structures called SARA (Structural Analysis and Reliability Assessment) is described.

INTRODUCTION

In the associated paper (Bergmeister et al. 2007) SARA project and its stages of evolution and development of extensions are explained. In this paper the keystones of the SARA system are presented in more detail, namely the nonlinear analysis of reinforced concrete structures, the stochastic package for treatment of random inputs as well as for evaluation of results, and a shell for an interactive control of the complex stochastic nonlinear analysis.

SIMULATION OF DAMAGE IN ENGINEERING STRUCTURES

Computer simulation based on finite element method is a recognized tool used in engineering practice to obtain structural resistance and load carrying capacity. Nonlinear analysis based on sophisticated material models helps to simulate realistic behavior and response of the modeled structure. Such an analysis (called also “virtual testing” of structures) and its results provide better insight into the structural properties and enable realistic assessment of the structure, prediction of its behavior under various effects and conditions. Under certain conditions damage in the structural model can be obtained. Nonlinear simulation can even trace the post peak behavior of the structure after exceeding its maximum load carrying capacity. In the case of reinforced concrete structures, cracks in tensile concrete will develop, reinforcements can yield, and concrete in compression can crush. All these phenomena, which can occur in the real structure, can be simulated and predicted by an appropriate computer model.

The nonlinear finite element simulation of a structure is a rather demanding process, consuming a lot of computer time and capacity. Therefore it is usually used in deterministic way, i.e. with an assumption, that all the necessary input parameters are well known in advance and they can be obtained as deterministic values.

In the SARA system the nonlinear finite element computer program ATENA is employed for the analysis of the modeled structure. The software ATENA is well established for realistic computer simulation of damage and failure of concrete and reinforced concrete structures in a deterministic way (Červenka 2000).

Finite Element Analysis

ATENA is based on finite element discretization and consequent numerical analysis of the modeled structure (Fig. 1). Isoparametric finite elements are used for meshing of the geometrical model of the structure. Alternatively linear or quadratic shape functions can be used in elements. The material volume is modeled by brick, tetrahedral or pyramidal elements. Two-dimensional models are meshed by quadrilateral or triangular finite elements. Plates and shells can be efficiently represented by special layered elements. Reinforcement can be treated in the form of reinforcing bars, pre-stressing cables or as smeared reinforcement given by reinforcement ratio and direction. The discrete reinforcement is fully independent on the finite element mesh. Bond-slip law can be prescribed for reinforcing bars.

Model parts can be connected by contact elements with defined properties. The entire model can be supported by local supports in points, along lines or surfaces, or connected to the background by contact springs.

The structure can be loaded with various actions: body forces, nodal or linear forces, supports, prescribed deformations, temperature, shrinkage, pre-stressing. These loadings are combined into load steps, which are solved stepwise utilizing advanced solution methods: Newton-Raphson, modified Newton-Raphson or arc-length. Secant, tangential or elastic material stiffness can be employed in particular models. Line-search method with optional parameters accelerates the convergence of solution, which is controlled by energy-based and residua-based criteria.

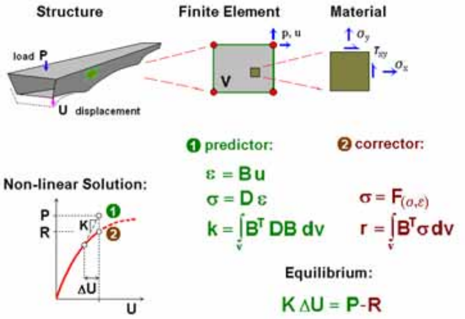

Figure 1. Scheme of the nonlinear finite element method

This efficient finite element solution is accomplished by a user-friendly graphical environment (ATENA GUE), which supports the user during pre- and post-processing and enables real-time graphical tracing and control during the analysis. The pre-processing includes sophisticated input of structural and material properties, an automatic finite element meshing procedure etc. The interactive solution control window enables graphical as well as numerical monitoring of the actual task (e.g. tracing of load-displacement response) and supports user interventions during the analysis (user interrupt, restart). The graphical post-processing can show all important values from the analysis. In the concrete structure, cracks are of the main interest. They can be drawn with their thickness, shear and residual normal stresses. User-defined crack filter is available for obtaining realistic crack patterns. Other important values (strains, stresses, deflections, forces, reactions etc.) can be represented graphically as rendered areas, iso-areas, iso-lines, in form of vector or tensor arrow plots. All values can be also obtained in well-arranged numerical form.

