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# ANALYSIS MODEL DEVELOPMENT FOR A SUSPENSION BRIDGE BASED ON FIELD-MEASURED DATA

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## Abstract

Previously published shape-finding analysis methods or results are mostly based on the idealized structural system during design, need is to pay particular attention on the exploitation of field-measured data when establishing or updating the analysis model for maintenance purposes of the bridge in service. This study intends to propose an analytic model for an actual 3-span suspension bridge utilizing the survey of its configuration. Based on the shape-finding analysis results relative to the dead loads, a linearized finite displacement FE model is proposed and, an attempt to extend the measurement-based shape-finding analysis method from conventional methods by reflecting progressively the measured configuration and temperature is presented. Analysis results revealed that the current configuration of the bridge in service corresponds relatively well with the target configuration. This study proposed a model updating method which could provide theoretical basis for the establishment of analysis model for suspension bridges based on diversified maintenance data.

## INTRODUCTION

A suspension bridge is a structural system of where stiffness is influenced sensitively by the horizontal tension, $H_w$, of the main cables. Since this horizontal tension can be approximated as $H_w = wl^2 / 8f$, the main variables determining the stiffness of a suspension bridge can be seen as the dead load ($w$), the span length ($l$) and the sag of the main cable ($f$).

Except for the sag, the shape of the main cable and tension of the dead load are basically unknown. Various shape-finding analysis methods were proposed together with the evolution of the finite element analysis (Karumi, 1999; Kim et al., 2001; Kim et al., 2002).

However, the shape-finding analysis methods reported to date are relevant to idealized structural systems for design purpose, and there is a concern that the development of analytic models for maintenance purpose with regard to suspension bridges actually under operation is necessary.

Most of the suspension bridges under service are equipped with expensive monitoring instrumentation from which long-term data is being acquired. In addition, measurement of the hangers' tension through the vibrational method and measurement of the geometry of the main cables are now becoming common items during the periodic inspection process. Therefore, in order to reflect such field data in the shape-finding analysis model for maintenance purposes like the evaluation of the bearing capacity of a bridge, it seems necessary to propose updating the measurement-based shape-finding analysis process using the idealized shape-finding analysis for design purposes.
Considering this background, this study suggests a technique for the establishment of an analytic model based on measurement. This technique is applied to Gwangan Bridge, a suspension bridge under service in Korea, in order to propose an analytic model for maintenance purposes, reflecting the actual structural state of Gwangan Bridge.

## EXAMPLE BRIDGE

The actual bridge selected for the application is illustrated in Figure 1. The bridge is a 900 m long suspension bridge.

### Dead Loads

Figure 3 arranges the dead load data of the selected bridge obtained through a survey of previous documents. These data are: (1) detailed design data, (2) calculation data from contractor, (3) construction design data for the computation of fabrication camber and geometry control. Among these data, the latter appears to present the highest reliability and is exploited in this study.

### Measured hanger tension

The authors reported results of field measurements of hanger tensions performed in 2004 through vibration method for hanger ropes with length exceeding 10 m. Figure 3 plots the distribution of the measured tensions.

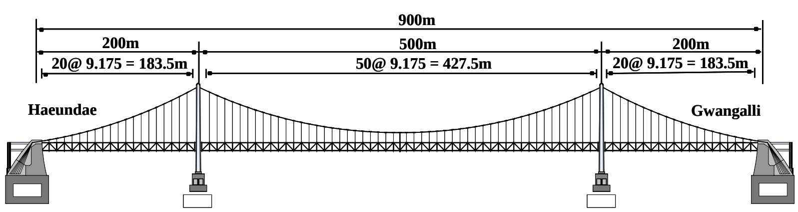



Figure 1. Dimensions of the selected bridge.

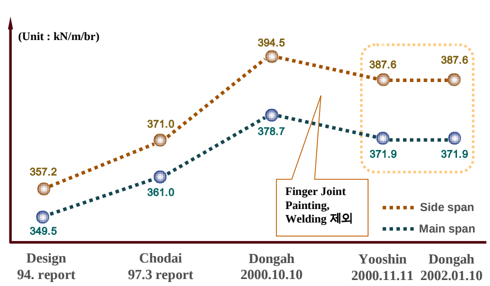



Figure 2. Sectional and dead loads of the selected bridge.

