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CALCULATING DEFLECTIONS FROM OBSERVED STRAINS

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Abstract

Deflections of structural components, such as bridge girders, are often the most difficult to monitor. Strain measurement is relatively simple with the use of electronic strain gauges, fiber optic sensors, or other strain measuring devices. This paper investigates two different methods for predicting or monitoring the deflection of a simply-supported full-scale bridge girder subjected to a partially distributed uniform load using strain measurements. A full-scale pre-stressed concrete bridge girder was instrumented and tested under a static monotonic load in the linear elastic range. This paper compares the experimentally measured deflections along half the length of the girder and compares to theoretically predicted deflections and deflections calculated from observed experimental strains by two different methods. Experimental test results show that estimating deflections from observed strains is feasible within the linear-elastic range of such girders. The methods outlined for predicting deflections of full-scale pre-stressed concrete bridge girders from observed strains are a valuable tool for structural engineers and for the periodic and continuous monitoring of civil structures such as bridges.

INTRODUCTION

In periodic and/or continuous structural health monitoring of structures or structural members, such as bridge girders, deflections tend to be the most difficult to measure and monitor [1]. Traditional devices such as displacement transducers are subject to substantial drift and have to be connected to an “immovable” reference point. Laser-controlled displacement measuring devices are not subject to these disadvantages but are very expensive.

On the other hand, strains are the easiest to measure. This paper investigates two different methods for calculating the deflection of a simply-supported full-scale bridge girder subjected to a partially distributed uniform load using strain measurements. Both methods rely on determining the curvature of the girder from the observed strains. In one of the methods, deflection from theoretical and observed strains are determined using numerical integration. The other method uses harmonic analysis to determine deflection of the girder from theoretical curvatures and curvatures calculated from measured strains. Deflections were continuously measured throughout the test in order to validate deflections predicted from the two models.

EXPERIMENTAL PROGRAM

Girder details

The full-scale pre-stressed concrete bridge girder that was tested was a channel-type girder. It measured 12000 mm in length and had end blocks at both ends for stress transfer from the pre-stressing strands. The overall dimensions of the cross-section were 1200 mm in width by 650 mm in depth. The girders two webs as shown in Figure 1. The reinforcement consisted of 20 - 13 mm diameter low-relaxation pre-stressing strands, 7 - 15M deformed steel bars located in the flange, and 15M deformed steel stirrups. The depth of the flange was 120 mm and both of the webs measured 150 mm in width at the extreme tension fiber with slight tapers on each side.

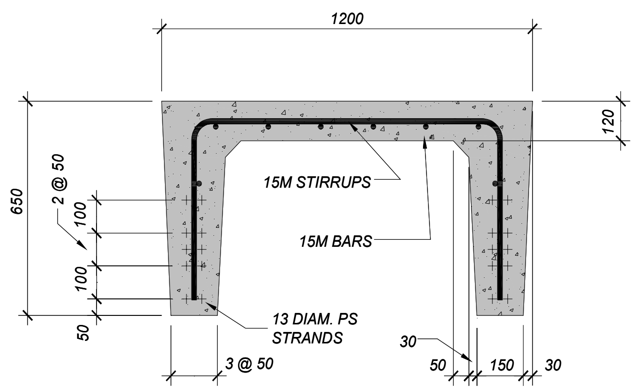


Figure 1: Typical cross-section of full-scale pre-stressed concrete bridge girder (all dimensions shown in mm)

Test set-up

The girder was simply-supported on two concrete block supports with steel rollers, allowing for free rotation, with a center-to-center spacing of 1100 mm (Figure 2a). The girder was subjected to 3-point bending with a steel beam apparatus at mid-span to simulate a partially distributed uniform load over a length of 1670 mm. The spacing of the steel beams was 500 mm center-to-center and each was placed above a neoprene pad. The steel loading apparatus was loaded by two independently controlled hydraulic jacks that were attached to a steel loading frame erected after the girder was placed on the supports (Figure 2b).

