Evaluation of shrinkage and cracking in modern concrete structures at early-age

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ABSTRACT: Early age shrinkage and cracking of cement-based systems is receiving renewed attention with the development of High-Strength/High-Performance Concrete (HSC/HPC). Modern concretes are more sensitive to cracking immediately after setting, which is due to material characteristics (lower water/binder ratio and higher cement content) and external environmental fluctuations (humidity and temperature change). Based on the degree of hydration concept, early age shrinkage of concrete structures is discussed in this paper, taking into account environmental humidity and concrete creep at early age. Internal stress development of concrete structures is analyzed applying linear viscoelastic model. The risk of cracking can be estimated for HSC/HPC structures, so that it will ensure the structures safety and reliability as well as improve the durability of concrete structures.

1 INTRODUCTION

Concrete is a partially saturated porous material composed of aggregates, cementitious material and water. The water in concrete reacts chemically with the cementitious materials forming a cement paste that binds the aggregates together. The process of chemical reaction increases with time resulting in the development of the mechanical properties of the concrete. The chemical reaction generally induces intrinsic autogenous and thermal shrinkage of the concrete. Environmental fluctuations in humidity and temperature also cause volume changes of the concrete. Restraining the concrete will result in the development of restrained stresses. Cracking due to the restrained shrinkage stress has been frequently observed at early age in concrete construction. For High-Strength/High-Performance Concrete (HSC/HPC) in modern structure, it is extremely serious due to its lower water/binder ratio and higher cement content. The early-age shrinkage cracks reduce the structural performance of concrete and may accelerate deterioration to shorten the service life. Therefore, it is of great importance to control the level of restrained shrinkage stress and cracking in early-age concrete.

Based on the degree of hydration concept, early age shrinkage of concrete structures is discussed in this paper, taking into account environmental humidity and concrete creep at early age. A linear viscoelastic model for early-age concrete is proposed to analyze the restrained stress analyze the restrained stress development of concrete structures.

2 EARLY-AGE SHRINKAGE OF CONCRETE

Early-age shrinkage of concrete involves several different phenomena described with different terminology, such as thermal, plastic, chemical, autogenous, and drying shrinkage. Generally, early-age deformation leads to excessive distortions, damage and even cracking in young concrete. In this study, thermal effect is minimized and is not considered by maintaining isothermal conditions to the possible extent. Therefore, the total shrinkage counted from the time of casting consists of plastic deformation, autogenous shrinkage, and drying shrinkage. However, the plastic deformation can be regarded as stress free movement even in restrained conditions. Therefore, for the purpose of stress analysis, it is required to determine the magnitude of potential free shrinkage including autogenous shrinkage and drying shrinkage.

2.1 *Autogenous shrinkage*

It is known that autogenous shrinkage is caused by self-desiccation as a result of the hydration process. In normal strength concretes the early-age autogenous shrinkage is negligible when the water–cement ration is higher than 0.4 (Jin-Keun & Chil-Sung 1999, Miyazawa & Tazawa 2001). For HPC or HSC, the early-age autogenous shrinkage is much greater due to its higher cementitious material content and lower water-cement ratio.

Degree of hydration α is a more fundamental parameter, defined as the cement fraction that has reacted. Due to difficulties in experimentally determining the degree of hydration α, it is often defined as the fraction of the heat of hydration that has been released. The degree of hydration α and the strength of concrete can be described as follows:

$$\alpha(t) = \frac{Q(t)}{Q_{\max}} \tag{1}$$

$$S = k \cdot (\alpha - \alpha_p) \tag{2}$$

where, α, α_p = the degree of hydration at the time of interest t and the percolation threshold for the degree of hydration from which value on strength starts to develop, Q(t), Q_{max} = the total heat developed at time t and the total heat development corresponding to complete hydration, k is constant.

Realizing that the decrease of pore humidity is the physical cause of autogenous shrinkage, a direct relationship can be developed between the autogenous shrinkage and the state or pore humidity. Based on the fact that the self-desiccation is a result of hydration process, it is assumed that the degree of self-desiccation or the loss of pore humidity can be expressed as a function of the degree of hydration (Seungwook 2002).

$$\Delta h_s(t) = \frac{a + \alpha(t)}{b} \tag{3}$$

where, $\Delta h_s(t)$ = humidity reduction due to self-desiccation or autogenous drying, α(t) = degree of hydration at the time of t, a, b = experimental coefficients.

2.2 *Drying model of concrete*

The physical mechanism of drying shrinkage is similar to that of autogenous shrinkage. The difference is that the capillary and gel pore water is driven to move by the external evaporation in drying shrinkage, while it is consumed internally by the hydration in autogenous shrinkage. The humidity profile in drying concrete generally involves a gradient from the exposed surface towards the core of concrete.

In concrete, the interactions between the cement matrix and interstitial liquid are strong. When relative humidity is lower than 40%, surface tension at the liquid-solid interface is predominant, and when 50%<RH<100%, there is coexistence between disjoining pressures and capillary attraction (Hubert et al. 2003). At macroscopic level these phenomena are quite important and induce shrinkage of concrete. Drying of concrete is also a complex phenomenon. There are three types of hydric transfers: Darcy flow of liquid water, Darcy flow of vapor, and diffusive transfer of vapor. Mainguy (2001) analyzed the influence of the three kinds of transfers on mass loss kinetics of one concrete. He showed that for vapor, the Darcy movements are negligible compared to diffusive movements. Then, water transfers in liquid form with evaporation to the sides of the structure can be described, it is sufficient to correctly describe kinetics of mass loss during time by using a non-linear diffusion equation expressed as a function of saturation degree or water content.

