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BRIDGE CONDITION ASSESSMENT USING A MOVING VEHICLE AS AN ACTUATOR

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Abstract

As for the assessment of bridge health condition, this paper describes an inverse problem solving for unknown material parameters using a moving vehicle as an actuator for bridge vibration. An object function for the element stiffness index which indicates the ratio of damaged flexural rigidity of the finite element of a beam to undamaged one is subtracted from a pseudo-static formulation of equations of motion for a bridge-vehicle interactive system. The Damped Least Squares (DLS) and Total Least Squares (TLS) minimizations are adopted for optimization methods to solve the inverse problem. An analytical study suggests feasibility of the proposed method for the bridge-condition assessment.

INTRODUCTION

The potential economic and life-safety implications of early diagnosis investigation in structures have motivated a considerable amount of research in structural health monitoring (SHM) based on vibrational data. Structures in many engineering fields are examined through periodic monitoring with the intention of minimizing the safety risk on the one hand and lowering maintenance costs to the greatest extent possible on the other hand by carrying out rehabilitation at appropriate times. For countries located in earthquake prone regions, after earthquakes, structural health monitoring is useful for rapid condition screening. It is also intended to provide reliable information regarding structural integrity very rapidly.

An important problem that must be solved in bridge health monitoring (BHM) using vibration measurements is how to excite the bridge economically, reliably and rapidly. Ambient vibrations induced by traffic and wind have been adopted as dynamic data for BHM (e.g. [1, 2]).

The use of a moving vehicle or train as a source of bridge vibration is especially valuable for bridge structures. An advantage of using a moving vehicle as a dynamic source for BHM might be ready excitement of the bridge. Another important point is that some highway bridges are tested before and during in-service periods through moving vehicle tests. Those traffic-induced vibration data of bridges are available for their condition screening.

This study presents an application of the methodology developed by the authors' [3, 4] for condition screening of bridges using a moving vehicle as a dynamic source of vibration. First, this paper describes a methodology for condition screening of bridges using a moving vehicle as a dynamic source of brief vibration. Damped Least Squares (DLS) [5] and Total Least Squares (TLS) [6] minimization methods are adopted as optimization tools to solve the inverse problem contaminated by noises. Feasibility of the proposed method is verified using a numerical example.

CONDITION ASSESSMENT METHODOLOGY

Equations of Motion for Bridge under Moving Vehicle

The combination of the interaction force at a contact point between the bridge and vehicle provides equations of motion for a bridge-vehicle interactive system [7]. The compact matrix formation of the interactive system is writable as

$$\begin{bmatrix} \mathbf{M}_{br} & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_{vv} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{q}}_r(t) \\ \ddot{\mathbf{q}}_v(t) \end{Bmatrix} + \begin{bmatrix} \mathbf{C}_{br} + \mathbf{C}_{cvb}(t) & \mathbf{C}_{bv}(t) \\ \mathbf{C}_{bv}^T(t) & \mathbf{C}_{vv} \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{q}}_r(t) \\ \dot{\mathbf{q}}_v(t) \end{Bmatrix} + \begin{bmatrix} \mathbf{K}_{br} + \mathbf{K}_{cvb}(t) & \mathbf{K}_{bv}(t) \\ \mathbf{K}_{bv}^T(t) & \mathbf{K}_{vv} \end{bmatrix} \begin{Bmatrix} \mathbf{q}_r(t) \\ \mathbf{q}_v(t) \end{Bmatrix} = \begin{Bmatrix} \mathbf{f}_{br}(t) \\ \mathbf{f}_{vv}(t) \end{Bmatrix} \quad (1)$$

