Assessment of the safety of continuous steel beams based on average strains from long gage optic sensors

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ABSTRACT: Although the strain distribution along the length of a beam in buildings or infrastructures is non-uniform, most fiber optic sensors are point sensors that can measure the strain only at a local point of a beam. Long gage fiber optic sensors that measure integrated strain over a relatively long length can consider strain variation. This type of sensor was found to be efficient and useful for monitoring large-scale structures. On the other hand, the maximum strain or stress in a beam can not be measured with long gage optic sensors. However, for the assessment of the safety of a continuous steel beam subjected to various vertical loads, the maximum strain or stress measured during monitoring is required for comparison with the allowable stress of the beam calculated by a design code. Therefore, in this paper, mathematical models are presented for determination of the maximum values of strains or stresses in continuous steel beams based on the average strains measured by long gage optic sensors.

1 INTRODUCTION

Structural health monitoring is concerned with the serviceability and safety of the structure. Especially for the case of building structures, serviceability of a building against lateral loads such as wind loads is evaluated in terms of two types of structural responses: lateral displacement and horizontal acceleration level. Excessive lateral displacement can cause structural problems as well as other diverse problems on non-structural elements such as damages to finishing materials, while excessive horizontal acceleration level can bring feelings of unpleasantness to building occupants. For these reasons various researches have been conducted on methods of measuring and controlling relative lateral displacements and horizontal acceleration of high-rise buildings (Tamura et al., 2002; Park et al., 2002).

In the consideration of safety of building and infrastructures, the strength or stress levels of connections and connected members due to live load, earthquake, wind, or other unexpected loadings must be checked to not exceed the design strength levels specified in design specifications (AISC, 2001). In allowable stress design of steel structures (AISC, 1989), if the maximum stress in a member reached the yield stress due to an unexpected overload, the member is considered to be analogous to failure. Although the steel will not fail at yield, excessively large deflections will deteriorate the serviceability of the structure. Therefore, to guarantee the safety of a structure and its users, the maximum stress in a member must be monitored.

Various fiber optic sensing systems based on different sensing mechanisms have been developed to assess the safety of structural members (Leung, 2001; Murukeshan et al., 2000). However, most existing fiber optic sensors used for health monitoring system can cover a relatively small range of structural members. Since the actual stress or strain distribution induced a continuous beam by varying amounts and types of loads is non-uniform, there are many difficulties in determining the maximum stress or strain in a beam with those point sensors. In this case, the reliability of evaluated safety depends on the number and location of point sensors. It is not also realistic to increase the number of sensors to overcome these drawbacks.

Utilization of the long gauge fiber optic sensors in health monitoring system can overcome those drawbacks involved in point sensors (Glisic and Simon, 2000; Tennyson et al., 2001; Ansari, 2005). The long gage sensors measure the relative displacement between two points on the structure. The distance between the two points along the fiber optic cable defines the gage length. Since the deformation measured is the average value measured over the gage length, the strain variation or stress distribution of a beam can be considered by long gage sensors. However, the maximum strain in a beam can not be measured from long gage optic sensors since the gage length is not localized. Park et al. (2005) presented simple mathematical models for determination of the maximum values of strains or stresses in single-span beams based on average strains measured by long gage optic sensors

In this paper, mathematical models are presented for determination of the maximum strain of a continuous beam from average strains measured by long gage optic sensors. In defining the relation between average and maximum strains, various types of loading and boundary conditions for typical beams in building structures or infrastructures are considered.

2 SENSING PRINCIPLES

This sensor consists of a length of ordinary telecom optical fiber bonded to or embedded into the structure. As the structure deforms, either by expansion or contraction, the fiber elongates or contracts accordingly. This displacement is measured using the methodology shown in the schematic of Figure 1.

This system works on the principle of low coherence interferometery using a short coherence length source. The light from the diode is split in two, travels two different path lengths and is then recombined at a photo detector. If the two paths are within approximately 10 microns of each other, the two recombined beams will start to interfere with each other. This interference pattern is monitored by the photo detector as the actuated mirror is moved through the region in which the two paths are approximately equal. The peak of the interference pattern occurs when the two light paths are exactly the same. The measurement obtained is the total displacement over the gauge length.

Since the long gauge sensor is a flexible optical fiber and, it can be attached to or embedded into many different configurations, and because of little limitation on length it is suitable for a large scale structure. In addition, this sensor is independent on EMI and stable chemically, so it suit long-term structure monitoring.

