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# BACK ANALYSIS TECHNIQUE FOR THE ESTIMATION OF TENSION FORCES ON HANGER CABLES

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## Abstract

In general, the tension forces of hanger cable in suspension bridges play an important role in evaluating the bridge conditions. The vibration method has been widely applied to estimate the tension forces by using the measured frequencies on hanger cables. However, the vibration method is not applicable to short hanger cables because the frequencies of short cables are severely sensitive to flexural rigidity. Thus, in this study, the tension forces of short hanger cables, of which the length is shorter than 10 meters, were estimated through back analysis of the cable frequencies measured from Gwangan suspension bridge in Korea. The univariate method is able to search the optimal tension forces without regard to the initial ones and has a rapid convergence rate. To verify the feasibility of back analysis, the results from back analysis and the vibration method are compared with the design tension forces. From the comparison, it can be inferred that back analysis results are in more reasonable agreement with the design tension forces of short hanger cables. Therefore, it is concluded that back analysis applied in this study is an appropriate tool for estimating tension forces of short hanger cables.

## INTRODUCTION

Recently, it is common for long span bridges to be designed and constructed by the development of cable materials that can endure high tension force, advanced design and construction techniques, and maintenance based on sensing techniques. A critical member of suspension bridges as a long span bridge is the suspended and hanged cables which can resist the dead and live loads. Thus, it is very important that not only introducing tension forces on the cables during construction, but also estimating tension forces after completion to verify the safety and stability of bridges.

As a nondestructive method, tension forces on hanger cables would be estimated based on theoretical equations. In a few cases, the static method (Ahn and Lee 2003) using the relationship between statically measured loads and displacements has been used for estimating tension force. The vibration method, known as the conventional method, has been widely applied to estimate the tension forces by using the measured frequencies on hanger cables. The vibration method can be normally classified into single mode method (Zui et al. 1996) and multi mode method (Shimada 1995). Both methods were established based on the cable equation of motion (Irvine 1981) considering the flexural rigidity of cable, but could not accurately estimate tension forces on the cables with large flexural rigidity. Therefore, the conventional vibration methods are not applicable to estimate tension forces on short hanger cables.

In this study, the hanger cables of Gwangan suspension bridge located in South Korea were used as a field example of estimating tension forces. The vibration method using multi modes (Shimada 1995) has been frequently applied to comparatively long hanger cables. However, in case of short hanger cables less than 10 meters, the back analysis technique proposed in this study is able to be applied to short hanger cables dominantly affected by flexural stiffness.

In the back analysis technique (Kim and Jeong 2005, Jeon and Yang 2005), forward analysis can be performed by using a commercial structural analysis program with a function of a common mathematical algorithm through simple modeling and modification. A difference between the calculated frequency from numerical model and the measured frequency from existing hanger cable is defined as error function, and the error function is minimized by using optimization algorithm and compensation factor. For unconstrained optimization, the univariate search method (Rao 1996) employed in this study is one of one-dimensional search methods in which a variable is varied at one time and the others are fixed for improvement of approximate value (Gioda and Maier 1980).

From the measured frequencies on the hanger cables, the flexural stiffness effect was discussed with the comparison resulting from back analysis and conventional vibration methods. As a result, it can be confirmed that the back analysis technique is more reliable for estimating tension force on short hanger cables dominated by flexural stiffness effect.

## CONVENTIONAL METHODS

### Vibration Method I (Using single mode)

The flexural stiffness effect on cables can be decreased according to the increase of $\xi$, which is a dimensionless parameter presenting flexural stiffness effect of cables. Vibration motion of cables with large value of $\xi$ is nearly similar with one of strings. In the case of $\xi = \sqrt{(T)/(EI)} \cdot L \geq 17$, influence of flexural stiffness can be negligible and cable vibration is dominated by tension force and length of cables. Therefore, ignoring the sag effect of cables, tension force of cables (Zui et al. 1996) could be estimated by equation (1), in which the measured first natural frequency can only be used.

$$T = \frac{4W}{g}(f_1 L)^2 [1 - 2.20\frac{C}{f_1} - 0.550\left(\frac{C}{f_1}\right)^2] \tag{1}$$

where $f_1$ is the measured first natural frequency and $C$ is $\sqrt{(EIg)/(WL^4)}$ .