Therein, $\mathbf{C}_{cvb}(t) \in \mathrm{R}^{nr\times nr}$ and $\mathbf{K}_{cvb}(t) \in \mathrm{R}^{nr\times nr}$ respectively denote the contribution of vehicle's damping and stiffness to those of the bridge. $\mathbf{C}_{bv}(t) \in \mathrm{R}^{nr\times nv}$ and $\mathbf{K}_{bv}(t) \in \mathrm{R}^{nr\times nv}$ are the coupled damping and stiffness matrices between the bridge and vehicle systems, where nr and nv respectively denote the DOFs of the bridge and vehicle models. The respective mass, damping and stiffness matrices for the vehicle are $\mathbf{M}_{vv}\in\mathrm{R}^{nv\times nv}$, $\mathbf{C}_{vv} \in \mathrm{R}^{nv\times nv}$ and $\mathbf{K}_{vv} \in \mathrm{R}^{nv\times nv}$. In addition, $\mathbf{q}_v(t) \in \mathrm{R}^{nv}$ and $\mathbf{f}_{vv}(t) \in \mathrm{R}^{nv}$ respectively indicate displacement and force vectors of the vehicle. The superscript T denotes the matrix transposition.

The system damping and stiffness matrices in Eq.(1) usually consist of time-varying coefficients [7]. Consequently, the conventional frequency domain approaches are not directly relevant to assess the current bridge's health condition based on the vibration data taken from a moving vehicle test.

Pseudo-Static Formulation

Important assumptions used through this study, to make the methodology easy to understand and the numerical example simple, are: damage causes a change of the bending rigidity to simplify the derivation; the mass and damping matrices of a bridge are assumed to be unaffected by damage; and parameters of the intact bridge and vehicle model are estimated initially through vibration experiments.

Subtracting linear equations for stiffness of a bridge from Eq.(1) of the bridge-vehicle interactive system yields Eq.(2) of a pseudo-static formulation to catch the change of stiffness of bridge structures [3, 4].

$$\mathbf{K}_{br}\mathbf{q}_r(t) = \mathbf{f}(t) \tag{2}$$

Therein, $\mathbf{f}(t) \in \mathbf{R}^{nr}$, and the change of stiffness $\mathbf{K}_{br}$ in Eq.(2) provides information about the bridge's current health condition, and to detect the change in $\mathbf{K}_{br}$ is the basic concept of the bridge condition assessment of this study.
The change of the element stiffness is obtainable using the element stiffness index (ESI) defined as

$$\mu_e = (E_e I_e)_d / (E_e I_e)_i \tag{3}$$

where $E_e I_e$ denotes the bending rigidity of the e-th element. The subscripts d and i respectively indicate the damaged and intact states. A noteworthy point is that the ESI value is unity for an intact bridge: $\mu_e = 1$ for $e = 1, \cdots, M$. M is the number of elements in a bridge model.

If matrix $\mathbf{L}_e \in \mathbf{R}^{2nf \times N}$ provides the assembly operator with an element that transforms the element stiffness matrix to a structural stiffness matrix in which nf denotes the number of DOFs at a node of an element, then the structural stiffness matrix can be written as

$$\mathbf{K}_{br} = \sum_{e=1}^{M} \mu_e \mathbf{L}_e^T \mathbf{K}_{be} \mathbf{L}_e \tag{4}$$

where $\mathbf{K}_{be} \in \mathbf{R}^{2nf \times 2nf}$ is the intact element stiffness matrix in global coordinates.

Substituting the relationship in Eq.(4) into Eq.(2) of the pseudo-static formulation gives the governing equation of μ_e as

$$\sum_{e=1}^{M} \mu_e \mathbf{h}_e(t) = \mathbf{f}(t) \tag{5}$$

where $\mathbf{h}_e(t) \in \mathbf{R}^{nr}$ is a coefficient vector of the e-th element at time t.