The system operates by scanning the attached sensor for an interference peak (maximum intensity of light), and recording the displacement. Any changes in the sensor's length can be detected by the shift in the signal peak to a different displacement location which is detected by the instrument.

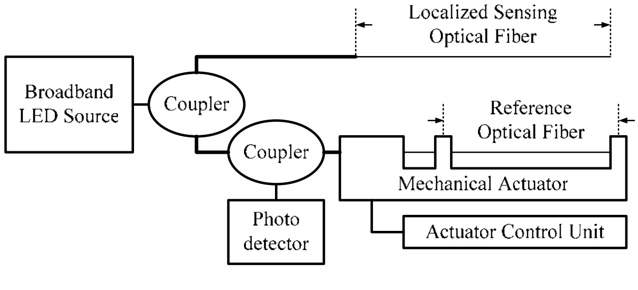


Figure 1. Schematic of long gauge instrument

3 MATHEMATICAL MODELS FOR BEAMS

3.1 *Average strain vs. maximum strain*

Beams are structural members that carry transverse loads applied at the right angles to the longitudinal axis of the member. The loads cause the beam to bend. For a linear elastic beam, the bending stress, σ_x, as a function of the position of x along the beam's axis can be found from the flexural formula in Eq. (1).

$$\sigma_x = \frac{M_x}{Z_x} = \varepsilon_x E \tag{1}$$

where M_x = bending moment due to loads; Z_x = elastic section modulus of a beam; ε_x = flexural strain; and E = the modulus of elasticity.

After the bending moment analysis is complete, it is necessary to determine the position of x on the beam where the maximum moment occurs. For the assessment of the safety of a beam, the maximum moment is then compared with the allowable stress of the beam based on design specifications.

In general, the maximum value of stress and the position where the maximum occurs in a beam structure in service can not be calculated by analysis due to uncertainties structural parameters required in structural modeling and variations of the intensities of loads required in structural analysis. Therefore, with the various types of strain sensors, the maximum strain measured is used in evaluation of the maximum stress as in Eq. (1). However, there are many difficulties in determining the maximum stress in a beam with point sensors since the actual stress distribution induced a beam by varying amounts and types of loads is non-uniform. In this case, the reliability of evaluated safety depends on the number and location of point sensors. It is not also realistic to increase the number of sensors to overcome these drawbacks.

Average strain, $\varepsilon_{ave.}$, defined by the ratio of the relative displacement between two points of x_1 and x_2 to the distance between the two points along the axis of a beam can be written as

$$\varepsilon_{ave.} = \frac{\int_{x_1}^{x_2} \varepsilon(x)}{l_{x2-x1}} \tag{2}$$

where l_{x2-x1} = the distance between the two points x_1 and x_2.

If the strain measured is the average value measured over the region from x_1 to x_2 of a beam, the strain variation or stress distribution of a beam can be considered. However, the maximum strain within the region from x_1 to x_2 of a beam can not be measured from the average strain measured. It is necessary to develop the mathematical models for determination of the maximum strain of a beam from average strains measured by long gage optic sensors.

3.2 *Mathematical models*

Mathematical models are derived by defining the relation between the average strain measured from the long gage sensor and the maximum strain of two span beams. Two span beams subjected to two types loading conditions such as point and distributed loads with variable magnitudes are considered in this study.

3.2.1 *Continuous beams subjected to point loads*

As shown in Fig. 2, a continuous beam with spans of lengths l_1 and l_2 is subjected to a point load of P_1 acting at the arbitrary distance *a* from the left-hand support on one span and a point load of P_2 acting at the arbitrary distance *b* from the left-hand support on the other span. The moment of inertias for the left and right spans are I_1 and I_2, respectively.

