



# ON THE USE OF A BIVARIATE REGRESSIVE ADAPTIVE INDEX FOR STRUCTURAL HEALTH MONITORING

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## Abstract

This study expands upon the concept of a Bivariate Regressive Adaptive INdex (BRAIN) for structural health monitoring. BRAIN utilizes a data-driven damage index that automatically adapts to the data set, extracting the most damage sensitive model features. The effectiveness of the dynamic index will be further validated herein using the Phase I IASC-ASCE Structural Health Monitoring Benchmark. Another key feature of BRAIN: its exploitation of heterogeneous sensor arrays to enhance detection capability is independently validated on repeated simulations of randomly-excited thin beams with modest levels of damage. The findings demonstrate BRAIN's enhanced damage detection capabilities, in comparison with other methods.

## INTRODUCTION

Initial efforts toward structural health monitoring (SHM) were largely focused on ascertaining the performance of structures under extreme events such as earthquakes. With rapid advancements in sensor technologies and increased cost effectiveness, SHM soon evolved from a "luxury" to a feasible concept for proactive maintenance and routine assessment of a wide variety of structures. Still, the objective of SHM has remained unchanged: to accurately detect, locate, and assess the severity of structural damage. Most commercial monitoring systems adopted the traditional cable-based hub and spoke architecture of these early applications, where distributed sensors are connected by cables to a centralized data acquisition unit; however, given the burdens associated with the installation and maintenance of multi-channel cable-based systems, research in wireless sensor networks (WSN) has been rapidly expanding over the last decade. Today, nodes have more robust communications and processing capabilities, and have been adapted for a diverse array of sensing elements, including strain, acceleration, pressure, and temperature. However, in order to practically realize WSNs, local processing capabilities at each sensor node must be fully exploited to minimize the amount of information transmitted, and thereby the power demands on the network. This often means that system identification frameworks originally intended for the classic hub and spoke architecture must be adapted or abandoned in favor of distributed identification schemes capable of reliably detecting damage using the limited on-board computational resources of the sensor nodes, in the presence of model, load and environmental variabilities.

The approach presented herein, the *Bivariate Regressive Adaptive Index* or BRAIN, responds to this need by building upon existing statistical signal processing techniques to minimize the impact of these variabilities. The approach belongs to a class of time-domain detection schemes using various forms of autoregressive models to reconstruct largely acceleration responses and extract damage features, either coefficient or residual-based. The

efforts to date have demonstrated the potential of such autoregressive approaches to provide effective damage diagnosis and the capability for embedment within wireless sensor networks<sup>[1-7]</sup>. Unfortunately, the model orders required for such autoregressive representations can strain the limited computational capability of the local processors and often requires simplified damage sensitive features (DSFs). This generally implies that DSFs are specifically tailored for the underlying autoregressive model, limiting their robustness and their ability to be extended to other applications. BRAIN circumvents this limitation through a data-driven DSF adapting to the most sensitive model coefficients in a given damage scenario, which vary with location, loading condition and damage severity. The performance of this data-driven DSF in comparison with comparable “static” DSFs will be presented in this study, using the Phase I IASC-ASCE Structural Health Monitoring Benchmark Problem.

Subsequently, it will be shown that this data-driven feature is capable of not only incorporating a wide variety of potential autoregressive models, but also a diversity of sensor data through bivariate autoregressive (BAR) models. This provision for heterogeneous sensing, when coupled with the data-driven DSF concept, yields the formal BRAIN framework and provides a dramatically enhanced detection capability, as demonstrated through repeated simulations of randomly-excited thin beams with minor levels of damage.

