The 3rd International Conference on Structural Health Monitoring of Intelligent Infrastructure
Vancouver, British Columbia, Canada
November 13-16, 2007

IDENTIFICATION OF MOVING LOADS THROUGH BRIDGE DYNAMIC RESPONSES

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Abstract

A force identification algorithm is proposed for estimating axle loads of trucks moving over a bridge. A bridge is modeled as a beam in the current study. Accelerations are measured at multiple locations along the bridge and time histories of displacements are computed from measured strains at the same locations. Stiffness and mass are considered as already identified, and damping is neglected due to a relatively short passing period of a vehicle over a bridge. Part of data measured while the truck is moving on the bridge is used. Nodal force vectors are computed by using a database of equivalent nodal force transformation matrix at different speeds. The algorithm was examined through laboratory experiments on a single span model bridge.

INTRODUCTION

It is acknowledged that correct identification of moving loads over a bridge is important for the adequate maintenance of bridges.[6] Many experimental studies on bridges have been carried out for similar purposes whose main issues are to identify correct wheel loads and to estimate live load effects on a bridge. WIM(weigh-in-motion) is a broad coverage over this area. WIM can be categorized by two types; embedded WIM and BWIM(bridge WIM). Different from embedded WIM as a direct measure of moving loads, BWIM estimates moving loads indirectly by considering a bridge as a balance. Wheel loads are identified inversely by measuring responses of a bridge at selected locations.[8,9] Most available BWIM algorithms have applied influence lines of moment or strain.[5,7] Since a bridge behaves dynamically under passing vehicles, however, such static algorithms definitely provide inherent errors.

The current study proposes an algorithm for identifying moving loads using the dynamic governing equation. Use of dynamic governing equation can alleviate the effect of vibration but still cannot resolve the problem related to the incomplete measurement in space, state, and noise.[8] The proposed algorithm utilizes acceleration time histories measured at limited locations and computes displacement analytically from dynamic strain measured at the same locations as acceleration. Time histories of velocity were not computed because damping was ignored in the algorithm by assuming that a vehicle passes a bridge in a relatively short period. The most distinguishable difference between available algorithms and the proposed one may be the use of measured acceleration time history.

The proposed algorithm utilizes a beam model identified through a system identification method using measured responses. Nodal force transformation matrix is also proposed to transform a moving load into an equivalent nodal force. Since vehicles pass a bridge at different speeds and the resulting equivalent nodal forces vary depending on the speed, a ready-made database of nodal force transformation matrix at each different vehicle speed was prepared and applied to estimate moving loads inversely. To examine the developed algorithm, laboratory tests were carried out with the cases of one vehicle and two vehicles moving at a constant speed.

FORCE IDENTIFICATION ALGORITHM

Dynamic Governing Equation

It has been assumed that mass and stiffness properties can be identified through an application of system identification. It has been also assumed that damping effects can be neglected by noticing that a truck usually passes over a bridge in a relative short period. Thus, the dynamic governing equation can be reduced to Eq.(1) with known structural matrices of **M** and **K**.

$$\mathbf{M}\ddot{\mathbf{u}}(t) + \mathbf{K}\mathbf{u}(t) = \mathbf{f}(t) \qquad (1)$$

where $\mathbf{M}(N\times N)$, $\mathbf{K}(N\times N)$ = mass and stiffness, $\ddot{\mathbf{u}}, \mathbf{u}, \mathbf{f}$ = acceleration, displacement, and force vector at time t, respectively.

Computation of Unmeasured Acceleration

To consider the incompleteness in space when accelerations are measured at limited DOF, acceleration can be divided into measured and unmeasured parts at each time step and can be related to the modal information as Eq.(2).

$$\ddot{\mathbf{u}}(t) = \left\{ \ddot{\mathbf{u}}_m(t) \quad \ddot{\mathbf{u}}_u(t) \right\}^T = \left[\mathbf{\Phi}_m \quad \mathbf{\Phi}_u \right]^T \ddot{\mathbf{q}}(t) = \mathbf{\Phi}\ddot{\mathbf{q}}(t) \qquad (2)$$

where $\ddot{\mathbf{u}}_m, \ddot{\mathbf{u}}_u$ = measured and unmeasured part of acceleration vector at time step t, $\mathbf{\Phi}_m, \mathbf{\Phi}_u$ = measured and unmeasured part of mode shape matrix $\mathbf{\Phi}$, $\ddot{\mathbf{q}}$ = double differentiation of generalized coordinate vector with respect to t, respectively. By the least squared, acceleration response at unmeasured DOF can be formulated as Eq.(3).

$$\ddot{\mathbf{q}} = \left[\mathbf{\Phi}_m^T \quad \mathbf{\Phi}_m \right]^{-1} \mathbf{\Phi}_m^T \ddot{\mathbf{u}}_m, \; \ddot{\mathbf{u}}_u = \mathbf{\Phi}_u \ddot{\mathbf{q}} \qquad (3)$$