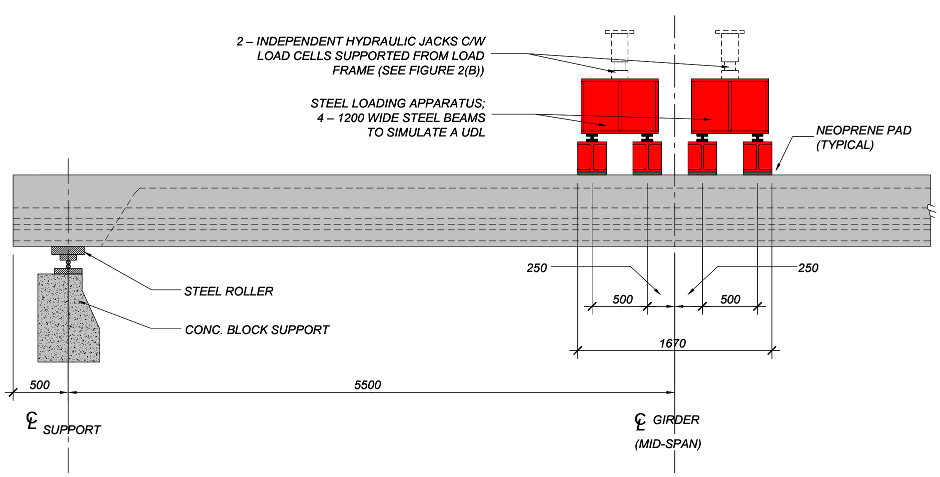


Figure 2a: Elevation of simply-supported beam and loading apparatus

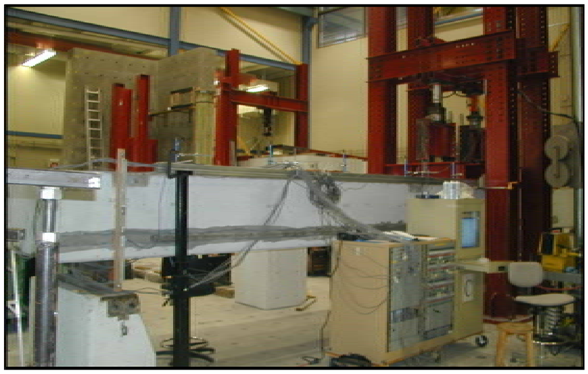


Figure 2b: Photograph of test set-up

Deflection Measurement

Deflections were measured at 6 locations along the left length of the girder. Linear variable displacement transducers (LVDTs) were supported from a free-standing rack and were placed at 500 mm from the support and then spaced every 1000 mm up to mid-span of the girder (Figure 3). They were placed above the girder and were freely resting near the edge of the flange. An LVDT was placed at the same distance from the support where a particular cross-section of strain gauges was located. All of the LVDTs were connected to a data acquisition system via extension cables. In order to minimize the quantity of instrumentation required, it was assumed that the girder would behave symmetrically under the applied load.

