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PERFORMANCE PREDICTION FUNCTIONS FOR RELIABILITY ASSESSMENT BASED ON STRUCTURAL HEALTH MONITORING

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Abstract

In general, monitoring systems consist of one or more sensor networks. These sensor networks produce a very large amount of information that has to be processed by data acquisition stations and decision routines. It is extremely difficult, and sometimes impossible, to include each individual obtained value in the assessment process. Therefore, there is a need for procedures and techniques to reduce the amount of monitored data.

Data processing by use of prediction functions seems to be a promising procedure, since only data violating or indicating a necessary update of the used prediction functions are needed for processing. In consequence, the prediction functions provide instead of raw monitored data the basis for the reliability assessment of engineering structures. Therefore, a clear defined procedure for the definition of prediction functions is required. In addition, Bayesian updating has to be used in order to update the parameters of the prediction functions.

The objective of this paper is to present a procedure for the initial set up of prediction functions, the definition of necessary monitoring periods, and the inclusion of additional monitored data by using Bayesian updating. The proposed approach is presented on monitored data of the I-39 Northbound Bridge over the Wisconsin River, obtained from the ATLSS Center at Lehigh University.

INTRODUCTION

In general, monitoring systems applied to bridges contain a network of sensors. A common feature of monitoring systems is the need of handling the continuous supplied data. Several authors proposed the use of Data Stream Management Systems DSMS (Appadwedula et al. 2005) to process the amount of unbounded data by running a continuous long-term monitoring program. DSMS have to channel the data obtained by the sensor networks.

Typical queries on monitoring of bridge structures are about constraints on values obtained by sensors, also called value constraints. Such constraints can be, for example, strains in a steel structure.

There is also a great interest in constraints evaluating the differences between the values measured by sensors. This type of constraint is called window-joint constraints (Grewal and Andrews 1993). They reflect a temporal relationship for two values to be joined. Measurements of very different values at approximately the same time could reveal the existence of violations of the observed physical quantity or malfunctioning sensors.

In general, for monitored structural systems it is possible to assign prediction functions to sensors. These functions can be updated only when they differ significantly from the predicted values. This concept allows the reduction of the monitoring frequency and the associated costs caused by query processing. For example, the corrosion of rebar in a concrete deck will probably evolve during the years following predictable patterns, Marsh and Frangopol (2007).

Predictions can be reasonably used in various structural applications. For example, an inclinometer on a structure could provide not only the current angle of a structural component but also its vector of movement or expected trajectory. Similarly, the values measured by a temperature sensor positioned on a structure are expected to change continuously during the years. The strains in a bridge girder and the corrosion in rebar are values that can be predicted by using prediction functions. The importance of prediction functions increases when the values to be measured are expected to change according to predefined patterns or when unexpected violations should be detected. Babu and Widom (2001) propose to develop prediction functions based on measured quantities by sensors. This concept could be used to reduce the data acquisition and number of communicated data by processing only the significant violating monitored data. Prediction functions could be obtained in a variety of ways (Miles and Shevlin 2001, Grewal and Andrews 1993, Teply et al. 2006, Stehno et al. 1987) and thresholds of structural properties can be specified by testing, by material laws or by users.

Sensors of bridge monitoring systems measure, in general, physical quantities continuously or at a certain sampling frequency (e.g., 10 times per day). The measurements are associated with a unique identifier. The prediction S_j is related to the measured data obtained from sensors as

$$S_j = <S_i,\ PhyQ_i,\ T_j\ ,\ f_{pj}(t)> \qquad (1)$$

where S_i is the identifier of a sensor, $PhyQ_i$ is the physical quantity it measures, T_j is the time of the update, and f_{pj}(t) is the prediction function that, given a certain time instant t, retrieves the expected monitored data (Appadwedula et al. 2005). Eq.(1) allows estimating monitor data based on the past measurements together with prediction functions. Since prediction functions are key factors of this method, location and updating procedures will be presented in the following.

PREDICTION FUNCTIONS BASED ON MONITORED EXTREME VALUES

As monitored data are random and the interest by the development of prediction functions f_p for structural characteristics is in most cases the description of the distribution of extreme values, there is no requirement to use all monitored data. Only extreme values of defined monitoring units P' (e.g. maximum daily values) are of interest. The grouping of extreme values to sample sets of monitored periods P already allows an approximated location of f_p with respect to a threshold of an investigated physical quantity.