Computation of Displacement

Displacements are computed from measured dynamic strain data locating at the same locations of accelerometers. From the beam theory, the following relationship of Eq.(4) can be easily formulated.

$$\boldsymbol{\varepsilon}_m(x,t) = -\frac{h}{2}\mathbf{u}_m''(x,t) = -\frac{h}{2}\mathbf{\Phi}_m''(x)\mathbf{q}_\varepsilon(t) \qquad (4)$$

where $\boldsymbol{\varepsilon}_m$ = strain data measured at the same locations of accelerometers, $\mathbf{u}_m'', \mathbf{\Phi}_m''$ = double differentiation of displacement vector and mode shape matrix with respect to the coordinate x in the longitudinal direction, and $\mathbf{q}_\varepsilon$ = generalized coordinate related to the strain, respectively.

Transformation of Moving Loads to Equivalent Nodal Force

The force vector **f** in Eq.(1) is an equivalent nodal force in a finite element model. Since actual loads are moving over a bridge, however, it is required to relate moving loads to the equivalent nodal force as Eq.(5). The actual loads can be obtained by the least squared formulation of Eq.(6).

$$\mathbf{f}(t) = \mathbf{T}(t,v)\mathbf{p}(t) \qquad (5)$$

$$\mathbf{p}(t) = \left[\mathbf{T}(t)^T \mathbf{T}(t) \right]^{-1} \mathbf{T}(t)^T \mathbf{f}(t) \qquad (6)$$

where **T** = transformation matrix between the equivalent nodal force **f** and actual loads **p** in a constant velocity v.

LABORATORY EXPERIMENTAL STUDY

To examine the applicability of the proposed algorithm, laboratory experiments were carried out. The model steel bridge of 6*m* in length and the model vehicle are shown in Figure 1. The model truck was controlled to move at a constant speed on the bridge. The distance between two axles of a model truck was 30*cm* which is too short as 1/20 of the length of the bridge to identify both axles reasonably. Accelerations were measured to the vertical direction at 5 locations of equal distance on both side surfaces. Longitudinal strains by fiber optic sensors were also measured at the same locations of the accelerometers below the side flanges.

Figure 1. Laboratory model steel bridge and model vehicle

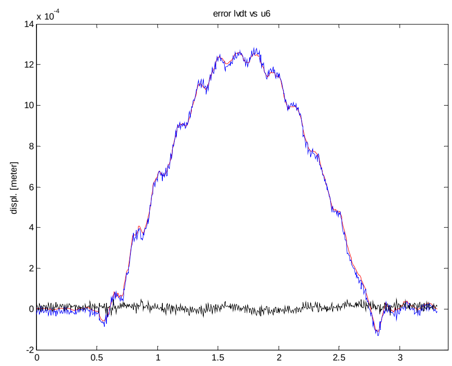
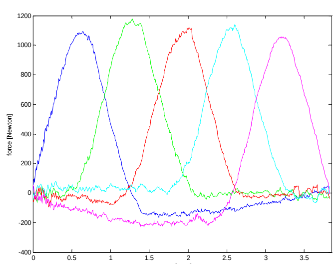


Figure 2. Comparison of displacements from LVDT and those computed from strains



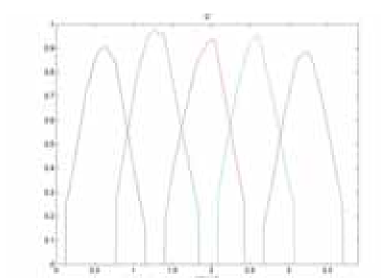
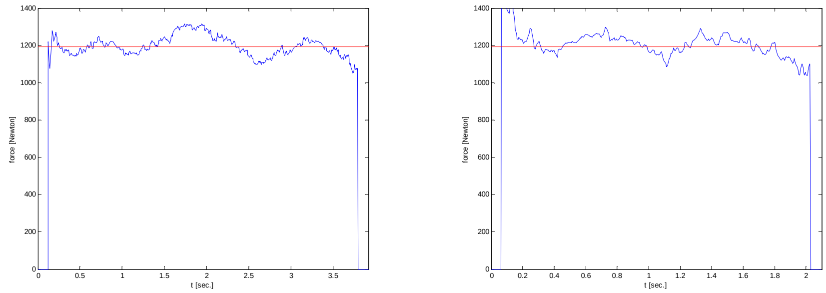


Figure 3. Computed nodal forces

Figure 4. Transformed nodal forces

Before carrying out the moving load tests, mass and stiffness properties of the beam model were identified by a system identification scheme using measured modal data obtained from impact hammer tests. Figure 2 compares the displacements measured from LVDT and those computed from measured strain data. The results demonstrate the usefulness of the strain measurements with narrow gap between the two displacements even though negligibly small fluctuation of displacements computed from strain data could be observed. Figure 3 shows the computed nodal forces from the dynamic governing equation of Eq.(1) and the nodal force data for the transformation matrix are illustrated in Figure 4.