If $\mathbf{x} = \{\mu_1; \mu_2; \ldots; \mu_M\}$, $\mathbf{x} \in \mathbf{R}^{M}$, gives the vector of ESI of the bridge and $\mathbf{H}(t) = [\mathbf{h}_1(t)\ \mathbf{h}_2(t) \ldots \mathbf{h}_M(t)]$, $\mathbf{H}(t) \in \mathbf{R}^{nr \times M}$, is a coefficient matrix of a bridge model at time t then Eq.(5) can be condensed as a matrix formation for measured data of mt samples taken from a moving vehicle test.

$$\hat{\mathbf{A}}\mathbf{x} = \hat{\mathbf{b}} \tag{6}$$

where, $\hat{\mathbf{A}} = [\mathbf{H}(t_0); \ldots; \mathbf{H}(t_{mt-1})]$, $\hat{\mathbf{A}} \in \mathbf{R}^{nq \times M}$, in which $nq = nr \times mt$, is the coefficient matrix and the observation vector $\hat{\mathbf{b}} \in \mathbf{R}^{nq}$ is defined as $\hat{\mathbf{b}} = [\mathbf{f}(t_0); \ldots; \mathbf{f}(t_{mt-1})]$.

Equation (6) denotes the pseudo-static formulation in the form of a linear system of equations subtracted from equations of motion for the bridge-vehicle interaction without any noise.

If the linear equation in Eq.(6) is contaminated by noise, then it is rewritable as

$$\mathbf{A}\mathbf{x} \approx \mathbf{b} \tag{7}$$

where $\mathbf{A} \in \mathbf{R}^{nq \times M}$, with $M < nq$, is the coefficient matrix with noise, $\mathbf{b} \in \mathbf{R}^{nq}$ is the observation vector with noise and $\mathbf{x} \in \mathbf{R}^{M}$ is the unknown vector.

OPTIMIZATION METHOD

A simple solution to the linear equation in Eq.(7) is to use the Tikhonov regularization [5] as one of the damped least-squares (DLS) minimization. The DLS minimization of $\mathbf{Ax} \approx \mathbf{b}$ is given as

$$\min \left\{ \|\mathbf{Ax}-\mathbf{b}\|_2 - \lambda \|\mathbf{x}-\mathbf{x}_0\|_2 \right\} \tag{8}$$

where, $\| \ \|_2$ denotes the 2-norm of a vector. The second term is the side constraint which stabilizes the problem and singles out a useful and stable solution. The regularization parameter λ controls the weight given to minimization of the side constraint relative to minimization of the residual norm. The L-curve method [8] is used in this study to choose the optimal regularization parameter. The vector $\mathbf{x}_0$ is a priori estimate of the solution $\mathbf{x}$. As a priori, unit vector is adopted because the ESI value of bridge is assumed to be less than or equal to the unit value.

The TLS approach [6] possesses superior noise rejection properties, differently from the DLS method, in the problem that all of the measurement uncertainty is associated with both data and observations. The TLS problems take into account independent noises or errors in both data and observations under the assumption of identical variances. The TLS problem seeks to

$$\min \left\| [\mathbf{A} \ \ \mathbf{b}] - [\hat{\mathbf{A}} \ \ \hat{\mathbf{b}}] \right\|_F \tag{10}$$

where $\| \ \|_F$ denotes the Frobenius norm of a matrix.

One of the easiest way to obtain the minimum norm solution might be the singular value decomposition (SVD) approach, and $[\hat{\mathbf{A}} \ \ \hat{\mathbf{b}}]$ is estimated as the matrix of rank l closest to $[\mathbf{A} \ \ \mathbf{b}]$. This is also a form of regularization. Once a minimizing $[\hat{\mathbf{A}} \ \ \hat{\mathbf{b}}]$ is found, then any $\mathbf{x}$ satisfying the following Eq.(11) is called a TLS solution.

$$\hat{\mathbf{A}}\mathbf{x} = \hat{\mathbf{b}} \tag{11}$$

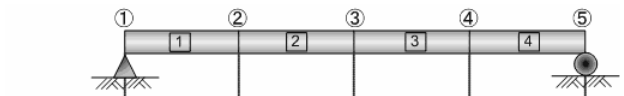
The health condition of bridge structures is detectable directly from investigating the ESI vector $\mathbf{x}$. Determination of the ESI vector can also provide information for damage location and its severity.