### Vibration Method II (Using multi modes)

In general, an equation of motion for cable structures considers flexural rigidity, tension force and mass distribution of cables. Thus, the tension force of existing cables could be estimated by the measured natural frequencies corresponding to cable vibration modes. In a cable model as shown in Fig. 1, equation (2) can be constituted by using dynamic displacement of gravity direction $v(x, t)$at time $t$ and length $x$ (Irvine 1981).

$$T\frac{\partial^2 v(x,t)}{\partial x^2} - EI\frac{\partial^4 v(x,t)}{\partial x^4} = \frac{w}{g}\frac{\partial^2 v(x,t)}{\partial t^2} \tag{2}$$

where $T$ is tension force of cable, $EI$ flexural rigidity, $w$ unit weight, and $g$ acceleration of gravity. Equation (2) as a partial differential equation can be solved with various conditions. Equation (3) (Shimada 1995) has been normally used for multi mode vibration method, which is to obtain the linear relationship between $(f_n/n)^2$ and $n^2$, where $n$ is the order of vibration mode, $f_n$ is the n$^{th}$ natural frequency. In the vibration method using multi modes, the relationship between $(f_n/n)^2$ and $n^2$ could be derived as follows:

$$(\frac{f_n}{n})^2 = \frac{Tg}{4wL^2} + \frac{n^2\pi^2 EIg}{4wL^4} = b + a \cdot n^2 \tag{3}$$

If a constant $b$ in equation (3) can be obtained from linear regression using the measured natural frequencies and the corresponding orders of vibration mode, tension force of cable could be estimated by using equation (4).

$$T = 4(w/g)L^2 \cdot b \tag{4}$$

## BACK ANALYSIS TECHNIQUE

As one of unconstrained optimization techniques, the univariate search method employed in this study is a one-dimensional search method in which a variable is varied at one time and the others are fixed for improvement of approximate value (Rao 1996). This method is an ordinal application of one-dimensional search as shown in Fig. 2. The unknown variable $X$ could be estimated by optimizing error function $f(X)$ and performing back analysis. The error function as shown in equation (5) can be presented as follows:

$$Minimizing = f(X) = \sqrt{\sum_{k=1}^{N} \left| U_k(X) - \overline{U_k(X)} \right|^2} \tag{5}$$

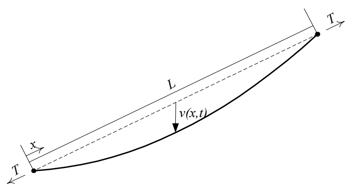



Figure 1. simply supported cable model

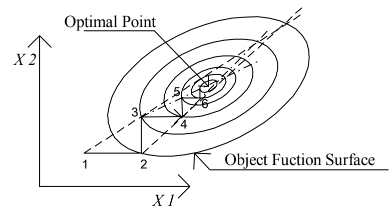



Figure 2. Optimal point searching process of one-dimensional search

where $\overline{U_k(X)}$ is the measured term, $U_k(X)$ the predicted term by back analysis. In this study, the unknown variable $X$ is the tension force of cable, $\overline{U_k(X)}$ is the measured natural frequency and $U_k(X)$ is the predicted natural frequency by back analysis. For minimizing the error function, the parameters can be used in equation (6):

$$X_{i+1} = X_i + \lambda_i^* S_i \tag{6}$$

where $X_i$ is tension force, $\lambda_i^*$ step length and $S_i$ is searching direction at each step.

Fig. 3 shows a flow chart of back analysis algorithm employed in this study. In equation (7), the unknown variable is $T_i$ as a cable tension force and the design tension can be assumed as an initial force. In equation (7), $\lambda_i^*$ is step length:

$$\lambda_i^* = T_i(\beta_i - 1) \tag{7}$$

As a compensation factor, $\beta_i$ is presented using the weighting factor and frequency ratio as shown in equation (8) :

$$\beta_i = \sum_{k=1}^{n} W_i \left( \overline{f_k} / f_k^i \right) / \sum_{i=1}^{n} W_i \tag{8}$$

where $W_i$ is the modal participation factor corresponding to the $i^{th}$ vibration mode. And searching direction $S_i$ can be defined as unity. Convergence criterion in this optimization algorithm could be defined as relative error of unknown parameter $T_i$ in equation (9) :

$$\varepsilon_T = \frac{\left| T_{i+1} - T_i \right|}{T_i} \tag{9}$$

According to Fig. 3, back analysis could be performed by comparing the predicted frequencies ($f_k$) resulted from structural analysis program with the frequencies ( $\overline{f_k}$ ) measured from existing cables.