### DAMAGE DETECTION IN HOMOGENEOUS SENSOR NETWORKS

As discussed previously, the time-series damage detection problem is completely reliant on the underlying model used to represent the time series, and this model was generally formulated with one type of response in mind, e.g., acceleration. For example, Nair et al.<sup>[5]</sup> advocated the use of an autoregressive moving averages (ARMA) model to represent a homogeneous response signal, i.e., acceleration. The performance of this model was contrasted with other common model types for basic signal representation in Su and Kijewski-Correa<sup>[7]</sup>, where it was shown that, for the same effective model order, an autoregressive (AR) reconstruction of a simulated acceleration signal has an RMS residual error only 6% greater than the ARMA approach. This loss of resolution may be justified for use in WSNs, given the reduced computational demands of the AR model, provided an appropriately sensitive DSF that is employed. Therefore, this study will apply an AR model for homogeneous time series, where the  $k^{th}$  standardized vibration signal, e.g., acceleration  $A$ , is represented at each time step  $n$  by  $na$  AR terms<sup>[8]</sup>:

$$\tilde{A}_k(n) = \sum_{i=1}^{na} \alpha_{ki} A(n-i) + \zeta_k(n) \quad (1)$$

Through extensive investigation, Nair et al.<sup>[5]</sup> found that the first three AR coefficients of an ARMA representation were the most sensitive to damage within their homogeneous detection framework. However, as mentioned previously, the departure from a more accurate ARMA approach can only be justified if an appropriate DSF is proposed to offset those losses. When using an AR representation, while the first few coefficients generally are the most affected by damage, this is not always the case. Since the intention is to have a high reliability DSF under a diverse range of response conditions, a new type of DSF was proposed<sup>[1]</sup> and subsequently validated<sup>[7]</sup>. This adaptive or *data-driven DSF* identifies the AR coefficient that has changed most significantly compared to the average AR coefficients stored in the reference database. The reference database is a common necessity in this type of detection problem and in this case houses the AR coefficients associated with numerous acquired signals from the structure in its initial, preferably undamaged state, under a wide range of environmental and operational conditions:

$$DSF1_k = \max \left[ \left| \frac{\alpha_{ki} - avg[\alpha_{ki}]_{ref}}{std[\alpha_{ki}]_{ref}} \right| \right]_{i=1:na} \quad (2)$$

This will be called *DSF1*, since a single response type (acceleration) is considered. As discussed by Su and Kijewski-Correa<sup>[7]</sup>, DSF1 has two advantages:

- 1) The original AR coefficients for each vibration signal in the reference database does not have to be stored at the node; only the mean and standard deviation of each coefficient are required. Thus only  $2na$  reference values are finally stored at each sensor node after some training period for the WSN. Again keep in mind

that  $na$  is relatively small ( $< 20$ ). This dramatically reduces not only the required on-board memory, but also any computation (and power drain) associated with a damage decision.

- 2) DSF1 is unaffected by the choice of underlying model (AR, ARMA, etc.), unlike other “static” DSFs that are tied to or have been validated with only a specific model type or sensor in mind. More importantly, this permits some of the trade-offs in accuracy discussed earlier. This also implies that if there is a location where higher order coefficients are more sensitive to damage, they will be exploited. Thus the DSF is data-driven and more reliable.

A Gaussian model can generally be applied to represent the DSF1 values associated with the reference pool, allowing the user to specify a desired percentile of statistical significance, e.g., 95%. Therefore, let  $DSF1_p$  represent the DSF1 from the reference pool at percentile  $P$ . Then damage is indicated with  $P$ -percent certainty whenever DSF1 value estimated from new measurements satisfies the following inequality:  $DSF1 > DSF1_p$ .

### 1.1 Applications to Phase I IASC-ASCE Benchmark Problem

The structure used in the Phase I IASC-ASCE Structural Health Monitoring Benchmark was a four-story, two-bay by two-bay frame<sup>[9]</sup>. Each bay is 1.25x1.25m in plan and 0.9 m high. Slabs are placed at each floor level to simulate the mass of the structure. Both a 12 degree of freedom (DOF) and 120 DOF model were developed for use in MATLAB. This study uses the lower order lumped mass model, driven by white noise at each floor, employing the assumption that each of the floors are perfectly rigid and free from out-of-plane translation and rotation. The six damage patterns (DP) specified by Johnson et al.<sup>[9]</sup> are summarized in Table 1. These damage patterns were assigned a relative damage level from minor to severe to facilitate discussions later in this paper.