But in the nonlinear analysis, the most crucial role plays the constitutive relation in a material point (constitutive model), see Fig. 1. It decides how well the structural model can represent the real behavior of the modeled structure. Since concrete is a complex material with strongly nonlinear response even under service load conditions, special constitutive models for the finite element analysis of concrete structures are employed.

Nonlinear Constitutive Models

For computer simulation of concrete and reinforced concrete structures including its interaction with neighborhood ATENA offers a variety of nonlinear material models for concrete, quasi-brittle materials, soils and metals, namely:

- damage-based material model
- fracture-plastic cementitious material
- microplane material model
- Drucker-Prager plasticity model
- Von Mises plasticity model
- plasticity with hardening for reinforcement
- etc.

Some of the main features for modeling of concrete are described in this chapter.

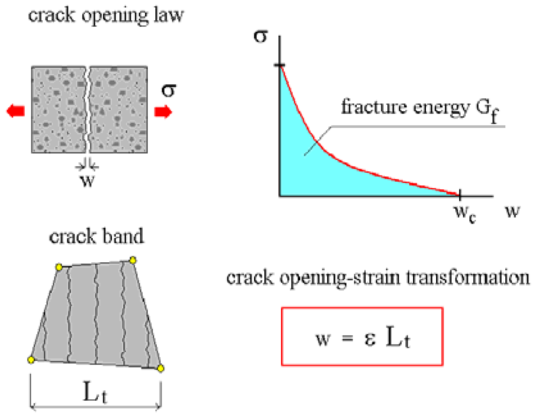

Figure 2. Smeared crack model for tensile behavior of concrete

Tensile behavior of concrete is modeled by nonlinear fracture mechanics combined with the crack band method (Bažant and Oh 1983) and smeared crack concept, Fig. 2. Main material parameters are tensile strength, fracture energy and shape of the stress-crack opening curve (Hordijk 1991).

A real discrete crack is simulated by a band of localized strains, Fig. 3. The crack strain is related to the element size. Consequently, the softening law in terms of strains for the smeared model is calculated for each element individually, while the crack-opening law is preserved. This model is objective due to the energy formulation and its dependency on the finite element mesh size is neglectable, which was confirmed by numerous studies (e.g. Červenka and Pukl 1995).

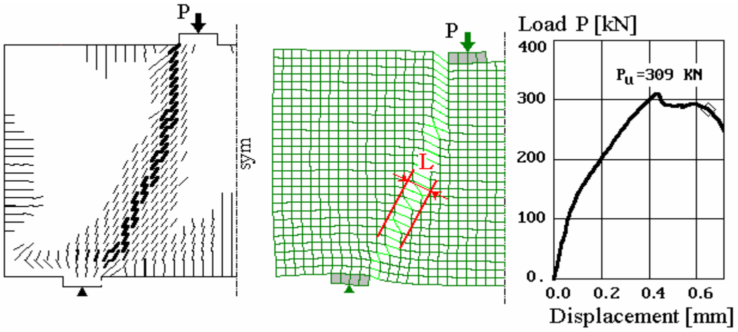

Figure 3. Crack band in a shear wall analysis

The compressive behavior of concrete is covered by the theory of plasticity with a non-associated flow rule according to Menétrey and Willam (1995). This model describes well the specific phenomena in concrete in compression, namely volume change after crushing or influence of the lateral stresses to the compressive strength, so called confinement effect.

However, the input parameters for the above described material models (and also further structural parameters) are not deterministic as usually assumed. They have random nature; they are uncertain and often even unknown, and can be only estimated. In order to account for the randomness and uncertainty of input parameters, probabilistic approach methods should be employed.

Furthermore, the deterministic calculation works optimally in terms of mean values. Safety issues at this level should be addressed by means of global safety factors. Application of appropriate stochastic methods allows statistic evaluation of the structural response (resistance) and direct assessment of structural safety.

PROBABILISTIC APPROACH

The stochastic engine of the SARA system is the probabilistic program FReeT - Feasible Reliability Engineering Tool. This probabilistic software for statistical, sensitivity and reliability analysis of engineering problems was designed with a special focus on the computationally intensive problems, which do not allow performing thousands of samples (Novák et al. 2003). Program FReeT is equipped with a user-friendly graphical user environment, which enables comfortable input of stochastic parameters as well as illustrative interpretation of results.