The variability and tendency of those sample sets can be computed by using chart methods as proposed by Levine et al. (2001). These methods indicate if the dispersion of the consecutive sample sets are caused by chance or by other processes, or in other words, if the monitored processes are stable or unstable. Since in engineering there are a lot of environmental factors influencing the monitored data (temperature, solar radiation, traffic etc.) this check will yield in most cases to unstable processes. Therefore, the application of only chart methods, as used for production control, for the derivation of prediction functions is not efficient. Extreme values E_p recorded during a monitoring period P of the previously mentioned sample sets provide a powerful information for the derivation of prediction functions.

There are numerous prediction functions for the description of degradation processes not taking into account the instantaneous information of monitored data. Most of them are based on advanced analytical formulations (Stehno 1987, Teply 2006). However, for the derivation of prediction functions f_p, from monitored data, polynomial approaches of 1st, 2nd or 3rd order seem to be useful

$$f_P = \sum_{k=0}^{w} a_k \cdot t^k \ldots w = 1, 2, 3 \tag{2}$$

where a_k = coefficients, w = order of the polynomial function, and t is the time.

The coefficients a_k can be obtained by using the following consecutive steps:

Step (a); computation of the *necessary duration of monitoring periods P* according to the allowable probability p per monitoring unit P' of violating the threshold established by the prediction function f_p by the monitored data with a confidence level C. Formulations for these constraints can be derived from the theory of acceptance sampling (Ang and Tang 2007) as follows:

$$\sqrt{P} = \frac{\Phi^{-1}(C)}{\Phi^{-1}(p) - \Phi^{-1}(S_p / P)} \tag{3}$$

where S_P = selected allowable violating samples of f_p per monitored period P and $\Phi^{-1}(.)$ = the inverse standard normal cumulative distribution function.

Step (b); determination of the *Pre-polynomial coefficients* a'_k (associated with a_k of Eq. (2)) by mean square fitting to the extreme values E_p of the monitoring units P' per monitoring period P;

Step (c); movement of the prediction function f_p towards the maximum value of E_p such that no more than S_p monitored extreme values E_p violates f_p to fulfill the *defined requirements of Step (a)*. This adaptation is carried out mostly via a correction of $a'_{k=0}$ to $a''_{k=0}$ which yields:

$$f_P = \sum_{k=0}^{w} a''_k \cdot t^k \ldots w = 1, 2, 3 \tag{4}$$

where a''_k = coefficients after adjusting the prediction function f_p with respect to the extreme values E_p.

Step (d); the above *defined criteria for the location* of f_p do not restrict the value of the difference ζ between the prediction function and the violating extreme values S_p. The restriction of these differences ζ within each P can be done by the previously mentioned chart methods discussed in Levine et al. (2001).

This procedure yields finally to the coefficients presented in Eq.(2), based on the demand that the difference between E_p (including S_p) and f_p is caused by chance. More details about the steps are provided in Strauss et al. (2007).

The adaptation of the prediction functions (coefficients a_k) can be done in each monitoring period by using the monitored data of former periods P (e.g., a prediction function of 1st order needs at least two monitoring periods to include the past developments of the monitored data). Fig. 1, for instance, shows the prediction function $f_p^{(1,2)}$ derived from the first and second monitoring period ($P = 22.3$ *days*) of the monitored data taken from the I-39 Northbound Bridge. This bridge, over the Wisconsin River, was built in 1961 in Wausau, Wisconsin, USA. The total length of the bridge is *196.04 m (643.2 ft)*. It is a five span continuous steel plate girder bridge. The monitoring of this bridge included the strain/stress behavior of specified structural components. Fig. 2 shows for the sensor CH15 the monitored extreme data per day. Details about the further targets and results of the monitoring program are given in Mahmoud (2005). As it can be seen from Fig. 1, f_p can be used (a) to predict the strain/stress distribution behavior at the end of the monitor period or even at the end of the lifetime, and (b) to select possible interruption periods of monitoring as discussed in Strauss et al. (2007).

Since strain/stress values are not providing structural reliability measures, the reliability assessment should be based on a more general information. Therefore, there is the need to include the monitored physical quantities (e.g., strains) into a reliability format such as the β index format.