NUMERICAL EXAMPLE

Numerical Simulation

Dynamic responses of the bridge and vehicle taken from the simulation are assumed as measured data. The simulation for the traffic-induced vibration of the bridge is based on the modal analysis. Dynamic equations for the bridge-vehicle interactive system are solved by using Newmark's β method as a direct integration method. The value of 0.25 is used for β. The solution is obtainable with a relative margin of error of less than 0.001. One-fifth of the natural period of the highest mode (third mode: 19.85Hz) considered in the simulation is used as the time interval in analysis; that is, 0.01 second is used as the time interval. The roadway surface roughness measured at the real bridge of the bridge model is used in the numerical simulation. During the simulation the speed of vehicle is assumed to be 20 km/h.

A bridge with span length of 40.4m, which is idealized as a beam element with 2 DOFs at each node, is adopted as the bridge model. The moment of inertia and weight per unit length are, respectively, 0.2197m^4 and 74.09kN/m. The mass per unit length is 7.552 ton/m. The damping constant of the bridge is assumed as 0.0253 for the first two modes. Figure 1 shows the FE model of the observation-bridge with damage scenarios. Therein the abbreviation SCN denotes scenario, and the values in scenarios stand for the expected normalized changes in the bending rigidity of each member such as the ESI value: for example 0.80 denotes the damaged element with 20% loss of the cross sectional moment of inertia compared with that of the intact one.



Scenario I:	1.00	**0.80**	1.00	1.00
Scenario II:	1.00	**0.80**	**0.80**	1.00
Scenario III:	**0.90**	1.00	1.00	**0.80**

Figure 1. Bridge model and damage scenarios.

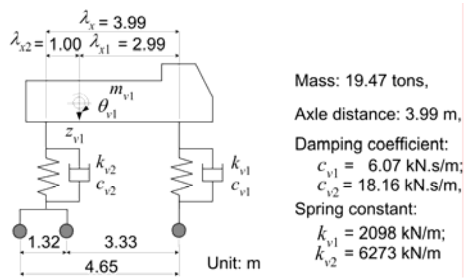


Figure 2. Vehicle model.

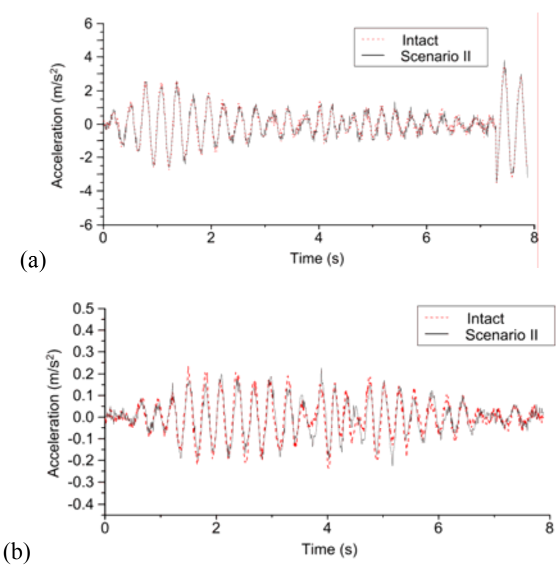


Figure 3. Example of dynamic responses of the vehicle and bridges taken from the simulation under a vehicle respectively running on the intact and damaged (Scenario II) bridges with speed of 20km/h; (a) acceleration of vehicle's bounce motion; and (b) acceleration of bridge at span centre.

A dump truck is idealized as 2DOFs system (see Figure 2) to simplify the numerical example. Properties of the dump truck used for this study are: the truck's gross weight is 191kN, and the respective axle and tandem axle distances are 3.99 m and 1.32 m; the respective spring constants of the front and rear axles are 2098kN/m and 6273kN/m; and, as the damping constant, 6.07kN s/m for the front axle and 18.16kN s/m for the rear axle are used. Dynamic responses of the damaged bridge are simulated by reducing the cross sectional moment of inertia of elements according to the damage scenario shown in Figure 1. To consider uncertainty in both **A** and **b** of Eq.(7), simulated Gaussian noises are added to calculated dynamic responses that contribute to both **A** and **b**. Noise levels of 10% and 20% are adopted, in which the noise level is defined as the ratio of root mean squares (RMS) of measurement noise to the RMS value of the calculated noiseless dynamic responses.