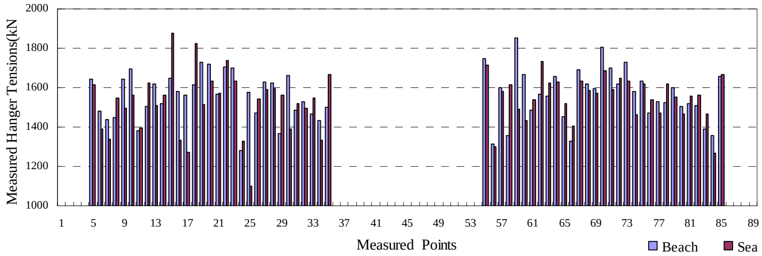



Figure 3. Measured hanger tensions in the selected bridge.

## SURVEY OF CONFIGURATION

### Selection of Survey Spots

Technically, in order to secure economic efficiency beyond exploitability, the survey spots have been determined as shown in Figure 4 and measurements have been taken.

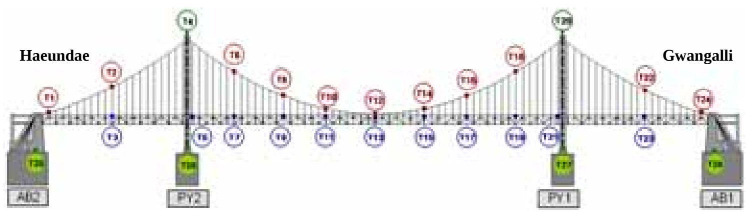



Figure 4. Location of the survey spots in the selected bridge.

### Interpolation of the Configuration Survey

Since the survey results correspond to the coordinates of the survey spots, of which the number is significantly smaller than the number of nodes in the analytic model, interpolation is required to obtain the coordinates of the nodes. Figure 5 illustrates the orders applied for interpolation according to the number of survey spots in each of the structural members.

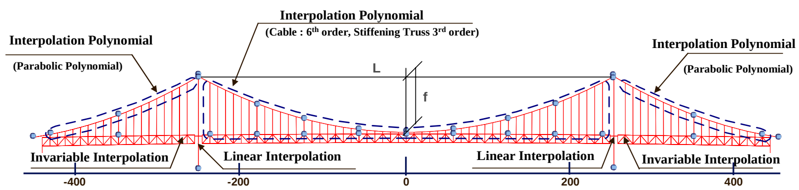



Figure 5. Interpolation orders in the main cables and stiffening truss of the selected bridge.

From the geometric coordinates generated through polynomial interpolation, the measured sag and length of the main span corresponding to major variables in a suspension bridge are represented in Figure 6.

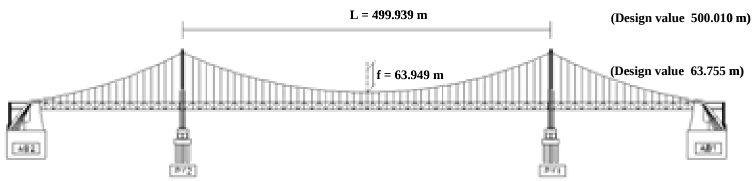



Figure 6. Main span length and sag measured in the selected bridge

## SHAPE-FINDING ANALYSIS THEORY

This study adopts the analysis theory proposed by Kim et al. (2002) for a self-anchored suspension bridge. Even if the proposed theory was developed for a self-anchored suspension bridge in which large compressive forces are applied in the stiffening truss, this theory is naturally adaptable to earth-anchored suspension bridges.
The following describes the shape-finding analysis applicable in design stage.

### Three-Dimensional Elastic Catenary Cable Element

An elastic catenary cable element is used for the main cables and the hangers. This element is derived from the exact solution of the elastic catenary cable equation deformed due to its self-weight.
Consider a cable suspended between points *i* (0,0,0) and *j* ($l_x$, $l_y$, $l_z$). The relative distances between the two nodes along the global *X*-, *Y*-, and *Z*-axis, denoted by $l_x$, $l_y$, and $l_z$, can be expressed as a function of the global nodal force $F_i$ ($i$ = 1,2,3) at node *i* as

$$l_x = \frac{F_1}{E_c A_c} L_0 - \frac{F_1}{w}\left\{\sinh^{-1}\left(\frac{F_3}{H}\right) - \sinh^{-1}\left(\frac{F_3 - wL_0}{H}\right)\right\} = f\left(F_1, F_2, F_3\right)$$

$$l_y = \frac{F_2}{E_c A_c} L_0 - \frac{F_2}{w}\left\{\sinh^{-1}\left(\frac{F_3}{H}\right) - \sinh^{-1}\left(\frac{F_3 - wL_0}{H}\right)\right\} = g\left(F_1, F_2, F_3\right) \qquad (1)$$