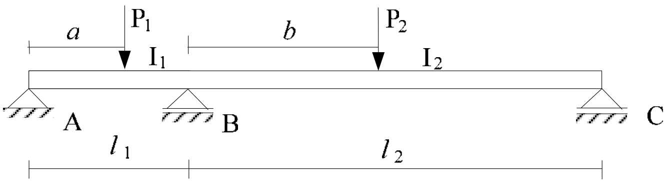
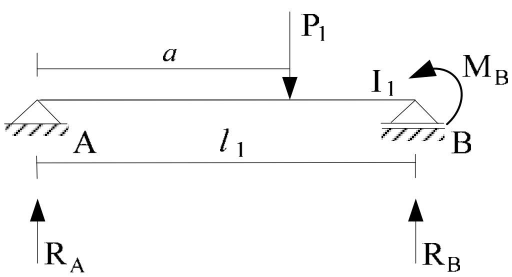
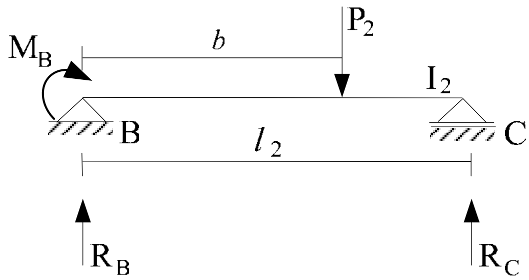


Figure 2. Two span beam subjected to with different point loads and moments of inertia



(a) Free-body diagram of the span A-B



(b) Free-body diagram of the span B-C

Figure 3. Free-body diagrams of the two span beam

From the free-body diagram of the left span shown in Fig. 3(a), the longitudinal strain of the beam, $\varepsilon_1(x)$, can be expressed as a function of the distance *x* from the left-hand support.

$$\varepsilon_1(x) = \frac{1}{EZ_1}\left(P_1 - \frac{a}{l_1}P_1 + \frac{M_B}{l_1}\right)x \tag{3}$$

where Z_1 = the elastic section modulus of the span and M_B = the bending moment at the support *B*.

The maximum strain in the beam having the concentrated load occurs at the point where the load is applied. The maximum strain of the left span, $\varepsilon_{max(1)}$, is calculated by

$$\varepsilon_{max(1)} = \frac{a}{EZ_1}\left(P_1 - \frac{a}{l_1}P_1 + \frac{M_B}{l_1}\right) \tag{4}$$

If the length of the sensor is set to the distance *a* from the left-hand support to the point where the load is applied, then the average strain over the gage length, $\varepsilon_{ave(1)}$, can be given by

$$\varepsilon_{ave(1)} = \frac{1}{a}\int_0^a \varepsilon(x)\, dx = \frac{a}{2EZ_1}\left(P_1 - \frac{a}{l_1}P_1 + \frac{M_B}{l_1}\right) \tag{5}$$

Thus, from the Eqs. (4) and (5), $\varepsilon_{max(1)}$ can be expressed in term of $\varepsilon_{ave(1)}$ can be measured from the long gage sensor.

$$\varepsilon_{max(1)} = 2\frac{a}{2EZ_1}\left(P_1 - \frac{a}{l_1}P_1 + \frac{M_B}{l_1}\right) = 2\varepsilon_{ave(1)} \tag{6}$$

Eq. (5) defines the relationship between the average strain measured from the long gage sensor and the maximum strain of a beam.

From the free-body diagram of the right span shown in Fig. 3(b), the longitudinal strain of the beam, $\varepsilon_2(x)$, can be expressed as a function of the distance *x* from the right-hand support.

$$\varepsilon_2(x) = \frac{1}{EZ_2}\left(\frac{b}{l_2}P_2 + \frac{M_B}{l_2}\right)x \tag{7}$$

where Z_2 = the elastic section modulus of the right span.

As we did for the left span, the maximum strain of the right span, $\varepsilon_{max(2)}$, is calculated by

$$\varepsilon_{max(2)} = \frac{1}{EZ_2}\left(\frac{b}{l_2}P_2 + \frac{M_B}{l_2}\right)(l_2 - b) \tag{8}$$

If the length of the sensor is set to the distance $l_2 - b$ from the support *C* to the point where the load is applied, then the average strain over the gage length, $\varepsilon_{ave(2)}$, can be given by

$$\varepsilon_{ave(2)} = \frac{1}{l_2 - b}\int_0^{l_2-b}\varepsilon(x)\,dx = \frac{l_2 - b}{2EZ_2}\left(\frac{b}{l_2}P_2 + \frac{M_B}{l_2}\right) \tag{9}$$

Thus, from the Eqs. (8) and (9), $\varepsilon_{max(2)}$ can be expressed in term of $\varepsilon_{ave(2)}$ can be measured from the long gage sensor.

$$\varepsilon_{max(2)} = 2\frac{l_2 - b}{2EZ_2}\left(\frac{b}{l_2}P_2 + \frac{M_B}{l_2}\right) = 2\varepsilon_{ave(2)} \tag{10}$$

Eq. (10) defines the relationship between the average strain measured from the long gage sensor and the maximum strain in the second span of the beam. In other words, for two-span continuous beam subjected to concentrated loads, the maximum strain in the each span is always two times the average strain from the long gage sensors regardless the location the concentrated load.