Time histories polluted by the noise of 20% noise level are shown in Figure 3, which demonstrates an example of dynamic responses of the vehicle and bridge attributable to bridge health conditions.

Bridge Condition Assessment

Estimated ESI values using simulated dynamic responses of the bridge and vehicle are summarized in Tables 1, 2 and 3, where the abbreviation NL stands for the Noise Level, and the Error is estimated from the difference between the estimated ESI value and the reference one shown in Figure 1.

Table 1. Estimated bridge health condition for damage scenario SCN-I

NL	Optimization method		Element 1	Element 2	Element 3	Element 4
10%	DLS	Estimated	0.9498	0.7994	0.9646	1.0213
		Error (%)	-5.02	-0.01	-3.54	2.13
	TLS	Estimated	0.9687	0.7947	0.9837	1.0255
		Error (%)	-3.13	-0.66	-1.63	2.55
20%	DLS	Estimated	0.9215	0.7307	0.9140	0.9247
		Error (%)	-7.85	-8.66	-8.60	-7.53
	TLS	Estimated	0.9620	0.7892	0.9618	1.0281
		Error (%)	-3.80	-1.35	-3.82	2.81

Table 2. Estimated bridge health condition for damage scenario SCN-II

NL	Optimization method		Element 1	Element 2	Element 3	Element 4
10%	DLS	Estimated	0.9897	0.8094	0.8138	0.9861
		Error (%)	-1.03	1.18	1.73	-1.39
	TLS	Estimated	0.9872	0.7995	0.8073	0.9784
		Error (%)	-1.28	-0.06	0.91	-2.16
20%	DLS	Estimated	0.9397	0.7629	0.7679	0.9323
		Error (%)	-6.03	-4.64	-4.01	-6.77
	TLS	Estimated	0.9916	0.7874	0.8012	0.9700
		Error (%)	-0.84	-1.58	0.15	-3.00

Table 3. Estimated bridge health condition for damage scenario SCN-III

NL	Optimization method		Element 1	Element 2	Element 3	Element 4
10%	DLS	Estimated	0.8705	1.0061	0.9784	0.7999
		Error (%)	-3.28	0.61	-2.16	-0.01
	TLS	Estimated	0.8929	1.0010	0.9979	0.7812
		Error (%)	-0.79	0.10	-0.21	-2.35
20%	DLS	Estimated	0.8165	0.9327	0.9158	0.7496
		Error (%)	-9.28	-6.73	-8.42	-6.30
	TLS	Estimated	0.9223	0.9659	1.0113	0.7519
		Error (%)	2.48	-3.41	1.13	-6.01

Observations from the tables show that the DLS approach gives a solution within the error less than about 5% in the case of considering perturbation with noise level of 10%. In contrast, the TLS approach provides a solution of error less than 3.2%.

In considering 20% noise level, the identification error grows up to about 9.3% by means of the DLS approach. The TLS approach gives a good solution with the error less than 3.8% even in the case of the noise level of 20%. This is because the TLS minimization possesses superior noise rejection properties in the problem of independent noises in both data and observations under the assumption of identical variances. However, it is noteworthy that the TLS approach adopted in this study is based on the SVD and becomes prohibitive when the dimensions of **A** become large because the SVD algorithm is of complexity.

An interesting result should be described is that the proposed method provides identification of the suspected damage members.

CONCLUSIONS

This study presents the feasibility of the methodology for condition screening of bridges using a moving vehicle as a dynamic source of vibration through a numerical example. Observations reveal that not only the suspected damaged member, but also severity of the damage are detectable using the proposed method. Investigations also suggest the feasibility of the proposed method for damage identification of bridges. It is also observed that, even though the DLS minimization that is one of the methods of Tikhonov regularization provides good identification results, the TLS approach gives the better estimate for the problem of the noise associating with both coefficient matrix and observation vector in comparison with those of DLS approach.

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