Sectional damage coefficient of R.C. beam with corroded steel bar in compressive area

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ABSTRACT: This paper presents a concept of sectional damage coefficient of reinforced concrete beam with corroded steel bar in compressive area. Factors on sectional damage coefficient have been analyzed to get some of important properties of it such as its curve shape of three-folded linear to rust which is confirmed by experiment. Theories results are compared with experimental ones. After the relationship of corrosion amount and model parameters of structure has been founded through sectional damage coefficient, a new dynamic damage identified method with corroded bars in compressive area of bending components has been presented. The corrosion amount of steel bar in concrete can also be gotten indirectly by this kind of method. Consequently, the sectional damage coefficient presented in this paper has a pretty good usage value.

1 INRODUCTION

when corrosion of bars occurs in compressive area of reinforced concrete, axial compressive stress of concrete combination of expansive stress of corrode bars makes concrete bearing bidirectional compressive and tensile stress, under this kind of situation, the compressive strength of concrete will decrease (Xide Zhang 2004), then influence the behaviour of the whole structure (Zhang 2003, Roberto Gapozucca & Cerrfi M. Nilde. 2000). The study on this damage is very little, one reason is that when this kind of destruction occurs, the ratio of reinforcement is very low and concrete does not crack until the rust reaches a certain value. therefore, only little attention has been paid on it, another reason is the complication of the study, although there are many models on destruction laws of concrete under complicated stresses and it is out of the problem to study the structural behaviour of concrete under complicated stresses by digital calculation method, but it's very difficult to set up relationship between corrosion amount and expansive force of steel bar, some scholars had set up the relationship based on the corrosion mechanism of steel bar, but rust is a material with very complicated mechanical performance, so other models are important to detect the corrosion of bar.

2. SECTIONAL DAMAGE COEFFICIENT

2.1 *Definition of Sectional Damage Coefficient*

Section damage comprises various aspects, with steel bars in compressive area corroded and affected by expansive force of steel bars, concrete will be in a condition of bidirectional different stresses, reduction of concrete leads to the shift of neutral axis, and increase of curvature of section. We frequently apply integrated index to appraise behaviour of structure, for example, relative neutral-axis depth in concrete beam can be adopted to appreciate the models of beam collapse and rate of steel bars, shear-span ratio is introduced to determine collapse model of structure .Referring to analogical method degree of sectional damage with corroded bar in compressive area can be estimated comprehensively through a coefficient without dimension.

To some extent, the deterioration reflects degree of section damage caused by increase of strain in the compressive area and shift of neutral axis, therefore, section stiffness is a integrated index of section performance. But it's disadvantage to take stiffness as integrated value index directly, for stiffness is not a coefficient without dimension, it's value is pertinent to section size, which is disadvantageous to the compare of different structure, therefore, stiffness ratio of pre-corrosion and post-corrosion is defined as a comprehensive target called sectional damage coefficient that reflects degree of section damage:

$$\chi = \frac{B_{co}}{B_0} \tag{1}$$

where χ =sectional damage coefficient; B_0 = actual section stiffness before the steel bar is corroded. B_{co} =effective stiffness when the steel bar has been corroded.

According to the definition, the sectional damage coefficient is a positive number less than 1. Rust of bar leads to the deterioration of sectional stiffness, moreover, sectional damage coefficient will reduce when the rust quantity increases which means seriousness of section damage. It is suitable to use this coefficient to appreciate the degree of section damage.

2.2 *Influence element*

2.2.1. *Influence of corrosion quantity*

A main effect on sectional damage coefficient is rust quantity, for section damage is induced by the rust bar. The relationship between sectional damage coefficient and rust quantity of bar should be consistent with that between effective section stiffness and rust quantity of bar.

The Figure1 gives the relationship between sectional damage coefficient χ and corrosion quantity ρ which is analyzed by computer. The relationship approaches a three-folded linear, and the shift point shows that strength of damaged concrete is equal to maximal stress caused by load, that is $\sigma_c = f_y$, and yield of tensile bar is produced by section damage.