Table 1. Damage Patterns of Phase I IASC-ASCE Benchmark Problem

Pattern	Description	Damage Level
DP0	Undamaged	None
DP1	All braces of 1 <sup>st</sup> floor removed	Severe
DP2	All braces of 1 <sup>st</sup> and 3 <sup>rd</sup> floor removed	Severe
DP3	One brace of 1 <sup>st</sup> floor removed	Moderate
DP4	One brace of 1 <sup>st</sup> and 3 <sup>rd</sup> floor removed	Moderate
DP5	Pattern 4 + floor beam partially loosened	Moderate
DP6	1/3 stiffness reduction, one brace, 1 <sup>st</sup> floor	Minor

To verify the relative performance of DSF1 in equation (2), it is now compared to a “static” DSF also based on AR coefficients<sup>[5]</sup>:

$$DSF1c_k = \frac{\alpha_{k1}}{\sqrt{\alpha_{k1}^2 + \alpha_{k2}^2 + \alpha_{k3}^2}} \quad (3)$$

This will be termed the comparison homogeneous damage sensitive feature or DSF1c. Note that DSF1c was intended for use with ARMA models<sup>[5]</sup>; however, for consistency, an AR model is used to represent the acceleration data evaluated by both DSF1 and DSF1c. This will allow the advantages of a data-driven DSF to be underscored.

Each damage pattern is simulated 10 times to explore the repeatability of the results. A  $P=97.5\%$  one-sided confidence interval is specified for distinguishing statistically significant damage in equation (2), while a two-sided confidence interval at 97.5% is used with equation (3). The damage detection rates at all floors are provided in Figure 1 and an example of DSF1 and DSF1c values for DP0-3 are provided for the 2<sup>nd</sup> floor in Table 2, where bold-faced values indicate that  $DSF1 > DSF1_p$  or that DSF1c was outside the two-sided confidence limits derived from the reference pool, thus signifying damage. Note DP0 damage case is provided to evaluate any tendency toward false positives. Detection rates for DP1-6 are plotted in Figure 1, and as an overall measure of detection capability, the detection rates averaged over all floors are presented in Table 3.

Several important conclusions can be drawn regarding overall damage detection (not localization) capability, meaning the ability to detect damage from any sensor output:

1. Neither DSF appears susceptible to false positives.

- For the most severe level of damage (DP2), both DSF1 and DSF1c can detect damage consistently based on the response at any of the floors. For the other severe damage case (DP1), DSF1c detects damage only in the first floor consistently, closest to the point of damage, and has an averaged detection rate of 35%, while DSF1 can again detect damage consistently at all 4 stories, for an average detection rate of 100%.