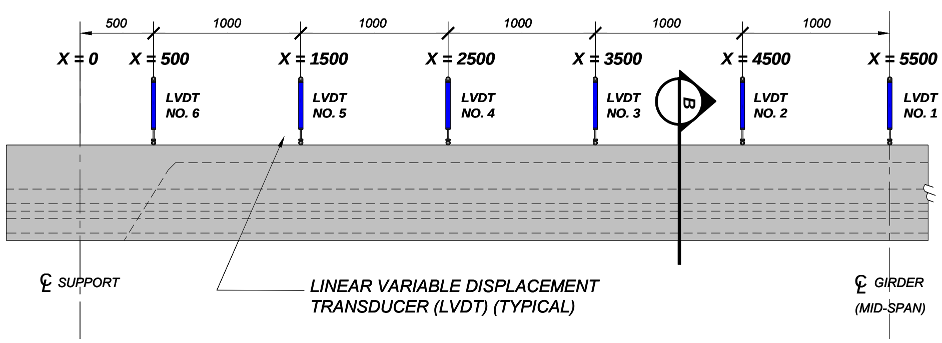


Figure 3: LVDT

Strain Measurement

A large number of strain gauges were required along the length of the girder in order to estimate the magnitude of deflection from recorded strains. The proposed methods for determining deflections from observed strain magnitudes required that strain gauges be placed at several intervals along the girder. A total of 34 electronic strain gauges were required and all of them were installed on the surface of the concrete. Three strain gauges were installed over the support and 500 mm from the support because of the end block. Four strain gauges were installed along the remaining 7 sections, equally spaced at 1000 mm center-to-center (Figure 4a). At each cross-section along the girder one concrete strain gauge with a, gauge length of 50 mm was placed in the longitudinal direction on the top extreme compression fiber of the flange and on the center-line of the girder in the transverse direction. Another strain gauge at the instrumentation section was placed in the same orientation on the underside of the flange and the remaining two gauges, at any given cross-section, were placed on the extreme tension fiber of each of the webs (Figure 4b). Each of the strain gauges was installed using appropriate strain gauge placement techniques and was connected to a data acquisition system.

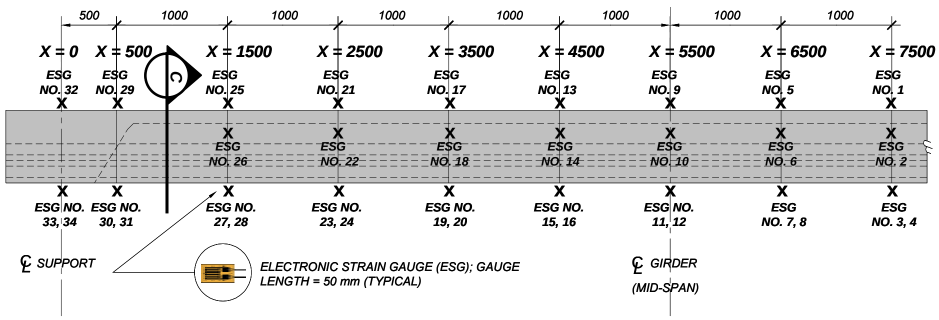


Figure 4a: Locations of electronic strain gauges along the girder

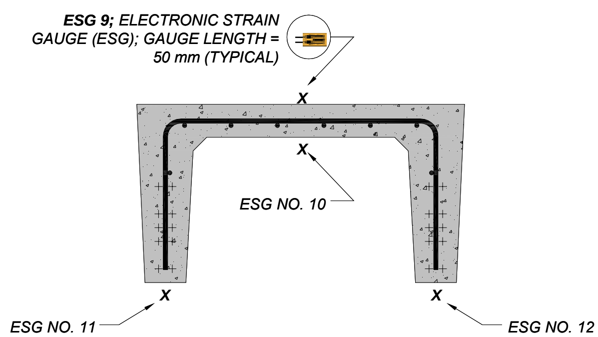


Figure 4b: Locations of electronic strain gauges on cross-section 'C' from Figure 4a

Test Procedure

Researchers felt that the mass of the steel loading apparatus was heavy enough to induce minimal strains and deflections in the girder; and to avoid any experimental error, the steel loading apparatus was removed prior to zeroing all of the instrumentation. After all of the instrumentation was zeroed and deemed to be functioning correctly, the steel loading apparatus was placed back onto the girder and the test was started. The load was applied in 25 kN increments up to a maximum load of 250 kN. At each 25 kN increment, the load was held constant in order to allow the data acquisition system to record several readings. After obtaining a maximum load of 250 kN, the load was removed and the procedure was repeated several times in order to confirm the readings from the instrumentation.

CLASSICAL BEAM THEORY, NUMERICAL THEORY, AND HARMONIC THEORY FOR PREDICTING DEFLECTION

It is noted that classical beam theory works well for specific load conditions, such as the one used in the experiment, and for well defined geometries. Structures such as bridges do not have well defined loading cases, traffic loads are completely random, and geometries can become a lot more complex than those in the laboratory. The other two methods outlined in this section use numerical integration and harmonic analysis to estimate deflections. Classical beam theory may be very difficult to apply to more complicated structures such as bridges and, therefore, the methods of numerical integration of curvatures along with harmonic analysis using curvatures are investigated.

Prediction of Girder Deflection Using Classical Beam Theory for a Simply Supported Girder Subjected to a Partially Distributed Uniform Load

The deflection curve can be derived from the moment curve for a simply supported beam. The set-up for the pre-stressed concrete girder can be simplified and is shown in Figure 5.