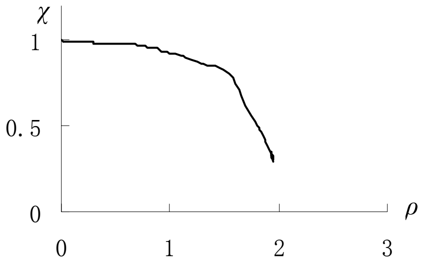

Figure.1 Influence of rust quantity

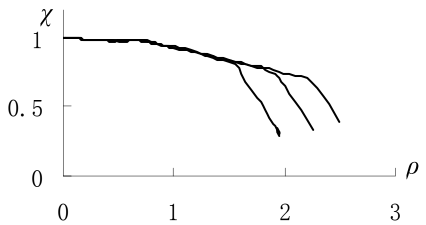

Figure.2 Influence of acting moment

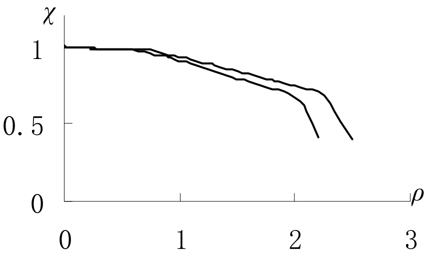

Figure.3 Influence of section dimension

2.2.2. *Influence of acting moment*

The moment of inertia of cracked bending concrete section is constant which is not only relevant to ratio of reinforcement but also the load. So the section stiffness changes with the value of acting load. But from the definition of sectional damage coefficient can be formulated as:

$$\chi = \frac{B_{co}}{B_0} = \frac{\phi_0}{\phi_{co}} \tag{2}$$

Where ϕ_0=section curvature pre-corrosion; ϕ_{co} = section curvature post-corrosion.

According to the formula (2) sectional damage coefficient has no thing to do with acting moment.

Figure.2 is drawn according to the relationship between sectional damage coefficient and rust quantity which is calculated through various acting moment. The curve in the Figure.2 which is calculated under three acting moment lap over each other when rust quantity is low, but if rust quantity is higher, different acting loads decide yield value of bar. The more acting moment, the smaller yield value of bar, which produces that curve shift point is early.

From what is related above, acting moment has no impact on the relationship between sectional damage coefficient and corrosion quantity before tensile bar yields, sectional damage coefficient just varies with corrosion quantity when other influence element is defined.

2.2.3. *Influence of section dimension*

Component has variety of shapes and sizes, depths of neutral axis and the height, so the section stiffness varies as well, which works on damage coefficient.

Fig.3 gives the relationship between coefficient and corrosion quantity with the same geometrical size but different section heights. Relationship between damage coefficient and rust quantity is irrelevant when the section geometrical sizes vary, but when the corrosion quantity is small, there is no effect on damage coefficient according to the computing as σ_{cc} less than σ_c.

2.2.4. *Influence of ratio of reinforcement*

Different ratio of reinforcement corresponds with various damaging coefficient. Damaging coefficient is defined by the ratio of section stiffness pre-rust and post- rust. Influence elements comprise not only the element enumerated above, but also the ratio of reinforcement.

When the dimension of section and acting moment is constant and ratio of reinforcement is different, the ratio of reinforcement decides the yield figure in the course of the corrosion of tensile bar, that is to say, the higher the ratio of reinforcement is, the more corrosion quantity is when tensile bar yields.

2.3 *Characteristic*

According to the analysis above, the characteristic of sectional damaging coefficient can be summarized as.

(1)Damage coefficient is a without dimension coefficient.
(2)Damage coefficient is a positive decimal fraction less than l. the less χ is, the more serious the damaging coefficient is. There is no damage if χ is equal to l.
(3)The relationship of damage coefficient and corrosion quantity can be considered as a three-folded-linear.
(4)Acting load has no effect on damage coefficient before tensile bar yields in the same section. The influence is outstanding after the tensile bar yields.
(5)Size of section has some influence on damage coefficient when the corrosion quantity is lower, load stress less than the concrete strength after concrete is damaged. There is no relationship between damaging coefficient and size of section when σ_c less than f_c. The damage coefficient is a function whose only parameter is corrosion quantity.

3. COMPARE BETWEEN THEORETICAL AND EXPERIENCE

3.1 *experience and result*

Four beams were made to validate characteristic of damage coefficient. The section is rectangular, size 100mm×120mm, length 2200m, span 2000mm, a bar of HPR335 that the diameter is 12mm was placed in the tensile and compressive area separately.