This can be done by limit state considerations such as (a) $g_1(\boldsymbol{X_1})$ = resistance threshold – prediction function of load effect, (b) $g_2(\boldsymbol{X_2})$ = resistance threshold – load effect based on all monitored data, or (c) $g_3(\boldsymbol{X_3})$ = resistance threshold – load effect based on maximum value of monitored data. Fig. 3 shows these three approaches for a mean resistance threshold $f_y = 377\ MPa$ and a coefficient of variation ($COV) = 0.07$. The standard deviations assigned to the prediction function, to the monitored data, and to the extreme monitored value are computed from the extreme values of one monitoring period P (e.g., $\sigma = 9.2\ MPa$ for $P^{(1)}$.)

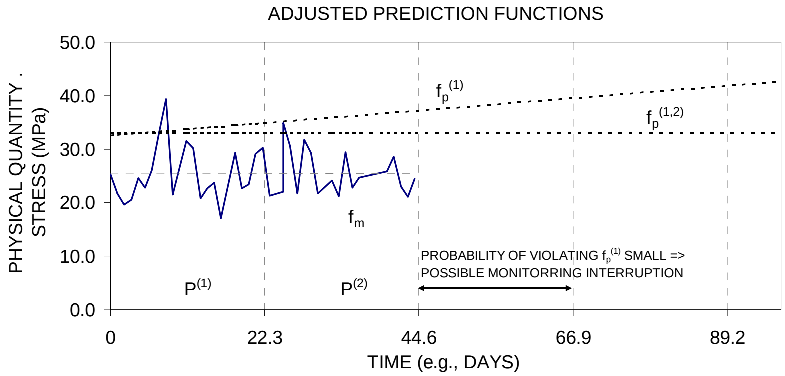

Figure 1. Prediction functions $f_p^{(1)}$ and $f_p^{(1,2)}$ of the monitoring periods $P^{(1)}$ and $P^{(2)}$ obtained from the monitored extreme values

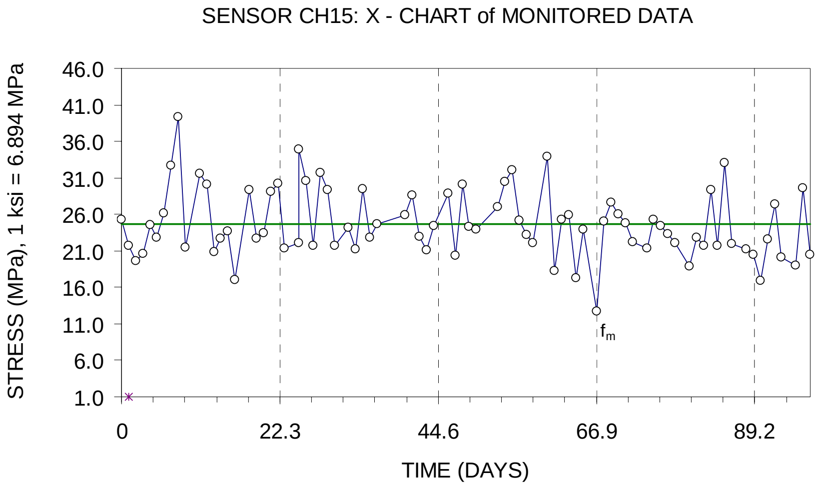

Figure 2. Monitored extreme values per day during the whole monitoring period (data taken from Mahmoud et al. 2005)

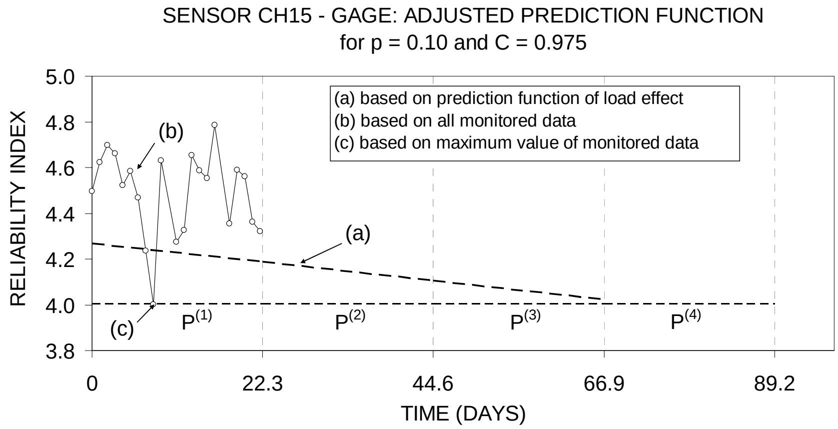

Figure 3. Reliability index computed from the extreme values of the monitored data, and the prediction reliability index function

BAYESIAN UPDATING FOR PREDICTION FUNCTIONS

Monitored data are similar to statistical data which can be more or less accurate. In consequence, prediction functions derived from monitored data for the reliability assessment cannot be considered as exact. Such functions and their specification factors derived from monitored data are therefore a priori information.