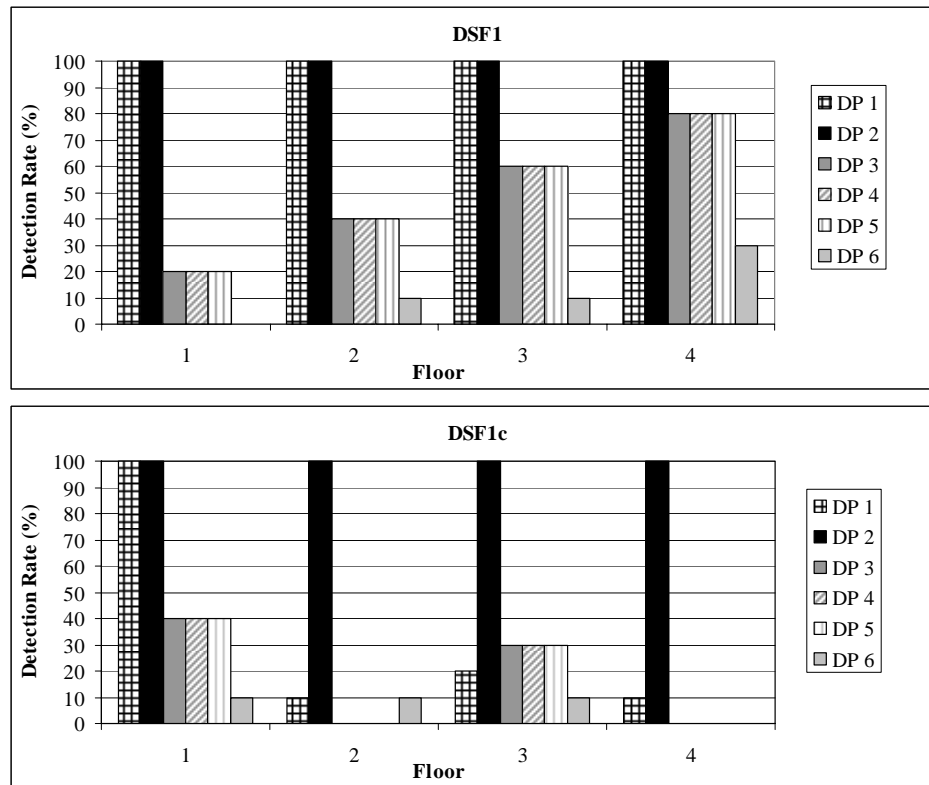


Figure 1. DSF1 and DSF1c detection rates for Phase I IASC-ASCE Benchmark

- For the moderate and minor damage cases (DP3-6), DSF1c is not as successful: with detection rates as high as 40% at floor one, but as low as 0% at the top floor, for an average detection rate of 17.5% for modest damage (DP3-5) and 7.5% for minor damage (DP6). Detection capability is strongest at floors 1 and 3, where damage has been imposed. This indicates that a static DSF is best suited for detection near the point of damage in instances where the severity is minor to modest.
- DSF1 demonstrates the inverse capability for minor to modest damage levels. It can detect modest damage (DP3-5) with up to 80% repeatability at the fourth floor, though the capability progressively diminishes down the building. Minor damage (DP6) shows a similar trend, with 30% detection rate at the top floor, dropping to only 10% at the lower floors. This results in average detection rates of 50% under moderate damage (DP3-5) and 12.5% under minor damage (DP6). Since the acceleration response increases up the building, the findings here may indicate that the homogeneous data-driven DSF performs better as the response amplitude increases, consistent with the findings of Su and Kijewski-Correa<sup>[7]</sup>. This makes this class of DSF well-suited for applications where dense sensor networks are not feasible and measurements may only be taken at limited locations.
- Note also that the DSF1 values increase with the damage level (Table 2), providing a means to directly quantify severity of damage.

Overall, the average detection rates for the data-driven DSF (DSF1) are up to nearly three times better than the static DSF (DSF1c). Still, these results do not consider the added flexibility of a data-driven DSF to incorporate multiple sensor outputs. The advantages of this capability are now explored.

Table 2. Damage detection results for DSF1 and DSF1c, 2<sup>nd</sup> Floor of Phase I IASC-ASCE Benchmark (DP0-3)

Threshold	DSF1				DSF1c			
	DSF1 <sub>97.5%</sub> =3.0463				DSF1c <sub>97.5%</sub> = (0.07,0.25)			
	DP0	DP1	DP2	DP3	DP0	DP1	DP2	DP3
Test 1	2.54	<b>13.8</b>	<b>18.4</b>	2.51	0.14	0.13	<b>0.69</b>	0.13
Test 2	1.46	<b>12.9</b>	<b>17.4</b>	2.72	0.24	0.15	<b>0.64</b>	0.19
Test 3	2.39	<b>13.9</b>	<b>18.4</b>	<b>4.04</b>	0.19	0.23	<b>0.71</b>	0.24
Test 4	1.53	<b>13.1</b>	<b>15.9</b>	2.58	0.18	0.17	<b>0.68</b>	0.15
Test 5	1.85	<b>14.6</b>	<b>18.4</b>	<b>3.54</b>	0.16	0.14	<b>0.67</b>	0.17
Test 6	2.58	<b>13.8</b>	<b>19.3</b>	2.68	0.21	0.11	<b>0.64</b>	0.13
Test 7	2.78	<b>14.0</b>	<b>18.7</b>	2.58	0.10	<b>0.25</b>	<b>0.68</b>	0.23
Test 8	2.72	<b>13.9</b>	<b>20.1</b>	<b>3.47</b>	0.17	0.08	<b>0.72</b>	0.11
Test 9	1.97	<b>13.7</b>	<b>18.6</b>	<b>4.26</b>	0.20	0.08	<b>0.67</b>	0.09
Test 10	1.39	<b>14.3</b>	<b>17.6</b>	2.72	0.17	0.19	<b>0.76</b>	0.15
Det. Rate	0%	100%	100%	40%	0%	10%	100%	0%