Table 1. Test loads

Number	limit-load kN/m	test-load kN/m	radio
PCB-1	8.3	2.97	0.36
PCB-2	8.3	3.43	0.41
PCB-3	8.3	4.80	0.58
PCB-4	8.3	6.4	0.77

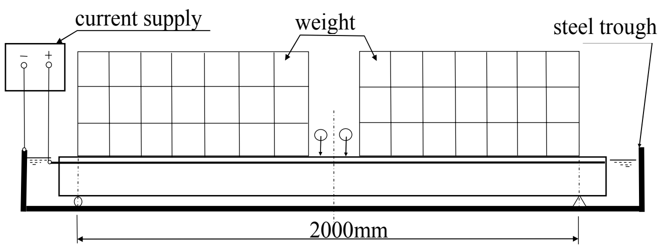

Figure.4 Test and load

Cover of bar in the compressive area is 20mm. Design strength of concrete is c20. $CaCl_2$ that amount proportion to concrete is 5% is put into the concrete, for the need of accelerating corrosion, conserving for 28 days. Actual strength of cubic concrete conserved under same condition is detected, mean strength value is 31.7Mpa.

Beams were tested as Fig.1. In order to study the influence of varying loads, different loads were used to act experimental beams. Loads were exerted as table 1 show. Weight which was uniform distributed on the top of experimental beam was adopted to keep load constant during the experimental. The load was ranked into 5-8 grades according to the design load when the beam and instrument are installed, hold the load constant and detected regularly until the data was tranquil, then, an instrument supplying direct current and constant voltage was used to accelerate bars corrode. Data were collected periodically until crack turned up in the compressive area.

3.2 *Compare of theoretical figure and experimental figure*

Damage coefficient was calculated according experimental result and definition of damage coefficient. Experimental beam stiffness was worked out in term to the follow formula：

$$B_{co} = \frac{384ql^4}{5f} \tag{3}$$

B_{co}= stiffness of rust bar in the compressive area of experimental beam; f = displacement of experimental beam; q= load acting experimental beam; l= span of experimental beam.

Stiffness of experimental beams before bar rust were deemed as initial stiffness B_0, then, calculated the stiffness of experimental beam according to formula (2) and stiffness of varying corrosion quantity by Farad law. Theoretical figures calculated by computer and experimental figures of four experimental beams were drawn together as Figures 5-8, where ρ is the corrosion quantity.