A priori predictions at specified points in time of a structure, e.g. at the end of lifetime, can be derived by short-time functions (i.e., difference between the time horizon considered and time when the data is processed Δt, say $\Delta t \leq 30$ *days*) or long-time functions (say $\Delta t > 30$ *days*). Long-time predictions can be improved by utilizing prior prediction functions described by mean μ'_Y and standard deviation σ'_Y and short-term monitored data. This procedure is called Bayesian statistical prediction or Bayesian Updating (BU).

The BU procedure also provides the possibility for a continuously adjustment of prediction functions, and finally for a realistic assessment of the physical quantity at the end of the life time. The procedure is briefly described in the following.

Let X be short-time values and Y long-time values of physical quantities generated from a prediction function at specified points in time of a structural lifetime (e.g. end of the lifetime). The associated prediction functions (e.g. polynomial r^{th} order) can be obtained by mean square fitting or other techniques applied to extreme values of monitored data as shown above. Since monitored data are restricted in time, the mean values μ'_X , μ'_Y and standard deviations σ'_X , σ'_Y describing prediction functions are prior statistical information.

Now suppose we are able to obtain a set of short-term monitored data for updating the prediction function. The objective is to use the monitored data to improve or update the long-time values X or Y. For this purpose a correction factor p_k is calculated by comparing the new monitored and prior values of the prediction functions as follows (Bazant 1985):

$$p_k = exp\left[-\frac{1}{2}\left(\frac{\mu_X - \mu_X'^{(k)}}{\sigma_X}\right)^2\right] \tag{5}$$

where $\mu_X'^{(k)}$ = the mean value of the k^{th} prediction function, and μ_X = mean value obtained by mean square fitting of the prediction function to the set of the new short-term monitored extreme data, and σ_X = standard deviation of the set of short-term monitored extreme data. The posterior (updated) mean values and standard deviations of the prediction functions can be obtained according to Bazant (1985) as

$$\mu_X'' = \frac{1}{\sum p_k} \sum_{k=1}^{K} p_k \cdot \mu_X'^{(k)} \tag{6}$$

$$\sigma_X'' = \sqrt{\frac{1}{\sum p_k} \sum_{k=1}^{K} p_k \cdot \left(\mu_X'^{(k)} - \mu_X''\right)^2} \tag{7}$$

$$\mu_Y'' = \frac{1}{\sum p_k} \sum_{k=1}^{K} p_k \cdot \mu_Y'^{(k)} \tag{8}$$

$$\sigma_Y'' = \sqrt{\frac{1}{\sum p_k} \sum_{k=1}^{K} p_k \cdot \left(\mu_Y'^{(k)} - \mu_Y''\right)^2} \tag{9}$$

Note that if data obtained by the measurements are too remote from the prior data then BU will not provide good results. The above mentioned formulas are valid for normal probability distributions (Bazant 1985).

This BU method was applied to the monitored data of the I-39 Northbound Bridge to evaluate the stress distribution, evaluated by using prediction functions, at the end of the monitoring horizon $t = 97$ *days*. The BU procedure was based on the continuously provided short-term monitored data of the monitoring periods $P = 22.3$ *days*. The procedure was as follows:

(a) a curve fitting for a polynomial of first order was performed to the monitored extreme data for the period $P^{(1)}$. The polynomial coefficients a_0 and a_1 as shown in Table 1 were obtained for the first prediction function $f_p^{(1)}$;

(b) the mean of the prior data $\mu_Y' = 34.454\ MPa$ was computed by using the polynomial function $f_p^{(1)}$ at the end of the period $P^{(1)} = 22.3$ *days*, and the associated standard deviation $\sigma_Y' = 6.64\ MPa$ was computed from the min and max values of the first period as shown in Table 1. The computation was based on a rectangular distribution.