## ESTIMATION OF CABLE TENSION FORCES

### Vibration measurement and analysis

Gwangan Bridge, as a suspension bridge located in Busan Korea, was selected for an example in this study and its main properties are summarized in Table 1. Long hanger cables are located near the main tower and short hanger cables are located around the center of the main span. In this study, not only long cables but also medium and short cables are chosen, as shown in Fig. 4, in order to verify the tension force estimation method which can be applied to most hanger cables not depending on cable length. 9 cables (No. 22, 29,

Table 1. Properties of Gwangan Bridge

| | |
|---|---|
| Dimension | Total length : 900m (main span : 500m side span : 200m)<br>total 3 spans 2 hinges, width : 24m |
| Type | Suspension bridge |
| Anchor block | Reinforced concrete block for strand settlement and maintaining cable tension force<br>Concrete 186,276m$^3$, rebar 18,467ton |
| Main Tower | Rigid tower supporting cables which are hanging reinforcing truss<br>Foundation : Bell Type pile foundation,<br>Height (from sea level) : 116.5m(octagon shape)<br>Size : 4×5 ~ 6.5×105m, material : steel weight : 6,480 ton |
| Main cable | Wire diameter : 5mm, 11,544 strips<br>Cable(diameter 60.6cm) → consist of 37 strands,<br>1 Strand : consist of 312 wires<br>Tension force : 24,500ton(12,250×2 Cables) |
| Reinforce Truss | Waren truss assembled square shape and cylindrical steel materials<br>Total steel weight : 23,708ton(steel deck 9,286ton, truss 14,422ton) |

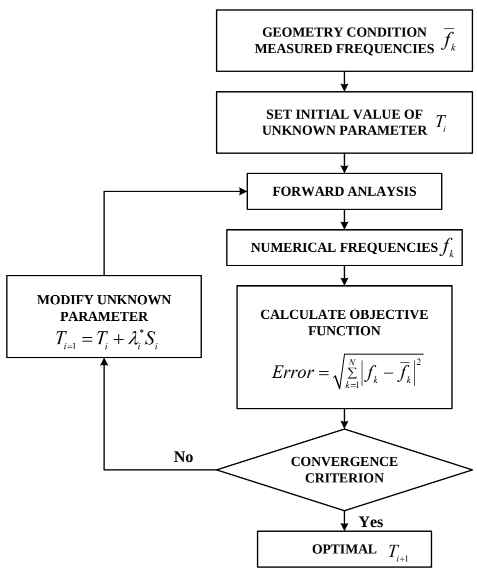



Figure 3. Optimization procedure of univariate search method

36~42) placed on beach side were selected as a target for cable tension force estimation. Each cable band is connected to two cable groups; one group consists of two hanger cables as shown in Figs. 5 and 6. One group of cables on each 9 cables, designated as group A, was applied to back analysis and vibration method. The properties of selected hanger cables are shown in Table 2.

Length of each hanger cable is presented in Table 2 and diameter of hanger cables is a representative of the area of hanger cable twisted by wires. For numerically modeling the hanger cables, it was assumed that boundary conditions at both ends are defined as hinge supports, and the design tension force is defined as an initial tension force of hanger cables.

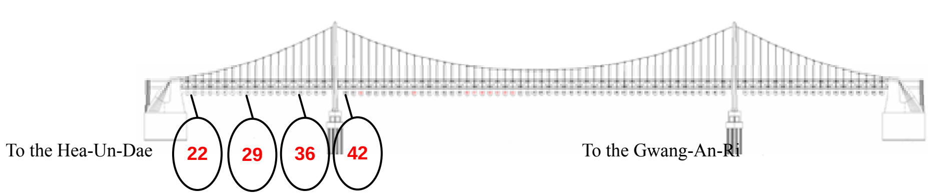



Figure 4. Hanger cables selected for tension force estimation

Table 2. Properties of selected hanger cables

| Cable ID | length (m) | $\xi$ | diameter (mm) | Young's modulus ($kN/mm^2$) | weight ($kN/mm^3$) | design tension (kN) |
|---|---|---|---|---|---|---|
| 22 | 50.850 | 155.2080 | | | | |
| 29 | 25.323 | 77.2934 | | | | |
| 36 | 8.833 | 27.5231 | | | | |
| 37 | 7.270 | 22.6850 | | | | |
| 38 | 5.891 | 18.4167 | 49.52 | 1.3734×102 | $8.0\times10^{-8}$ | 377.685 |
| 39 | 4.699 | 14.7143 | | | | |
| 40 | 3.691 | 11.5775 | | | | |
| 41 | 2.869 | 9.0048 | | | | |
| 42 | 2.232 | 6.9976 | | | | |