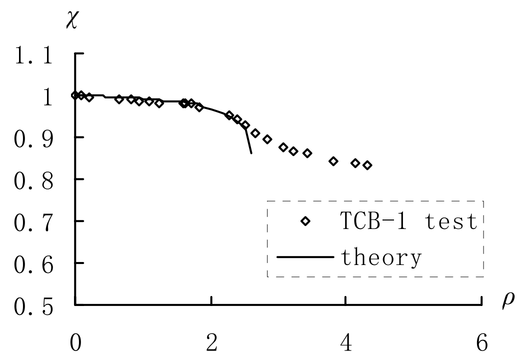

Figure 5 TCB-1 experimental and theoretical result

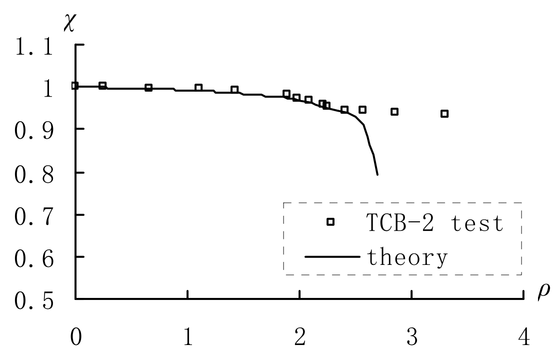

Figure 6 TCB-2 experimental and theoretical result

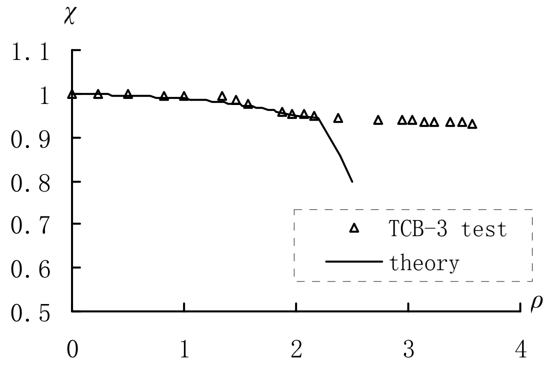

Figure 7 TCB-3 experimental and theoretical result

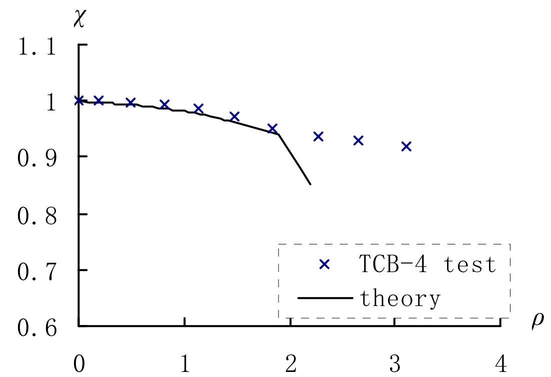

Figure 8 TCB-4 experimental and theoretical result

Damage coefficients of experimental beams are also a three-folded linear as the figures show above. But the third shift in three-folded-linear of experimental beam is not like the theoretical one to go down, but turn up, which is characteristic of the experimental damage coefficient. The theoretical and experimental figures answer for very well before concrete cracks. But after that, the damage coefficient of experimental beam doesn't change according to the corrosion quantity.

This should be rational, for tensile stress caused by corrosion will be released when concrete cracks. The increase of corrosion quantity doesn't produce the corresponding tensile stress in the concrete again. So theoretical calculation can only be used to situation before cracks caused by the corrosion of bar occur.

4. APPLICATION

While structure's own frequency can be acquired rapidly and accurately in the study of dynamic identification, the most ideal method is to detect with frequency of structure. Now there have been many experiments which had been done prove that there's tight relationship between structure's stiffness and its own frequency. Research (Agardh 1991; Kato, & Shimada 1986) done on experiments shows that there is certain relationship between reinforced concrete structure's own frequency and effective section moment of inertia, Through establishing the relationship between structure frequency and sectional damage coefficient, this article preliminarily sets up dynamic identification method of corrosion quantity of steel bar and damage degree of reinforced beam.

4.1 *Basic Assumption*

(1) Corrosion of steel bar along span of the beam is even. That is to say, once steel bar rusts, uniform corrosion will produce within the range of a certain length in beam, especially along the whole length of the beam.

(2)Whether steel bar rusts or not, the mass of the beam keeps constant.

(3)On condition that the same model and corrosion quantity of steel bar, the model function ratio of post-corrosion and pre-corrosion keeps constant.

$$\gamma = \frac{\varphi_{co}(x)}{\varphi(x)} \tag{4}$$

Fig 10 shows a variant stiffness beam, the average mass is $m(x)$, $M_1, M_2, \cdots, M_n$ are concentrated mass, at the coordinate x_1, $x_2, \cdots x_n$. When only taking into account the first stationary frequency, the beam's stationary frequency before corrosion is:

$$\omega_0^2 = \frac{\int_0^l EI(x)[\varphi''(x)]^2 dx}{\int_0^l m(x)\varphi^2(x)dx + \sum_{i=1}^n M_i \varphi^2(x_i)} \tag{5}$$

When steel bar in the compressive area of the beam produces uniform corrosion in the range of $l_1 \sim l_2$:

$$\omega_{co}^2 = \frac{\int_0^l EI_{co}(x)[\varphi''_{co}(x)]^2 dx}{\int_0^l m(x)\varphi_{co}{}^2(x)dx + \sum_{i=1}^n M_i \varphi_{co}{}^2(x_i)}$$
$$= \frac{\int_0^{l_1} EI(x)[\varphi''_{co}(x)]^2 dx + \int_{l_1}^{l_2} EI_{co}(x)[\varphi''_{co}(x)]^2 dx + \int_{l_2}^{l} EI(x)[\varphi''_{co}(x)]^2 dx}{\int_0^l m(x)\varphi_{co}{}^2(x)dx + \sum_{i=1}^n M_i \varphi_{co}{}^2(x_i)} \tag{6}$$

the ratio of the formula (6) and (5) is;

$$\frac{\omega_{co}^2}{\omega_0^2} = \frac{\int_0^l EI_{co}(x)[\varphi''_{co}(x)]^2 dx}{\int_0^l EI(x)[\varphi''(x)]^2 dx} \frac{\left(\int_0^l m(x)\varphi^2(x)dx + \sum_{i=1}^n M_i \varphi^2(x_i)\right)}{\left(\int_0^l m(x)\varphi_{co}{}^2(x)dx + \sum_{i=1}^n M_i \varphi_{co}{}^2(x_i)\right)} \tag{7}$$