$$l_z = \frac{F_3}{E_c A_c} L_0 - \frac{F_1}{2E_c A_c} - \frac{H}{w}\left[\left\{1 + \left(\frac{F_3}{H}\right)^2\right\}^{\frac{1}{2}} - \left\{1 + \left(\frac{F_3 - wL_0}{H}\right)^2\right\}^{\frac{1}{2}}\right] = h\left(F_1, F_2, F_3\right)$$

where $E_c$ is the elastic modulus, $A_c$ the cross-sectional area, $L_0$ the unstrained length, $H$ the in-plane horizontal tension, and $w$ the self-weight per unit length of the cable. The Newton-Raphson method is utilized as an iterative technique to solve Eq. (1) for the unknown nodal forces.

### Shape-Finding Analysis

Figure 7 shows the projected profile of the main cable on the vertical *X*–*Z* plane for a span. In the figure, $T$ ($T_x$, $T_z$) is the tension along the main cable, and $P_i$ is the hanger force at node *i*. The main cable is divided into *N* straight members in a piecewise manner if the number of hanger is *N*–1 in the span being considered. The force equilibrium in *X*–*Z* plane can then be expressed as

$$T_i \frac{d_i}{l_i} = T_{i+1} \frac{d_{i+1}}{l_{i+1}} \quad , \; i = 1,2,\cdots, N-1 \qquad (2)$$

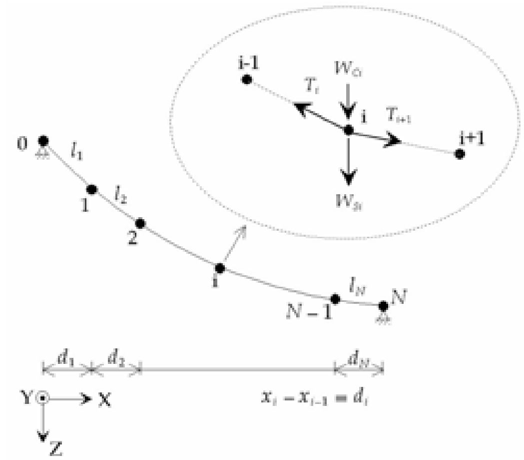



Figure 7. Force equilibrium of the main cable projected on the vertical plane.

where $T_i$ and $l_i$ are the tension and the length of the straight member between node $i$–1 and $i$, respectively, and $d_i = x_i - x_{i-1}$, in which $x_i$ is the $x$-coordinates of the main cable at node $i$. By applying Eq. (2) to all members, the following equation can be obtained.

$$T_1 \frac{d_1}{l_1} = T_2 \frac{d_2}{l_2} = \cdots = T_N \frac{d_N}{l_N} = T_x \tag{3}$$

From Figure 9, the force equilibrium at node $i$ can be expressed as

$$T_i \frac{z_i - z_{i-1}}{l_i} - T_{i+1} \frac{z_{i+1} - z_i}{l_{i+1}} = P_i + W_{Ci} \quad , \ i = 1,2,\cdots, N-1 \tag{4}$$

where $z_i$ the $z$-coordinates of the main cable, and $W_{Ci}$ the self-weight of the main cable allotted to node $i$.
Based on Eqs. (3) and (4), following can be obtained:

$$T_x \left( -\frac{z_{i-1} - z_i}{d_i} + \frac{z_i - z_{i+1}}{d_{i+1}} \right) = P_i + W_{Ci} = W_{Si} + W_{Ci} \quad , \ i = 1,2,\cdots, N-1 \tag{5}$$

where $W_{Si}$ denotes the vertical component of hanger tension at node $i$ which is the portioned dead weight of the stiffening truss supported by this hanger including the self-weight of hanger itself.
A total of $N$–1 simultaneous equations can be obtained from Eq. (5), while the unknowns are $z_i$ ($i$ = 1, 2, …, $N$–1) and $T_x$. One more equation is, therefore, necessary to determine the preliminary profile of the main cable. This supplementary constraint is provided by the mid-point sag in the center span $f$.

$$z_{\frac{N}{2}} = \frac{1}{2}(z_N + z_0) + f \tag{6}$$

From Eqs. (3), (5), and (6), the $z$ -coordinates and the tension of main cable at each node can be determined.

## MODEL UPDATING WITH FIELD-MEASURED DATA

### Designed Profile

The designed profile in this study denotes the calculated configuration of main cable according to the proposed shape-finding analysis with the weight distribution determined at the erection control stage. The shape-finding results are summarized in Table 1 for the critical parameters.