Under ambient vibration condition, cable responses were measured by using acceleration sensors which were attached in a transverse direction on hanger cable, as shown in Fig. 7. The measuring conditions are as follows: sampling rate is 200Hz, time interval is 0.005 second, minimum measurement duration is 100 seconds, number of data per channel is more than 20000, and frequency resolution ($\Delta f$) is about 0.012Hz.

For easily obtaining multi vibration modes of hanger cables, acceleration responses were acquired under ambient vibration condition. However, in searching all over signals measured under ambient vibration condition, there were several cases not satisfying with vibration mode range. And some cases are hard to define exactly the order of each vibration mode. Moreover because the acceleration sensors were attached near a cable support in field condition, the higher modes mainly affected by flexural stiffness are outstandingly measured. Therefore, for exactly distinguishing the order of vibration modes, an eigenvalue analysis was carried out by using the transverse responses measured from two cables connected by a hanger band under ambient vibration condition. As a result, vibration methods and back analysis can be performed with only transverse response of the two cables because of considering the effect of hanger clamp.

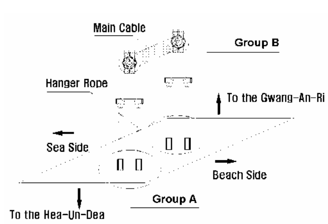



Figure 5. Hanger cables for measurement

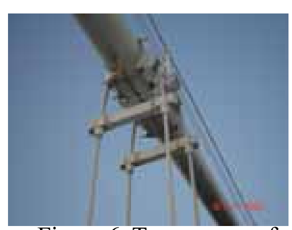


Figure 6. Two groups of hanger cables

Figure 7. Accelerometers attached on hanger cables

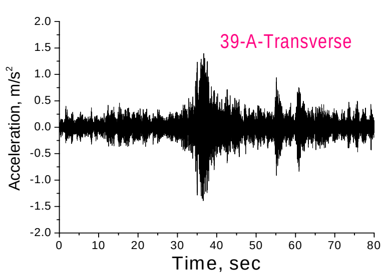



Figure 8. Acceleration response of No. 39 cable

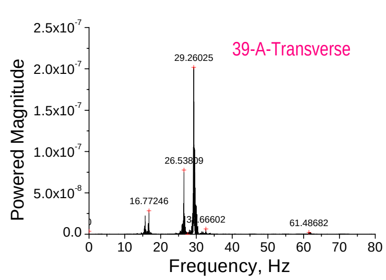



Figure 9. Power spectrum of Fig. 8

Figure 8 shows the transverse acceleration response of No. 39 cable under ambient vibration condition and its power spectrum is presented in Fig. 9. The measured natural frequencies of hanger cables under ambient vibration condition are summarized in Table 3.

**Back analysis results**

In this study, the hanger cables are modeled in beam element for considering flexural stiffness effect with cable lengths. Fig. 10 shows convergence processes of tension force on hanger cables by repetitive execution of back analysis. A difference between the calculated frequency from the numerical model and the measured frequency from the existing hanger cable is defined as an error function, and the error function is minimized by using the optimization algorithm and compensation factor. The modeling was limited in X-Z plane and both ends of hanger cables were assumed to hinge support for idealization of back analysis modeling. Even if there is no numerical error of cable tension at the convergence of back analysis as shown in Fig. 10, it is not evident that the converged tension values are feasible. Thus, in Fig. 11, the calculated frequencies on the hanger cables are compared with the measured ones.