For the assessment of the safety of the continuous beam, the negative maximum strain at the support *B* must be measured for comparison of the maximum strains $\varepsilon_{max(1)}$ and $\varepsilon_{max(2)}$. The negative maximum strain at the support *b*, $\varepsilon_{max(b)}$ can be calculated from the average strains $\varepsilon_{ave(1)}$ and $\varepsilon_{ave(2)}$ without using another sensor at the top of the section of the beam at the support.

The internal bending moment at the support *B*, M_B, can be expresses as a function of the sectional properties and applied loads

$$M_B = -(XP_1 + YP_2)$$

The parameters *X* and *Y* in Eq. (11) are given by

$$X = -\frac{\dfrac{a}{I_1}\left(\dfrac{a^2}{l_1} - l_1\right)}{\dfrac{l_1 I_2 + l_2 I_1}{I_1 I_2}} \tag{12}$$

$$Y = -\frac{\dfrac{b}{I_2}\left(3b - 2l_2 - \dfrac{b^2}{l_1}\right)}{\dfrac{l_1 I_2 + l_2 I_1}{I_1 I_2}} \tag{13}$$

Then, $\varepsilon_{max(B)}$ may be expresses as a functions of the average strains and applied loads.

$$\varepsilon_{max(B)} = \frac{M_B}{EZ_1} = \frac{2l_1}{a}\varepsilon_{ave(1)} - \frac{(l_1 - a)P_1}{EZ_1} \tag{14}$$

or

$$\varepsilon_{max(B)} = \frac{M_B}{EZ_2} = \frac{2l_2}{l_2 - b}\varepsilon_{ave(2)} - \frac{bP_2}{EZ_2} \tag{15}$$

The unknown values of applied loads P_1 and P_2 in Eqs (14) and (15) can also be estimated using the measured average strains $\varepsilon_{ave(1)}$ and $\varepsilon_{ave(2)}$.

$$P_1 = K\left(\frac{(l_2 - b)(Y - b)l_1 Z_1 \varepsilon_{ave(1)}}{a} + l_2 Y Z_2 \varepsilon_{ave(2)}\right) \tag{16}$$

$$P_2 = K\left(\frac{b l_1 X Z_1 \varepsilon_{ave(1)}}{a} + (l_1 - a + X)l_2 Z_2 \varepsilon_{ave(2)}\right) \tag{17}$$

The parameter *K* is given by

$$K = \frac{-2E}{D_1 + D_2 + D_3} \tag{18}$$

where $D_1 = l_1 l_2 (b - Y)$; $D_2 = l_1(-b^2 + bY - XY)$; and $D_3 = l_2(-ab - aY + bX)$.

As mentioned previously, the maximum values of the strains and stresses are needed when assessing the safety of the beam according to the structural code or specifications. For the continuous beam subjected to concentrated loads, the maximum strains of the beam $\varepsilon_{max(1)}$, $\varepsilon_{max(2)}$, and $\varepsilon_{max(B)}$ can be calculated by the average strains from the long gage sensors $\varepsilon_{ave(1)}$ and $\varepsilon_{ave(2)}$ regardless the location the concentrated loads. Furthermore, the magnitudes of the applied loads P_1 and P_2 can also be estimated using the measured average strains $\varepsilon_{ave(1)}$ and $\varepsilon_{ave(2)}$.

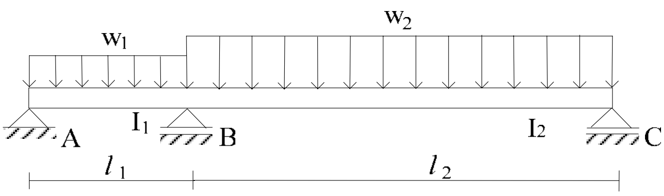


Figure 4. Two span beam with distributed loads

3.2.2 *Continuous beams subjected to distributed loads*

A continuous beam with spans of lengths l_1 and l_2 is subjected to a uniform load of w_1 acting on one span and a uniform load of w_2 acting on the other span (Fig. 4). The moment of inertias for the left and right spans are I_1 and I_2, respectively.