When there're no concentrated mass on the beam, for uniform cross section beam, EI=const, according to the 3td assumption:

$$k = \frac{\omega_{co}^2}{\omega_0^2} = \frac{\int_0^l EI_{co}(x)[\varphi''(x)]^2 dx}{\int_0^l EI(x)[\varphi''(x)]^2 dx}$$
$$= \frac{\int_0^{l_1} EI(x)[\varphi''(x)]^2 dx + \int_{l_1}^{l_2} EI_{co}(x)[\varphi''(x)]^2 dx + \int_{l_2}^{l} EI(x)[\varphi''(x)]^2 dx}{\int_0^l EI(x)[\varphi''(x)]^2 dx} \tag{8}$$

and then:

$$k = \frac{\omega_{co}^2}{\omega_0^2} = \frac{\int_0^{l_1}[\varphi''(x)]^2 dx + \chi\int_{l_1}^{l_2}[\varphi''(x)]^2 dx + \int_{l_2}^{l}[\varphi''(x)]^2 dx}{\int_0^{l}[\varphi''(x)]^2 dx} \quad (9)$$

When steel bar in compressive area along the whole span of the beam corrodes uniformly

$$k = \frac{\omega_{co}^2}{\omega_0^2} = \chi \quad (10)$$

in the formulas above, ω_0, ω_c = beam's stationary frequency corresponding to pre-corrosion and post-corrosion of steel bar; χ =beam's sectional damage coefficient; $EI(x)$, $EI_{co}(x)$=beam's stiffness corresponding to pre-corrosion and post-corrosion of steel bar; $\varphi(x)$, $\varphi_{co}(x)$ =beam's model function corresponding to pre-corrosion and post-corrosion of steel bar; k is the ratio of the beam's stationary frequency corresponding to pre-corrosion and post-corrosion of steel bar ,it's a parameter that reflects the damage of beam; when the bars of beams corrode uniformly along the span, according to formula (10), χ is the sectional damage coefficient. The relation between k and χ can also be learned from formula (9); As the sectional damage coefficient is relevant to corrosion quantity of steel bar, so the sectional damage coefficient plays the role of connecting beam's model parameter with corrosion quantity of steel bar and then makes it possible that detecting the corrosion quantity of steel bar in compressive area with dynamic detection method.

4.3 *detection method of damage*

During the course of working with dynamic identification method, firstly accelerative sensors should be placed on the detective beam .for the first model, at least one accelerative sensor should be placed on the half span and the quarter span separately. With dynamic detection and analysis system, the first frequency of the beam corresponding to pre-corrosion and post-corrosion can be detected. Consequently, sectional damage coefficient should be solved, if the uniform corrosion takes place along the whole lengthen of the beam, sectional damage coefficient can be achieved by formula (10); if the uniform corrosion takes place in a certain range of the beam, sectional damage coefficient after the corrosion of steel bars can be achieved by formula (9); in turn, taking use of sectional damage coefficient, corrosion quantity of reinforcing bar can be calculated. Furthermore, beam's deflection, carrying capacity and other targets after the corrosion of steel bar can be achieved.

When making use of the dynamic detection method the frequency of the beam before the corrosion of steel bar should be known, firstly, this requires detecting the structure intermittently. What's more, before detecting the damage of steel bar corrosion, whether the steel bar rusts and where the corrosion takes place should be fixed, then it's more effective to detect with this method. Some literates have researched the dynamic detection method and the location of damage, but in actual engineering, some other damage such as structural overload, sectional crack and disrepair also can take place in the structure; this will lead to the decrease of the structure's frequency. So this detection method must be applied to tell different kind of damage, otherwise it will lead to the error of the outcome. It is relatively reliable to adopt the method of this article, although the detection instrument of steel bar corrosion today can't detect the material corrosion quantity of steel bar, it does be accurate to tell whether steel bar rusts. After measuring whether steel bar rusts or the location of the corrosion with the detection instrument of steel bar corrosion, corrosion quantity and extent of damage can be detected by this method. The basic steps of the method are shown in the Fig 9.