Small-Sample Simulation

For time-intensive calculations like nonlinear mechanics of concrete, the small-sample simulation techniques based on stratified sampling of Monte Carlo type represent a rational compromise between feasibility and accuracy. Therefore Latin hypercube sampling (LHS) was selected as a key fundamental technique.

The method belongs to the category of stratified simulation methods (e.g. Mc Kay et al. 1979). It is a special type of the Monte Carlo simulation which uses the stratification of the theoretical probability distribution function of input random variables. It requires a relatively small number of simulations to estimate statistics of response – repetitive calculations of the structural response (tens or hundreds).

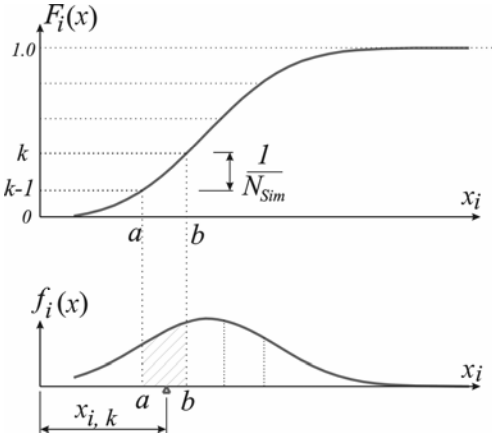

Figure 4. LHS - division of cumulative probability distribution function into layers

The basic feature of LHS is that the probability distribution functions for all random variables are divided into N_{Sim} equivalent intervals, Fig. 4 (N_{Sim} is a number of simulations); the values from the intervals are then used in the simulation process (random selection, middle of interval or mean value). This means that the range of the probability distribution function of each random variable is divided into intervals of equal probability. The samples are chosen directly from the distribution function based on an inverse transformation of distribution function. The representative parameters of variables are selected randomly, being based on random permutations of integers *1, 2, ..., j, N_{Sim}*. Every interval of each variable must be used only once during the simulation. Being based on this precondition, a table of random permutations can be used conveniently, each row of such a table belongs to a specific simulation and the column corresponds to one of the input random variables.

It has been proven that best LHS strategy, which simulates the means and variances very well, is the approach suggested by Keramat and Kielbasa (1997) and Huntington and Lyrintzis (1998). The mean of each interval should be chosen as (Fig. 4):

$$x_{i,k} = \frac{\int_{y_{i,k-1}}^{y_{i,k}} x \cdot f_i(x)\,dx}{\int_{y_{i,k-1}}^{y_{i,k}} f_i(x)\,dx} = N_{Sim} \cdot \int_{y_{i,k-1}}^{y_{i,k}} x \cdot f_i(x)\,dx \qquad (1)$$

where f_i is PDF of the variable X_i, and the integration limits are:

$$y_{i,k} = F_i^{-1}\left(\frac{k}{N_{Sim}}\right) \qquad (2)$$

The estimated mean value is achieved accurately and the variance of the sample set is much closer to the target one. For some probability density functions (inclusive e.g. Gaussian, Exponential, Laplace, Rayleigh, Logistic, Pareto, etc.) the integral (1) can be solved analytically. For others, the extra effort of doing the numerical integration is definitely worthwhile.

Statistical Correlation

Once samples are generated, the correlation structure according to the target correlation matrix must be taken into account. There are generally two problems related to the statistical correlation: First, during sampling an undesired correlation can occur between the random variables. For example, instead of the correlation coefficient zero for the uncorrelated random variables, i.e. an undesired correlation, can be generated. It can happen especially in the case of a very small number of simulations (tens), where the number of interval combinations is rather limited. The second task is to introduce the prescribed statistical correlation between the random variables defined by the correlation matrix. The columns in LHS simulation plan should be rearranged in such a way that they may fulfill the following two requirements: to diminish the undesired random correlation and to introduce the prescribed correlation. It can be done by using different techniques published in literature on LHS (e.g. Huntington and Lyrintzis 1998) but we found some serious limitations while using them.