Fig. 11 shows that the calculated natural frequencies are close to the measured natural frequencies through the convergence of back analysis. In some cases, the calculated frequencies resulted from initial analysis are a little closer to the measured frequencies than the calculated frequencies at convergence of tension force. However, it may be reasoned

Table 3. Measured natural frequencies of hanger cables under ambient vibration condition

| Mode | Measured Frequency (Hz) | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 22-A | 22-B | 29-A | 29-B | 36-A | 36-B | 39-A | 39-B |
| 1st | 1.57 | 1.55 | 3.22 | 3.05 | 9.40 | 9.20 | 16.77 | 16.72 |
| 2nd | 3.13 | 3.11 | 6.45 | 6.09 | 16.01 | 15.47 | 29.29 | 30.31 |
| 3rd | 4.75 | 4.66 | 9.62 | 9.14 | 23.12 | 22.60 | 60.00 | 63.09 |
| 4th | 6.33 | 6.23 | 12.66 | 12.01 | 34.78 | 34.54 | 96.70 | 94.37 |
| 5th | 7.93 | 7.81 | 15.36 | 14.58 | 49.17 | 48.73 | - | - |
| 6th | 9.46 | 9.38 | 18.09 | 17.21 | 65.01 | 65.75 | - | - |
| 7th | 11.20 | 10.96 | 21.37 | 20.37 | 84.11 | 85.44 | - | - |
| 8th | 12.61 | 12.38 | 25.06 | 23.95 | - | - | - | - |
| 9th | 13.96 | 13.72 | 28.71 | 27.66 | - | - | - | - |
| 10th | 15.42 | 15.18 | - | - | - | - | - | - |

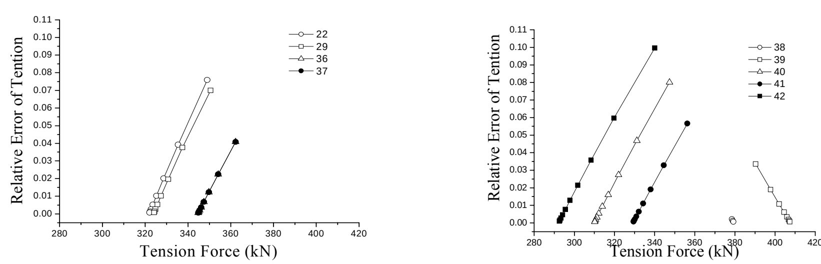
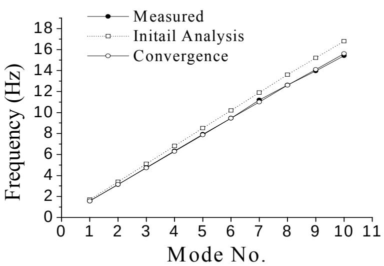
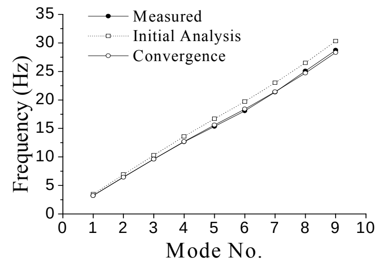


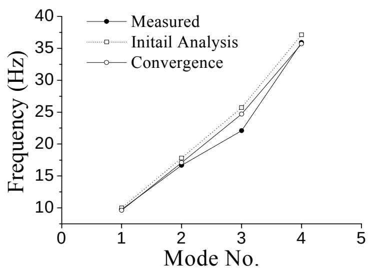

Figure 10. Convergence Processes of tension force on hanger cables



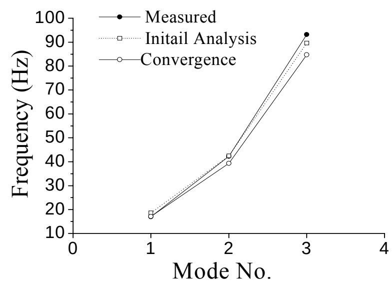

(a) No. 22 cable



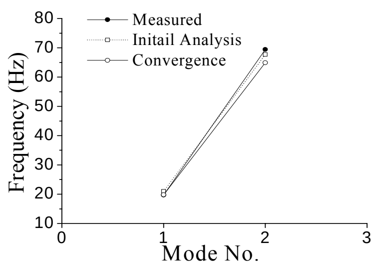

(b) No. 29 cable



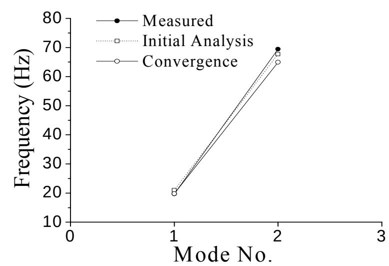

(c) No. 36 cable




(d) No. 40 cable




(e) No. 41 cable




(f) No. 42 cable

Figure 11. Comparison of the calculated and measured frequencies

that in the first natural frequency having a dominant vibration mode, the calculated frequencies are nearly coincided with the measured ones. Considering the modal participation factor in most cases of back analysis, the frequencies at convergence of tension force by back analysis are well conformed to the measured frequencies.