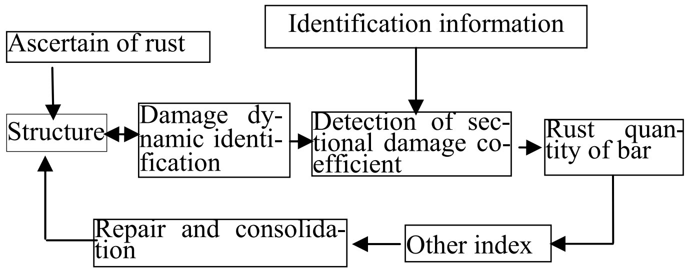

Figure. 9 Approach of damaging identification by dynamics

5. EXAMPLE

There is one simply supported beam of reinforced concrete, the span of the beam is 6000mm, there are no concentrated masses on the beam, the sectional size is 300×800mm, and the strength level of the concrete is C30, through detecting steel bar in compressive area in the range of 3000 to 7000mm from the tip of the beam（l_1=3000mm, l_2=3000mm）. At the same time, the first stationary frequency before corrosion is ω_0=3.812 and that after corrosion is ω_0=3.685. the beam's sectional damage coefficient. and the corrosion quantity can be achieced by the method of this article..

Assume the first model function is :

$$\varphi(x) = \sin\frac{\pi x}{l}$$

so: $$\varphi''(x) = -(\frac{\pi}{l})^2 \sin\frac{\pi x}{l}$$

in formula (9), each integral calculus is :

$$\alpha_1 = \int_0^{l_1}(\frac{\pi}{l})^4 \sin^2\frac{\pi x}{l}dx = (\frac{\pi}{l})^3(\frac{\pi l_1}{2l} - \frac{1}{4}\sin\frac{2\pi l_1}{l})$$
$$= 2.49668 \times 10^{-11}$$

$$\alpha_2 = \int_{l_1}^{l_2} (\frac{\pi}{l})^4 \sin^2 \frac{\pi x}{l} dx$$
$$= (\frac{\pi}{l})^3 \left[\frac{\pi}{2l}(l_2 - l_1) - \frac{1}{4}(\sin\frac{2\pi l_2}{l} - \sin\frac{2\pi l_1}{l}) \right]$$
$$= 6.89739 \times 10^{-11}$$

$$\alpha_3 = \int_{l_2}^{l} (\frac{\pi}{l})^4 \sin^2 \frac{\pi x}{l} dx = (\frac{\pi}{l})^3 \left[\frac{\pi}{2l}(l - l_2) + \frac{1}{4}\sin\frac{2\pi l_2}{l} \right]$$
$$= 1.18531 \times 10^{-12}$$

$$\alpha_4 = \int_0^l (\frac{\pi}{l})^4 \sin^2 \frac{\pi x}{l} dx = \frac{\pi^4}{2l^3} = 9.51261 \times 10^{-11}$$

from formula(9),the average sectional damage coefficient in the corrosion range is:

$$\chi = \frac{\alpha_4 \frac{\omega_{co}}{\omega_0} - \alpha_1 - \alpha_3}{\alpha_2} = 0.9096$$

When corrosion takes place along the span of the beam:

$$\chi = \omega_{co}/\omega_0 = 3.685/3.812 = 0.9667$$

According to reference (Zhang 2004) α=-0.0604, β=1.041.so the quantity of the corrosion is :

$$\rho = (\chi - \beta)/\alpha = (0.9667 - 1.041)/(-0.0604) = 1.23(\%)$$

It can be inferred from the example that dynamic identification method is feasible in theory and easy to perform. Before the beam cracks in compressive area which is caused by the expansive force of the corrosion, quantity of the corrosion is commonly little and the range of the influence is limited, otherwise, the structural frequency is in proportion to the square root of the stiffness, that is to say, the structural frequency is not very sensitive to the change of the stiffness, so it is necessary to adopt high sensitive sensors and software that can collect data and analyse frequency. Otherwise the error caused by the instrument will be very great. In this example the difference of the frequency of pre-corrosion and post-corrosion is about 3.3%, but the error of the quantity of the corrosion has reached 2.175%。

6. CONCLUSION

In order to do quantitative analysis about the damage degree of the member performance, concept of sectional damage coefficient has been presented in the article. Some important characteristic have been found, for example the relationship between corrosion quantity and sectional damage coefficient can be described as a three-folded linear. Sectional damage coefficient plays the role of connecting the parameter of the model with corrosion quantity of steel bar, which makes it possible to detect the corrosion quantity of steel bar in compressive area by dynamic detection method. Therefore, a new dynamic identification method that may be a feasible and easy to perform one, has found to detect the corrosion quantity of steel bar and damage degree of steel beams.

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