From the free-body diagram of the left span *A-B* shown in Fig. 5, the longitudinal strain of the beam, $\varepsilon_1(x)$, can be expressed as a function of the distance *x* from the left-hand support.

$$\varepsilon_1(x) = \frac{1}{EZ_1}\left\{\left(\frac{w_1 l_1}{2}+\frac{M_B}{l_1}\right)x-\frac{w_1}{2}x^2\right\} \quad (19)$$

The maximum strain in the continuous beam having the distributed loads occurs at the point where the bending moment is maximized. The maximum value of the bending moment occurs where the shear force equals to zero. This point can be found by setting $dM/dx = 0$ and solving for the value *x*.

$$x = \frac{l_1}{2}+\frac{M_B}{w_1 l_1} \quad (20)$$

By substituting x into the expression for the strain in Eq. (19), the maximum strain of the left span, $\varepsilon_{max(1)}$, is calculated by

$$\varepsilon_{max(1)} = \frac{1}{2w_1 EZ_1}\left(\frac{w_1 l_1}{2}+\frac{M_B}{l_1}\right)^2 \quad (21)$$

If the length of the sensor is set to the distance $l_1/4$ from the left-hand support to the point where no change in the signs of the strain is expected, as usually the case with a continuous beam with distributed loads, then the average strain over the gage length of $l_1/4$, $\varepsilon_{ave(1)}$, can be given by

$$\varepsilon_{ave(1)} = \frac{4}{l_1}\int_0^{l_1/4}\varepsilon_{(x)}\,dx = \frac{1}{96EZ_1}\left(5w_1 l_1^2+12 l_1 M_B\right) \quad (22)$$

Then, from the Eqs. (21) and (22), $\varepsilon_{max(1)}$ can be expressed in term of $\varepsilon_{ave(1)}$ can be measured from the long gage sensor.

$$\varepsilon_{max(1)} = \frac{1}{2w_1 EZ_1}\left(\frac{w_1 l_1}{2}+\frac{8EZ_1\varepsilon_{ave(1)}}{l_1}-\frac{5w_1 l_1}{12}\right)^2 \quad (23)$$

For the continuous beam subjected to loads, as we did for beams subjected to concentrated loads, the maximum strain of the right span of the beam, $\varepsilon_{max(2)}$, and the maximum strain at the support *B*, $\varepsilon_{max(B)}$ can be estimated by the average strains from the long gage sensors $\varepsilon_{ave(1)}$ and $\varepsilon_{ave(2)}$.

$$\varepsilon_{max(2)} = \frac{1}{2w_2 EZ_2}\left(\frac{w_2 l_2}{2}+\frac{8EZ_2\varepsilon_{ave(2)}}{l_1}-\frac{5w_2 l_2}{12}\right)^2 \quad (24)$$

$$\varepsilon_{max(B)} = 8\varepsilon_{ave(2)}-\frac{5w_2 l_2^{\,2}}{12} \quad (25)$$

4 TEST MODEL

The schematic diagram of the test model is shown in Fig. 5. The simple beam model consists of a simply supported H-200×200×8×12 section a length of 3 m. The beam carries two equally spaced concentrated loads as shown in the figure. The load was increased in three steps

Measurements during static testing were performed with both a long gage fiber optic sensor and five electrical strain gauges. To measure the average strain of the beam, a 4 m-long fiber optic sensor was continuously attached to the outer surface of the bottom flange of the beam. The commercially available FT optic sensors and FTI-3300 scanner has been used in this study (FOX-TEC, 2003). To get strain distributions along the length of the beam and measure strains at local points, 5 mm-long electrical strain gauges were attached parallel to the long gage optic sensor. The locations of the gauges are shown in Fig. 5.

Table 1. Estimated and measured maximum strains

Load (kN)	Estimated maximum strain based Eq. (26) (mm/m)	Measured maximum strain from strain gage (mm/m)
122.13	0.565	0.609
185.21	0.923	0.954
246.73	1.284	1.331

When the load applied by means of a hydraulic jack, the beam deflects downward and the maximum compressive strain occurs at the center of the span. The mathematical relationship that relates ε_{max} and ε_{ave} of the test model can be given by

$$\varepsilon_{max} = \frac{8}{7}\varepsilon_{ave} \quad (26)$$

The maximum strain at the center of the beam, the average strain from the long gage sensor, and estimated maximum strain based on the mathematical model in Eq. (26) are given in Table 1. The maximum differences are found to be 4.67 % in average.