(c) period $P^{(2)}$ provided the short-term monitored data for the first BU. The required μ_Y and σ_Y for the computation of p_k according to Eq. (6) at the end of the Period $P^{(2)}$ were obtained as in steps (a) and (b)

(d) the prior data of monitoring period $P^{(1)}$ together with the short-term monitored data of period $P^{(2)}$ provided according Eqs. (5), (8) and (9) the posterior data $\mu_Y'' = 34.454\ MPa$ and $\sigma_Y'' = 0.000\ MPa$ at the end of the monitoring horizon $t = 97$ *days*. Tables 2 and 3 show the results of consecutive steps of the BU procedure for the used short-term data $P^{(2)}$, $P^{(3)}$ and $P^{(4)}$.

The behavior of the computed posterior data (monitoring horizon $t = 97$ *days*) for BU performed at *44.6 days*, at *66.9 days* and *89.2 days* can be seen in Fig. 4. It is apparent that the updating process for the monitored data results in safe posterior data compared to the stress distribution monitored at the end of the monitoring horizon. In consequence, BU based on the p_k formulation allows the reliable updating of prediction functions and provides basis elements for the definition of possible monitoring interrupting periods, discussed in Strauss et al. (2007).

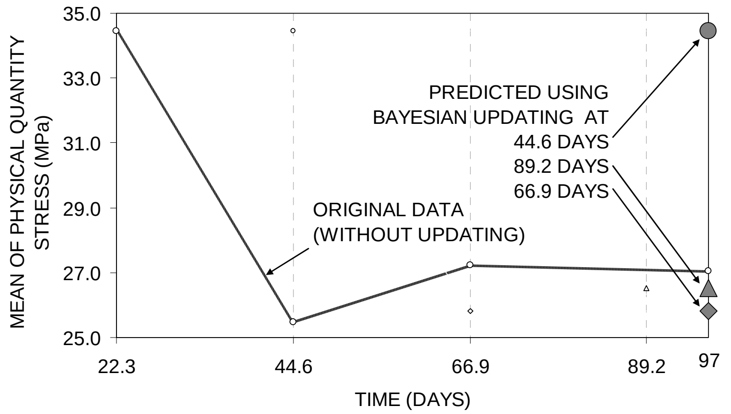

Figure 4. Effects of Bayesian Updating on the mean stress predicted at 97 days

Table 1. Characteristics of the monitored data and the prediction functions

Monitoring Period	Polynomial Coefficients a_0	Polynomial Coefficients a_1	Monitored Stress (MPa) max	Monitored Stress (MPa) min
$P^{(1)}$	24.3	0.1042	40.0	17.0
$P^{(2)}$	25.5	0.0000	35.0	20.0
$P^{(3)}$	27.2	0.0000	33.0	17.0
$P^{(4)}$	29.3	-0.0750	32.0	12.0

Table 2. Prior characteristics of the Bayesian Updating procedure applied to the monitoring data of the I-39 Northbound Bridge

Statistical Predictors				End of the Prior Monitoring Period
Original f_p		Prior Data		
μ'_Y	σ'_Y	μ'_Y	σ'_Y	Days
34.45	6.64	34.454	6.640	22.3
25.48	3.46	34.454	0.000	44.6
27.23	4.62	25.808	8.647	66.9
27.04	5.77	26.530	5.579	89.2

Table 3. Posterior characteristics of the Bayesian Updating procedure applied to the monitoring data of the I-39 Northbound Bridge

Statistical Predictors				
Short-Term Monitored Data		Posterior Data		Updating Time
μ_X	σ_X	μ''_Y	σ''_Y	Days
25.5	3.5	34.454	0.000	44.6
27.2	4.6	25.808	8.647	66.9
27.0	5.8	26.530	5.579	89.2

CONCLUSIONS

Prediction functions are essential elements for monitoring systems. They allow the rational treatment of monitored data and the reduction of the monitoring effort, by using only significant violating data and/or by predicting possible monitoring interruption times. There are different kinds of prediction functions. Most of them are based on sophisticated analytical functions not taking into account the monitoring results. This paper presents an approach for the inclusion of monitored data in prediction functions and, therefore, in the reliability assessment. The Bayesian

Updating procedure, applied to existing monitored data of the I-39 Northbound Bridge in Wisconsin, demonstrates the possibility to update the prediction functions and the determination of monitoring interruption periods.

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