(1.54m/sec) (2.85m/sec)

Figure 5. Identified forces of a moving truck with two different speeds

Table 1. Identification error corresponding to truck velocity

case	velocity (m/sec)		AE (%)	RMS (%)
I (one truck)	1.5		0.92	4.41
	3.0		1.26	3.70
II (two trucks)	1.0	truck 1	3.15	7.93
		truck 2	2.00	4.07
	2.0	truck 1	0.33	7.55
		truck 2	0.89	3.84

Case I: One Model Truck

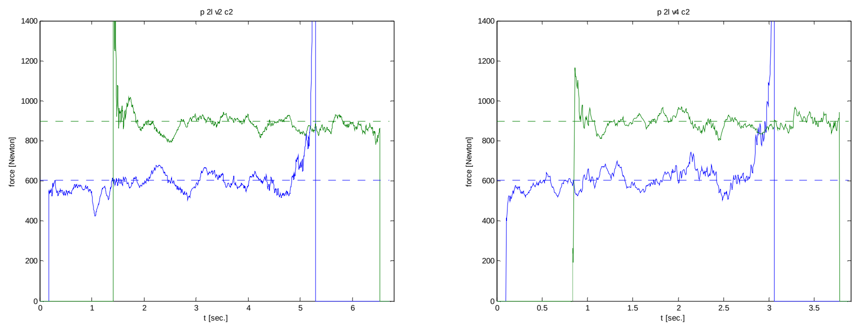
As the first trial case, one model truck of 1.2kN moved on the bridge with various speeds. Dynamic responses of accelerations and strains were measured and the truck load was identified as a single load along the beam. The identified results are drawn in Figure 5 for the speed of 1.54m/sec and 2.85m/sec, respectively. Regardless of the truck speeds, the trends of the two cases are almost identical. In both speeds, the identified forces gradually decrease as the truck approaches to the end of the bridge. The identification errors are summarized in Table 1 with errors defined by Eq.(7).

$$AE = \left| \frac{\sum_i p(t_i) - \sum_i \tilde{p}(t_i)}{\sum_i \tilde{p}(t_i)} \right| \times 100(\%)\,, \quad RMS = \frac{\sqrt{\sum_i \left(p(t_i) - \tilde{p}(t_i) \right)^2}}{\sqrt{\sum_i \tilde{p}(t_i)^2}} \times 100(\%) \qquad (7)$$

where $p(t_i), \tilde{p}(t_i)$ = estimated and actual loads at time step t_i.

Case II: Two Model Trucks

Two trucks were linked together by a buckle so that two trucks moved at the same constant speed over the bridge. Since the axle distance of each truck is too small, two trucks were identified as two concentrated loads. The weight of the front and the rear trucks were 0.6kN and 0.9kN for the experiments, respectively. The identified results are drawn in Figure 6 for the speed of 1.11m/sec and 1.91m/sec, respectively. From the figure, we can observe that the identification of the front load sharply jumped when the first load moved out of the bridge while that of the rear load sharply jumped when the rear load entered the bridge. Both numerical jumps occurred when the number of estimated loads was suddenly changed. The identification errors are also summarized in Table 1 with *AE* and *RMS* errors quantitatively. The results demonstrate that the truck loads could be reliably identified within a tolerable error even with those sharp jumps.



(1.11*m*/*sec*) (1.91*m*/*sec*)

Figure 6.Identified forces of two moving trucks with two different speeds

CONCLUSIONS

A new BWIM algorithm is presented based on the dynamic motion equation and have been examined through laboratory experimental studies. It has been assumed that an analytical model for the bridge and its mass and stiffness properties were reasonably identified before applying the algorithm. Damping was neglected by assuming that the passing period of moving loads over a bridge is relatively short to take into account of damping effects on the dynamic responses. Incompleteness of measured data in space and state was considered in developing the algorithm. Since a discretized analytical model has been used to express the dynamic behaviors of a bridge, a relationship between actual wheel loads and equivalent nodal forces on the model had to be formulated. A database of transformation matrix between wheel loads and equivalent nodal forces were constructed as a function of the speed of moving loads.

The usefulness of the algorithm could be demonstrated through laboratory tests with reliable results. Regardless of the number of trucks running over the model bridge with various constant speeds, the truck loads could be identified with *AE* error less than 3% and *RMS* error less than 8% in the trial cases.

ACKNOWLEDGEMENT

The study was supported by Korea Science and Engineering Foundation (KOSEF) under the Grant No. R01-2004-000-11009-0.

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