Table 3. Damage detection rates for DSF1 and DSF1c of Phase I IASC-ASCE Benchmark, averaged over all floors

	DP0	DP1	DP2	DP3	DP4	DP5	DP6
DSF1	0%	100%	100%	50%	50%	50%	12.5%
DSF1c	0%	35%	100%	17.5%	17.5%	17.5%	7.5%

## DAMAGE DETECTION IN HETEROGENEOUS SENSOR NETWORKS

Kijewski-Correa et al.<sup>[1]</sup> proposed the use of multiple vibration signals from different sensing elements, e.g., acceleration  $A$  and strain  $S$ , realizing the unique information regarding damage that can be carried by each. Various formulations that model the interrelation between these two measured quantities have been offered<sup>[1]</sup>, though the present study specifically focuses on the *bivariate autoregressive (BAR) model*<sup>[7]</sup> between strain and acceleration. In this representation, the  $k^{\text{th}}$  standardized strain and acceleration data pair ( $A, S$ ) is fit by a  $na+nb$  order model:

$$\tilde{A}_k(n) = \sum_{i=1}^{na} \alpha_{ki} A(n-i) + \sum_{j=0}^{nb} \beta_{kj} S(n-j) + \zeta_k(n) \quad (4)$$

where  $\alpha_{ki}$  is the  $i^{\text{th}}$  AR acceleration coefficient and  $\beta_{kj}$  is the  $j^{\text{th}}$  AR strain coefficient and  $\zeta_k$  is the residual error.

Since it has been shown that the combination of surface strain and acceleration data through the BAR model not only provides a more accurate signal representation, but also enhances damage detection in comparison with the use of either strain or acceleration alone<sup>[1,7]</sup>, DSF1 is modified for a heterogeneous representation to better exploit the most sensitive bivariate AR coefficients, through the BRAIN concept:

$$DSF2_k = \max \left[ \left| \frac{\alpha_{ik} - \text{avg}[\alpha_{ik}]_{ref}}{\text{std}[\alpha_{ik}]_{ref}} \right|_{i=1:Na}, \left| \frac{\beta_{jk} - \text{avg}[\beta_{jk}]_{ref}}{\text{std}[\beta_{jk}]_{ref}} \right|_{j=0:Nb} \right] \quad (5)$$

### 1.2 Applications to Cantilever Beam Model

Unfortunately, the Phase I IASC-ASCE benchmark provides only acceleration responses for a discrete system, so the utility of heterogeneous sensing employing strain cannot be demonstrated using it. However, this utility was previously demonstrated via a finite element model of a slender cantilever beam driven by Gaussian white noise input at its free end<sup>[7]</sup>. Acceleration and surface strain time history pairs were repeatedly simulated at the four locations at the quarter lengths of the beam and varying degrees of damage were subsequently introduced to the beam through a transverse cut, at 3L/8 from the fixed end. To demonstrate the performance of DSF2, damage

detection results for this beam were compared to those by DSF1. While full details of the analysis and a more comprehensive assessment of the findings are provided in Su and Kijewski-Correa<sup>[7]</sup>, a summary of results is shown in Figure 2 for very modest damage scenarios of 0.5% cross sectional area lost and 1.5% cross sectional area lost.

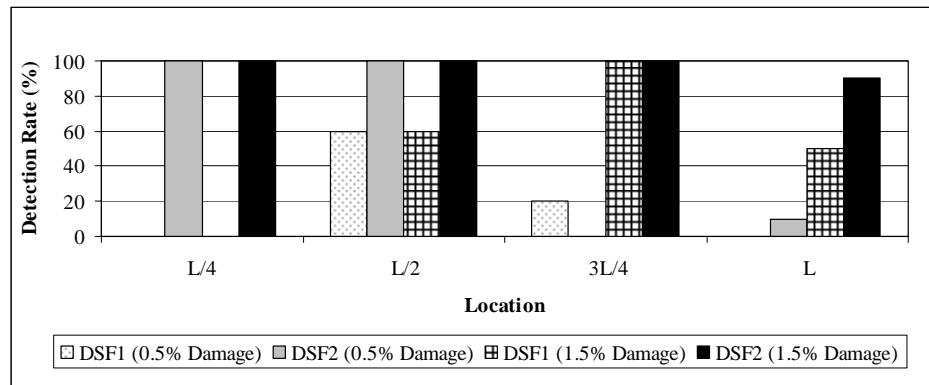


Figure 2. DSF1 and DSF2 detection rates for cantilever beam model

From Figure 2, several important conclusions can be drawn about the benefits of a heterogeneous formulation:

1. The larger of the two damage scenarios can be identified reliably at all measurement locations using DSF2. The vast improvement in detection capability in the vicinity of damage can be largely credited to the heterogeneous framework that adapts to the response component most critical at a given location, i.e., strain near the base and acceleration near the free end.
2. Consistent with DSF1, DSF2's detection rate falls off further from the damage location for the smaller of the two damage scenarios. The contrast between this finding and that for the benchmark building is largely associated with the small level of damage considered here and thus proximity to damage is critical.

It should be reemphasized that the BRAIN concept employing DSF2, not only provides more reliable detection, but requires significantly less computational effort and available memory, again due to the limited number of reference pool parameters stored locally.

## CONCLUSIONS

A time domain, Bivariate Regressive Adaptive INdex (BRAIN) for damage detection was explored in this study. Its novel feature is a data-driven DSF operating on heterogeneous sensor data within the computational constraints of wireless sensor networks. This study focused on the utility of the data-driven DSF using the Phase I IASC-ASCE Benchmark and demonstrated the enhanced sensitivity to damage facilitated by the data-driven DSF, even in a homogeneous context. Complementary examples underscored the added benefit of the heterogeneous philosophy of BRAIN. This performance was achieved while simultaneously reducing the amount of reference pool data stored locally at each sensor node to insure computational ease. Although further validation on more complex, experimental assemblies is presently underway, the results to date suggest that the BRAIN concept offers a more reliable and robust means of damage detection, whose computational demands are consistent with the limited resources available within wireless sensor networks.

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## REFERENCES

1. Kijewski-Correa, T., Su, S., Abittan, E. and Antsaklis, P.J., 'On the use of heterogeneous, wireless sensor networks for damage assessment in bridges under unknown excitations', Proceedings of 4th World Conference on Structural Control and Monitoring, San Diego, July 2006, CD-ROM.
2. Kijewski-Correa, T., Haenggi, M., and Antsaklis, P., 'Wireless sensor networks for structural health monitoring: a multi-scale approach', Proceedings of 2006 ASCE Structures Congress, 17th Analysis and Computation Specialty Conference, St. Louis, May 2006 (ASCE) CD-ROM.
3. Y. Lei, Y., Kiremidjian, A.S., Nair, K.K., Lynch, J.P. and Law, K.H., Kenny, T.W., Carryer, E., and Kottapalli, A., 'Statistical damage detection using time series analysis on a structural health monitoring benchmark problem', Proceedings of 9th International Conference on Applications of Statistics and Probability in Civil Engineering, San Francisco, July 2003.
4. Lynch, J.P. Sundararajan, A. Law, K.H. Kiremidjian, A.S. and Carryer, E., 'Embedding damage detection algorithms in a wireless sensing unit for operational power efficiency', Smart Mat. and Str. 13 (2004) 800-810.
5. Nair, K.K., Kiremidjian, A.S., and Law, K.H., 'Time series-based damage detection and localization algorithm with application to the ASCE benchmark structure', J. of Sound Vib. 291(3)(2006) 349-368.
6. Sohn, H., Farrar, C.R., Hunter, N.F. and Worden, K., 'Structural health monitoring using statistical pattern recognition techniques', J. Str. Eng. 126(11) (2000) 1356-1363.
7. Su, S., and Kijewski-Correa, T., 'Performance verification of bivariate regressive adaptive index for structural health monitoring', Proceedings of *SPIE Smart Structures and Materials & Nondestructive Evaluation and Health Monitoring*, San Diego, March 2007 (SPIE) CD-ROM.
8. Ljung, L., 'System Identification Theory for the User' (Prentice Hall PTR, New Jersey, 1999).
9. Johnson, E.A., Lam, H.F., Katafygiotis, L.S., and Beck, J.L., 'Phase I IASC-ASCE structural health monitoring benchmark using simulated data', J. Eng. Mech. 130 (1)(2004) 3-15.