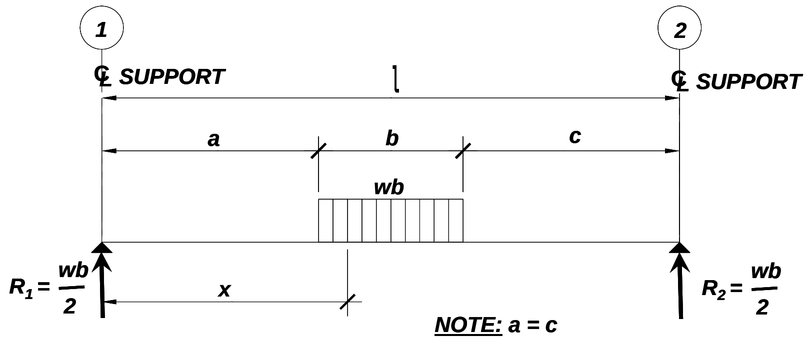


Figure 5: Idealized view of simply supported pre-stressed girder test set-up

The curve for the bending moment of the simply supported girder subjected to a partially distributed UL is given by:

$$EI\frac{d^2y}{dx^2} = M(x) \tag{1}$$

where

$$M(x) = \frac{wbx}{2}; 0 \le x \le a \tag{2}$$

$$M(x) = \frac{wbx}{2} - \frac{w(x-a)^2}{2}; a \le x \le \frac{\ell}{2} \tag{3}$$

Applying double integration to the left and right terms of Equation (1) yield the following equation for deflection:

$$EIy = \delta(x) = \frac{wbx^3}{12} - \frac{w(x-a)^4}{24} - w(\frac{b^3}{24} + \frac{ba^2}{4} + \frac{ab^2}{4})x; a \le x \le \frac{\ell}{2} \tag{4}$$

By substituting the desired distance *x* along the girder, the magnitude of applied load *wb*, the modulus of elasticity *E*, and transformed moment of inertia *I*, the slope and deflection at any point along the girder can be determined.

Prediction of Girder Deflection by Numerically Integrating Theoretical Curvatures

Figure 6 illustrates a simplified view of the girder during its non-deflected and deflected states. Strain gauges were located at 7 different cross-sections.

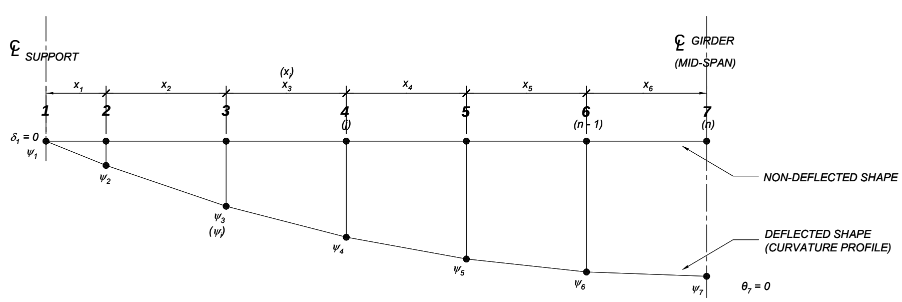


Figure 6: Simplified profile of simply supported girder illustrating principle of numerical integration

The curvature ψ_i can be determined at any cross-section i by the following expression:

$$\psi_i = \frac{\varepsilon_{i(top)} - \varepsilon_{i(btm)}}{d} \tag{5}$$

where $\varepsilon i_{(top)}$ and $\varepsilon_{i(btm)}$ are the top compression and bottom tensile measured strains respectively and d is the distance along the depth of the girder between them. Equation (5) will be used later to determine experimental curvatures. The slope at any given cross-section is simply the numerical integration (area under the curvature curve) of the calculated curvatures at that cross-section from the observed strains. For a simply supported girder, the slope at mid-span is equal to zero. Thus, the equation for slope at any given cross-section can be written as follows:

$$\theta_j = \sum_{i=j}^{n-1} \frac{\psi_i + \psi_{i+1}}{2} x_i \tag{6}$$

where ψ_i and ψ_{i+1} are the calculated curvatures at the different cross-sections along the girder and x_i is the horizontal distance between them. After the slope has been calculated at the respective cross-sections, the slope curve can be numerically integrated over its length starting at the support. The deflection for a simply supported girder at its support is zero and therefore the expression can be written as:

$$\delta_j = \sum_{i=1}^{j-1} \frac{\theta_i + \theta_{i+1}}{2} x_i \tag{7}$$

where θ_i and θ_{i+1} are the slopes at the different cross-sections determined from the calculated curvatures and x_i is the horizontal distance along the girder between the various cross-sections. The deflections were calculated using the theoretical curvatures determined by beam theory outlined in the previous section and were used to validate the method of estimating deflections using numerical integration.

Predicting Deflections Using Harmonic Analysis and Theoretical Curvatures

Estimating deflections using harmonic analysis also uses the same principle for determining curvatures from observed strains as noted in the previous section. The advantage of this method is that it optimizes the number of sections required, i.e. strain gauge locations. If the sections or strain measurement locations are symmetric with respect to mid-span then the central estimate of deflection is excellent, whether the loads are symmetric or asymmetric. It can be shown that the deflection of the girder is approximated by a sine function and is given by:

$$y = c \sin\left(\pi x / L\right) \tag{8}$$

where c is the amplitude of the sine function determined by minimizing the error using a least squares method and providing a best fit using curvatures. The distance along the girder at the location the deflection is to be calculated is represented by x and L is the length of the girder. The magnitude or amplitude of the sine function, defined by the variable c is given by the following equation, where ψ_i is curvature determined at any given section as illustrated in Figure 7.

$$c = \frac{L^2 \sum_{i=1}^{n} \psi_i \sin \pi\left(\frac{x_i}{L}\right)}{\pi^2 \sum_{i=1}^{n} \sin^2 \pi\left(\frac{x_i}{L}\right)} \tag{9}$$

EXPERIMENTAL RESULTS

The experimental noted outlined in this paper only deal with the girder behavior in the linear elastic range. The given are outlined in terms of measured deflections and observed strains.

Deflections

An initial deflection of 0.762 mm was measured due to the weight of the steel loading apparatus which consisted of 4 steel beams across the width of the girder. The maximum observed deflection of 11.318 mm was observed at the mid-span of the girder under a total applied load of 200 kN. The load deflection relationship was linear up to a load of 200 kN. The maximum deflection increased approximately 2.6 mm for every additional 50 kN of total applied load distributed over the width of 1670 mm. The deflections for the girder are listed in Table 1 for loads of 50, 100, 150, and 200 kN.

Table 1. Deflections for the total loads of 50, 100, 150, and 200 kN

	x = 0	x = 500	x = 1500	x = 2500	x = 3500	x = 4500	x = 5500
P = 50 kN	0.000*	0.305	1.122	1.995	2.792	2.991	3.189
P = 100 kN	0.000*	0.530	2.024	3.576	5.069	5.587	5.894
P = 150 kN	0.000*	0.770	2.636	4.913	7.034	7.968	8.357
P = 200 kN	0.000*	1.060	3.551	6.568	9.511	10.758	11.318

*Deflections over support were assumed to be zero

Note: Assuming symmetry, deflection profiles shown are for one half of the pre-stressed concrete girder. Symmetry was assumed.

Strains

At the mid-span, Initial strains of -43, -28, 3, and 9 με were measured on the extreme compression fiber on the underside of the web and at the bottom of the two flanges of the girder respectively, due to the weight of the steel loading appartatus. The strain magnitudes change significantly above the load of approximately 220 kN. A hairline crack at mid-span was observed when the load was at this levelat that time. For the purposes of this paper, methods of determining deflection from observed strains will be limited to the elastic range of the girder, up to a load of 200 kN. The maximum observed compressive strain measured on the extreme compression face of the girder was -289 με and the maximum strain observed on the extreme tension face was 456 με. Figure 7 shows that at a total applied load of 200 kN, the strain distribution over the depth of the girder is linear and the assumption that a plane section remains plane is correct. The experimental top and bottom strains for the extreme compression and tension taces respectively are noted in Table 2.