**Comparison of back analysis with vibration method result**

Table 4 and Fig. 12 show the estimated tension forces and differences between vibration methods and back analysis. The estimated values that resulted from all methods are obtained near the design tension force when the flexural stiffness effect of hanger cables could be ignored ($\xi \geq 17$). When the natural frequencies of cables are largely affected by the flexural stiffness ($\xi < 17$), the estimated values of vibration methods are deviated from the design tension value. On the other hand, the tension values estimated by back analysis are reasonable at the design tension level.

## CONCLUSIONS

In this study, the conventional cable tension estimation methods as single mode and multi mode vibration methods and the back analysis technique newly proposed were used for estimating the tension force hanger cables. The concluding remarks can be drawn as follows: In the case of comparatively long cables($\xi \geq 17$), as No. 22($\xi$ =155.21) ~ No. 35 ($\xi$ = 27.52) cables, the estimated tension forces of all methods are similar with design tension force. Therefore, all three methods could be used to estimate tension force for comparatively long cables. However, in the case of comparatively short cables, as No. 40 ~ No.42 cables, vibration methods could not be applied because differences are more than 50% from design tension. On the other side, the back analysis results are efficient to apply to comparatively short cables, as differences of back analysis are less than 22% from design tension force. Moreover, back analysis has much room to improve accuracy depending on updating the numerical models and measurement techniques. It can be concluded that the back analysis technique considering numerical models and modal participation factors is more applicable to estimate the tension force of hanger cables than

Table 4 . Estimated tension and difference from vibration methods and back analysis

| Cable ID | $\xi$ | Design tension | Vibration Method | | | | Back Analysis | |
|---|---|---|---|---|---|---|---|---|
| | | | Single Mode | | Multi Modes | | Multi Modes | |
| | | | Estimated Tension | Difference (%) | Estimated Tension | Difference (%) | Estimated Tension | Difference (%) |
| 22 | 155 | | 389.76 | 3.20 | 412.32 | 9.17 | 322.07 | -14.73 |
| 29 | 77 | | 395.38 | 4.69 | 410.75 | 8.75 | 324.14 | -14.18 |
| 36 | 28 | | 415.58 | 10.04 | 317.91 | -15.83 | 344.87 | -8.69 |
| 37 | 23 | | 505.86 | 33.94 | 559.01 | 48.01 | 451.90 | 19.65 |
| 38 | 18 | 377.69 | 387.22 | 2.53 | 368.14 | -2.53 | 379.20 | 0.40 |
| 39 | 15 | | 298.26 | -21.03 | 290.09 | -23.19 | 407.36 | 7.86 |
| 40 | 12 | | 135.38 | -64.15 | 149.01 | -60.55 | 310.13 | -17.89 |
| 41 | 9 | | 63.920 | -83.08 | 64.50 | -82.92 | 329.52 | -12.75 |
| 42 | 7 | | -10.64 | -102.82 | -180.80 | -147.87 | 292.42 | -22.57 |

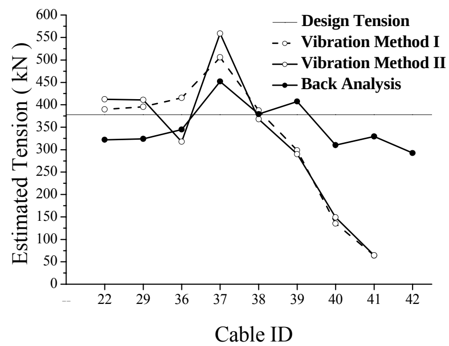



Figure 12. Comparison of estimated tension forces from vibration methods and back analysis

the conventional methods, as vibration methods, because the frequencies of short cables are severely sensitive to flexural rigidity.

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