A robust technique to impose statistical correlation based on the stochastic method of optimization called simulated annealing has been proposed recently by Vořechovský and Novák (2003). The imposition of the prescribed correlation matrix into the sampling scheme can be understood as an optimization problem: The difference between the prescribed *K* and the generated *S* correlation matrices should be as small as possible. A suitable measure of quality of the overall statistical properties can be introduced, e.g. a norm which utilizes the deviations of all correlation coefficients:

$$E_{oveall} = \sqrt{\sum_{i=1}^{N_V - 1} \sum_{j=i+1}^{N_V} \left(S_{i,j} - K_{i,j}\right)^2} \tag{3}$$

The norm *E* has to be minimized from the point of view of the definition of the optimization problem using simulated annealing optimization approach, N_v random variables realizations are related to the ordering in the sampling scheme.

Sensitivity Analysis

An important task in the structural reliability analysis is to determine the significance of random variables. With respect to the small-sample simulation techniques described above the straightforward and simple approach uses the non-parametric rank-order statistical correlation between the basic random variables and the structural response variable. The sensitivity analysis is obtained as an additional result of LHS, and no additional computational effort is necessary.

The relative effect of each basic variable on the structural response can be measured using the partial correlation coefficient between each basic input variable and the response variable. The method is based on the assumption that the random variable which influences the response variable most considerably (either in a positive or negative sense) will have a higher correlation coefficient than the other variables. Because the model for the structural response is generally nonlinear, a non-parametric rank-order correlation is used by means of the Spearman correlation coefficient or Kendall tau.

Reliability Assessment

In cases when we are constrained by a small number of simulations (tens, hundreds) it can be difficult to estimate the failure probability. The following approaches are therefore utilized here; they are approximately ordered from elementary (extremely small number of simulations, inaccurate) to more advanced techniques.

- Cornell´s reliability index - the calculation of reliability index from the estimation of the statistical characteristics of the safety margin
- The curve fitting approach - based on the selection of the most suitable probability distribution of the safety margin.
- FORM approximation (Hasofer-Lind´s index)
- Importance sampling techniques
- Response surface methods

These approaches are well known in reliability literature and also providing all details is beyond the aim of this paper. In spite of the fact that the calculation of the failure probability (or/and reliability index) using some of these techniques does not always belong to the category of very accurate reliability techniques (first three in the list), they represent a feasible alternative in many practical cases.

STOCHASTIC NONLINEAR ANALYSIS

In the randomized nonlinear simulation both the above described programs must be combined in an appropriate way. Input parameters of the deterministic model which was prepared and verified in ATENA are firstly randomized using LHS method in FReET. A set of randomized input files is consequently calculated in ATENA. Selected results from analyses are collected, and evaluated in FReET: statistical parameters of the structural response are obtained, sensitivity to the particular input parameters is estimated, and reliability of the structure under defined load is assessed.

In order to assure well-arranged data exchange and management, as well as control the keystone programs and additional supporting tools (e.g. the integrated DATABASE of statistical parameters for various materials), an interactive graphical shell called SARA Studio was developed. It enables the user to control interactively the whole process of the stochastic nonlinear analysis by efficient commands and useful program interfaces. The graphical desktop shows continuously (already during the particular analyses) histogram of selected results (ultimate load, deflection, maximum crack width etc.) and set of load-deflection diagrams. The actually running analysis can be controlled in a standard way through the original ATENA graphical user environment.

Several methodologies were developed as anextension of this basic concept for stochastic nonlinear analysis and virtual reliability assessment of structures with damage. They combine the SARA Studio and FReET with additional tools (e.g. neural network engine) in order to utilize it as a core for more complex investigations like inverse analysis of structural parameters, dynamic damage identification, stochastic degradation modeling or health monitoring of structures. Several such developments and practical applications are described in consequent papers in these proceedings (labeled “SARA – Part III” to “SARA – Part VII”) in more detail.

CONCLUSIONS

The presented software system integrates nonlinear finite element modeling with advanced stochastic and reliability technology into a powerful tool for virtual reliability assessment of structures with damage. It is ready to use in engineering practice and can also serve as a core for complex systems for structural damage identification and structural health monitoring. In this way it can efficiently support the decision-making process in maintenance of structures and can lead to considerably higher efficiency and cost savings.

ACKNOWLEDGEMENT

The related research was partially supported by Autobrennero Motor Highway A22, Italy, and by grant No. 1ET409870411 VITESPO of Czech national research program “Information society”. These financial supports are gratefully acknowledged.

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