Table 2: Top (extreme compression fiber) and bottom (extreme tension fiber) strain magnitudes for the total loads of 50, 100, 150, and 200 kN.

Distance Along Girder From Support (x) [mm] / Total Applied Load (P) [kN]	x = 0		x = 500		x = 1500		x = 2500		x = 3500		x = 4500		x = 5500		x = 6500		x =7500	
	ε_{TOP} [με]	ε_{BTM} [με]	ε_{TOP} [με]	ε_{BTM} [με]	ε_{TOP} [με]	ε_{BTM} [με]	ε_{TOP} [με]	ε_{BTM} [με]	ε_{TOP} [με]	ε_{BTM} [με]	ε_{TOP} [με]	ε_{BTM} [με]	ε_{TOP} [με]	ε_{BTM} [με]	ε_{TOP} [με]	ε_{BTM} [με]	ε_{TOP} [με]	ε_{BTM} [με]
P = 50 kN	-26	-19	-28	-3	-38	20	-53	47	-55	74	-84	101	-99	102	-50*	100	-86	81
P = 100 kN	-29	-18	-33	6	-57	48	-86	99	-55	147	-143	199	-160	210	-78*	200	-136	161
P = 150 kN	-32	-18	-39	15	-77	76	-120	153	-141	223	-204	302	-224	325	-106*	304	-188	244
P = 200 kN	-35	-17	-43	25	-96	106	-154	208	-184	299	-266	410	-289	456	-137*	428	-245	336

**The strain gauge located on the extreme compression fiber at a distance of x = 6500 mm from the support was not functioning properly. Therefore, the values shown are strain magnitudes on the underside of the web.*

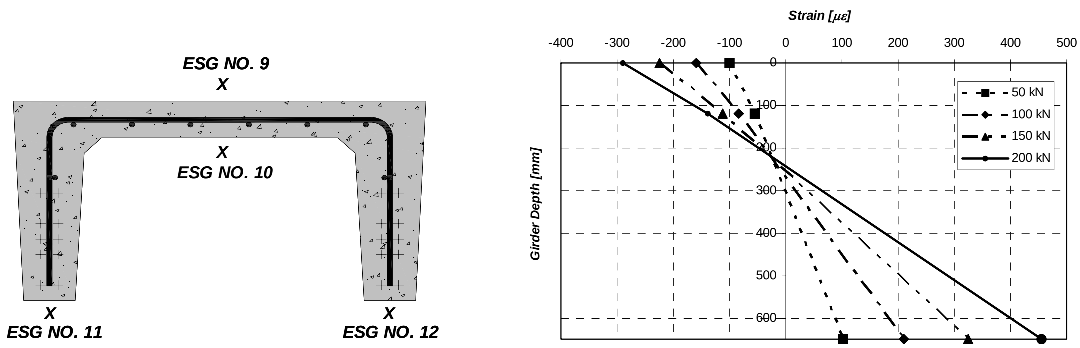


Figure 7: Plot of strain versus depth of girder for strain gauges located at mid-span of the pre-stressed concrete girder.