Fig. 6 shows the comparison between the estimated maximum strains obtained through the present model and local strain distributions measured directly by electrical strain gages. When the equally spaced point loads are applied, the distributions of measured strain from electrical gages are expected to be symmetric about the center of the beam. However, Fig. 6 clearly shows that the strain distributions are not symmetric about the point C. In other words, as usually the case with in experimentation, the magnitudes of the point loads applied are not the same.

Load (kN)	Estimated maximum strain based Eq. (26) (mm/m)	Measured maximum strain from strain gage (mm/m)
122.13	0.565	0.609
185.21	0.923	0.954
246.73	1.284	1.331

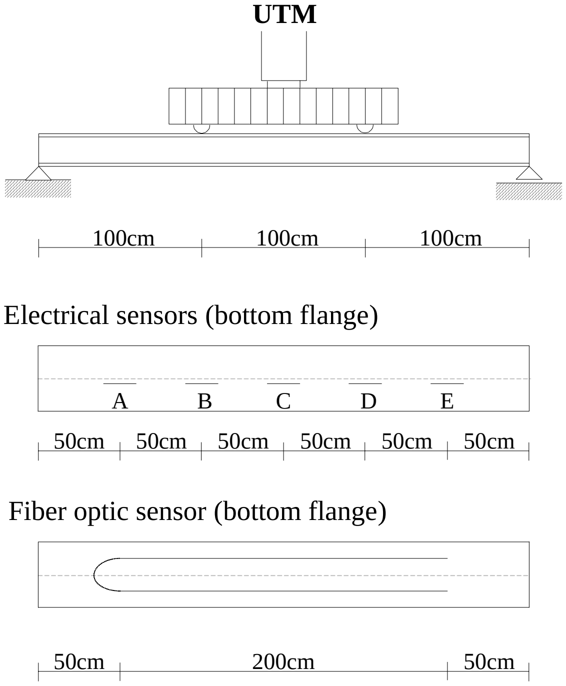


Figure 5. The schematic diagram of the test model

Table 2. Estimated and measured maximum strains with modification of loading conditions

It is found that magnitudes of the left and right concentrated loads are 43.4 % and 56.6 % of the applied concentrated load, respectively. After modification of the loading conditions, the estimated maximum strains obtained from long gage sensor and local strain distributions measured directly by electrical strain gages are compared in Table 2. The maximum differences are reduced to 2.28 % in average.

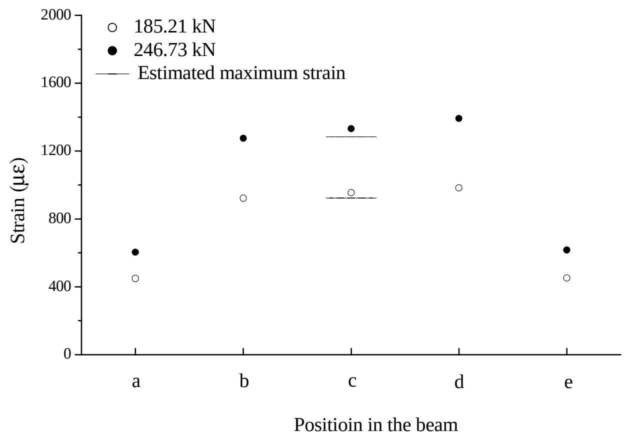


Figure 6. Comparison between the estimated maximum strains and measured strains

5 CONCLUSIONS

In this paper, mathematical models for estimations of the maximum strains in continuous beam structures are derived to overcome drawbacks involved in point sensors.

In the estimation models, the maximum strains are defined as functions of measured average strains from long gage optic sensors. This means that the strain variation or stress distribution of a beam can be considered by long gage sensors.

The model was tested on an experiment by comparing the maximum strain directly obtained from electrical gages and the estimated maximum strain based on the average strain from long gage optic sensors. The maximum values of strains estimated from the presented model agreed quite well with the directly measured values from electrical strain gauges. The estimated values of the maximum strains or stresses in beam can be used in assessment of the safety of beams in building and infrastructures.

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