Table 1. Designed profile

| | Parameters | Location | Results |
|---|---|---|---|
| Displacement (cm) | End link at stiffening truss | PY1 | $2.011\times10^{-6}$ |
| | | PY2 | $2.927\times10^{-8}$ |
| | Top of main tower | PY1 | $1.403\times10^{-5}$ |
| | | PY2 | $1.109\times10^{-6}$ |
| | Mid-point sag (m) | | 63.7550(=0.0) |
| Tension (kN) | Horizontal tension of main cable | | 90,120 |
| | Hanger | Entrance | 1515 |
| | | Side span | 1520~1591 |
| | | Main span | 1503~1570 |

Surveyed profile denotes the shape-finding analysis results considering measured data including surveyed configuration of the bridge structure. The definitions are as follows.
Surveyed profile 1: Update the designed profile with surveyed configuration
Surveyed profile 2: Update the surveyed profile 1 with measured hanger tension
Surveyed profile 3: Calibrate the surveyed profile 2 to the designed temperature

**Surveyed Profile 1**

The surveyed profile 1 basically follows the conventional shape-finding analysis procedure as is done for the designed profile. The surveyed sag at mid-span is used instead of designed or target sag in Eq. (6).

The surveyed profile 1 resulted in 63.949 m for the main cable sag and 89.77 MN for the horizontal tension of main cable.

**Surveyed Profile 2**

The surveyed profile 2 can be determined by updating the surveyed profile 1 with measured hanger tension. It can be achieved by using the measured hanger tension for the $P_i$ in Eq. (5).

The surveyed profile 2 resulted in 63.949 m for the main cable sag and 89.90 MN for the horizontal tension of main cable.

**Surveyed Profile 3**

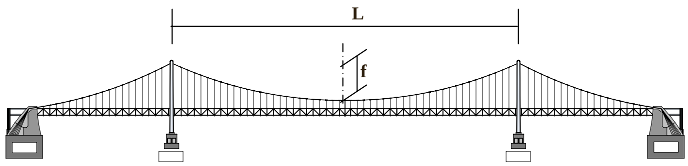
The surveyed profiles 1 and 2 contain the thermal deformation due to the difference of temperature between survey site and assumed value for the design. Therefore, temperature calibration is needed to compensate the possible difference between the designed profile and the surveyed profile 2.

Since a series of thermometers were installed along the main structural members of the bridge, the temperature of each member during the survey was also provided. The difference of temperature between the designed and measured was applied for each member and finally the survey profile 3 could be obtained.

The surveyed profile 3 resulted in 63.795 m for the main cable sag and 90.28 MN for the horizontal tension of main cable.

## RESULTS

Figure 8 compares the reviewed profiles in this study.




| | Designed Profile | Surveyed Profile, Measured Hanger Tension (Surveyed Profile 2) | Temperature Calibrated (Surveyed Profile 3) |
|---|---|---|---|
| L | 500.010 m | 499.939 m (-0.01%) | 499.977m (-0.01%) |
| f | 63.755 m | 63.949 m (+0.30%) | 63.795 m (+0.06%) |
| Horizontal Tension | 90.125 MN | 8990.05 tonf (-0.25%) | 90.280 MN (+0.17%) |

Figure 8. Determined profiles of the selected bridge.

Application of the proposed method to a selected bridge revealed that the current configuration of the bridge in service corresponds relatively well with the target configuration. In particular, with the calibration of temperature, it can be said that the surveyed profile shows good agreement with the target profile.

AB2 PY2

## CONCLUSIONS

An analysis model updating method exploiting measured configuration and temperature, hanger tension and calibrated weight is proposed using the analytic model of a suspension bridge established through conventional initial-shape analysis method. Since this is an analysis model for maintenance purpose, this study intended to propose a linearized finite displacement model. Application of the proposed method to a selected revealed that the current configuration of the bridge in service corresponds relatively well with the target configuration. Accordingly, the analytic model adopted during design can be seen to exhibit relatively high reliability, which does not necessitate its replacement with the measurement-based linearized finite displacement FE model. However, this study proposed a model updating method which could provide theoretical basis for the establishment of analysis model for suspension bridges based on diversified maintenance data. The contribution of this paper can be found in the minimization of the error that may occur between the analytic model and the actual structure based on shape-finding analysis theory by exploiting measured data at the most. Even if practically no difference could be observed between the measurement-based model and the model assumed during design in the case of the selected bridge, SI techniques proposed in previous researches may be additionally applied for bridges with conflicting models. By adopting such approach, optimal improvement of predictable variables would be possible. This study is expected to be a preparatory stage minimizing errors for model updating using SI technique.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