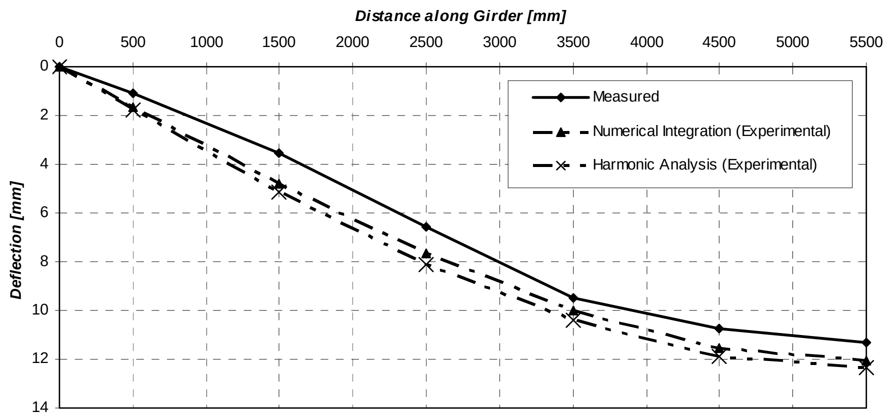
COMPARISON OF EXPERIMENTAL AND ANALYTICAL RESULTS

Numerical integration of theoretical curvatures and harmonic analysis using theoretical curvatures gave results accurate to about 1 % of those given by the classical beam theory. This section notes the results obtained using the numerical integration and harmonic analysis techniques of the theoretical curvatures and experimental curvatures calculated from observed strains and compares them to those measured experimentally. Classical beam theory estimates of deflection along with predictions of deflection using numerical integration and harmonic analysis of theoretical curvatures are compared to measured deflection in Table 3.

Table 3 and Figure 8 compare the experimentally measured deflections to those calculated from numerical integration and harmonic analysis of the experimentally observed strains. Both methods predict deflections with reasonable accuracy given the difficulty of predicting such deflections. Harmonic analysis using experimental curvatures determined from observed strains overestimated the mid-span deflection by 7.6 % and numerical integration also overestimated the actual measured deflection by 6.6 %.

Table 3. Comparison of measured deflections and theoretical deflections

Classical Beam Theory (Total Load = 200 kN)			
Distance Along Girder [mm]	**Deflection, δ (Measured)**	**Deflection, δ (Theoretical)**	**% Difference**
0 (Support)	0	0.000	0.0
500	1.06	1.900	79.3
1500	3.551	5.574	57.0
2500	6.568	8.867	35.0
3500	9.511	11.525	21.2
4500	10.758	13.295	23.6
5500 (Mid-span)	11.318	13.927	23.1
Numerical Integration of Theoretical Curvatures (Total Load = 200 kN)			
Distance Along Girder [mm]	**Deflection, δ (Measured)**	**Deflection, δ (Theoretical)**	**% Difference**
0 (Support)	0	0.000	0.0
500	1.06	1.886	77.9
1500	3.551	5.515	55.3
2500	6.568	8.764	33.4
3500	9.511	11.377	19.6
4500	10.758	13.102	21.8
5500 (Mid-span)	11.318	13.710	21.1
Harmonic Analysis using Theoretical Curvatures (Total Load = 200 kN)			
Distance Along Girder [mm]	**Deflection, δ (Measured)**	**Deflection, δ (Theoretical)**	**% Difference**
0 (Support)	0	0.000	0.0
500	1.06	1.998	88.5
1500	3.551	5.834	64.3
2500	6.568	9.196	40.0
3500	9.511	11.813	24.2
4500	10.758	13.474	25.2
5500 (Mid-span)	11.318	14.043	24.1



Note: Deflection profiles shown are for one half of the pre-stressed concrete girder. Symmetry was assumed.

Figure 10: Measured, numerical integration of experimental curvature, and harmonic analysis using experimental curvature deflection profiles for the total applied load of 200 kN

CONCLUSIONS AND RECOMMENDATIONS

Experimental test results obtained from the static testing of a full-scale pre-stressed concrete bridge girder subjected to a partially distributed uniform load indicated that estimating deflections from observed strains is feasible within the linear-elastic range of such girders. Beam theory for a simply supported predicted a mid-span deflection of 13.927 mm compared to the actual measured deflection of 11.318 mm. The proposed methods of predicting deflections along the length of the girder due to an applied load from observed strains also predicted deflections with smaller accuracy given the difficulty of predicting such deflections. Mid-span deflections of 12.068 and 12.178 mm were estimated using numerical integration and harmonic analysis of calculated curvatures from observed strain magnitudes respectively. Both methods depend upon curvatures determined from measured strains suggesting that the accuracy of strain measuring devices used for the test do have a direct effect on the accuracy of deflection predictions. Although fiber optic sensors were also used to measure strains, the results are not included in this paper.

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REFERENCES

1. Kim, N.-S. and Cho, N.-S. 'Estimating deflection of a simple beam model using fiber optic Bragg-grating sensors', *Experimental Mechanics*, 44(4)(2004) 433-439.