A classical model of hydric diffusion was proposed by Bazant and Kim (1991), the expressions translating the conservation of the water mass and of the flux of relative humidity are:

$$\frac{\partial w}{\partial t} = -div(J); \qquad J = -C \cdot grad(h) \tag{4}$$

where, t = time, h = relative humidity inside the pores, w = the water content, C = the coefficient of hydric diffusion which depends on h.

The sorption-desorption isotherm is written under the w = g(h) shape, and thus:

$$\frac{\partial h_d}{\partial t} = \frac{1}{g'(h)} \frac{\partial w}{\partial t} \tag{5}$$

in which

$$\frac{\partial w}{\partial h} = g'(h) \tag{6}$$

with g(h) hydric capacity.

Based on the experimental results, g(h) can be assumed to a constant k, the non-linear diffusion equation can be obtained:

$$\frac{\partial h_d}{\partial t} = k \cdot div(C(h) grad(h)) \tag{7}$$

The moisture diffusion equation of concrete can be expressed as follows:

$$\frac{\partial h_d}{\partial t} = div(D(h) grad(h)) \tag{8}$$

where, D(h) = the coefficient of moisture diffusion, D(h)=k·C(h).

2.3 *Early-age shrinkage of concrete*

The previous studies (Hubert et al. 2003, Torrenti et al. 1999) proposed a direct relationship between the loss of water content and early-age shrinkage of concrete. Based on the experimental results, early-age shrinkage of concrete can be assumed as to be proportional to loss in mass when relative humidity is higher than 50% (Granger et al. 1997a,b). This study develops a simple model to calculate the early-age shrinkage of concrete according to the change of humidity profiles.

2.3.1 *Humidity profile in concrete*

Humidity change of concrete at early age is shown in Figure 1, it can be written:

$$\Delta h = \Delta h_d + \Delta h_s \tag{9}$$

where, Δh = total humidity reduction, Δh_d = humidity reduction due to external drying, and Δh_s = humidity reduction due to autogenous drying.

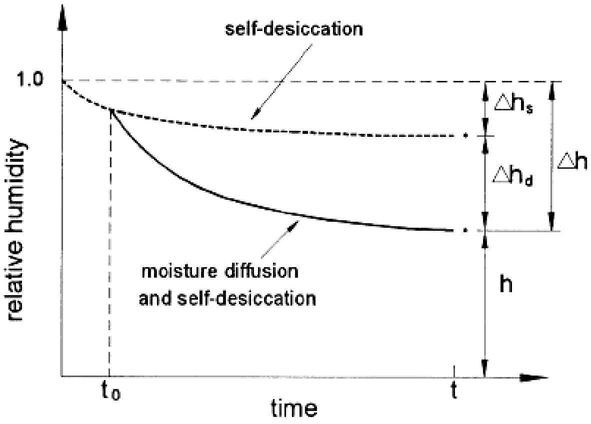

Figure 1. Humidity change of concrete at early age

Experimental results showed the distribution of relative humidity of the specimens subjected to drying and self-desiccation (Jin-Keun & Chil-Sung 1999). For high strength concrete with low water/cement ratio, self-desiccation may be more active and have a considerable influence on moisture distribution. However, for low strength concrete with high water/cement ratio, the initial internal relative humidity is fully saturated at the start of the experiment, the variation of humidity due to self-desiccation is very small.

Relative humidity changes in concrete are expressed as the variation of relative humidity due to moisture diffusion and self-desiccation of concrete:

$$\frac{\partial h}{\partial t} = \frac{\partial h_d}{\partial t} + \frac{\partial h_s}{\partial t} \tag{10}$$

According Fick's second law, moisture diffusion equation can be written:

$$\frac{\partial h_d}{\partial t} = div(D(h) \cdot grad(h)) \tag{11}$$

Humidity distribution in concrete can be obtained:

$$\frac{\partial h}{\partial t} = div(D(h) \cdot grad(h)) + \frac{\partial h_s}{\partial t} \tag{12}$$

where, D(h) denotes the moisture diffusion coefficient, In CEB-FIP (1990) model code, the moisture diffusion coefficient for isothermal conditions is expressed as a function of the pore relative humidity:

$$D(h) = D_{\max}\left[\phi + \frac{1-\phi}{1+[(1-h)/(1-h_c)]^n}\right] \tag{13}$$

$$\phi = {}^{D_{\min}}\!/\!_{D_{\max}} \tag{14}$$

where, h represents the internal relative humidity, $D_{\max}$ is the maximum of D(h) for h = 1.0, $D_{\min}$ is the minimum of D(h) for h = 0, h_c = the pore relative humidity at D(h) = $0.5D_{\max}$, n = an exponent.

2.3.2 *Concrete volume deformation due to humidity reduction*

The unrestrained deformation of concrete due to humidity changes at early age includes autogenous shrinkage and drying shrinkage, it can be written:

$$\varepsilon_{sh}(x,y,t) = \varepsilon_{AS}(t) + \varepsilon_{DS}(x,y,t) \tag{15}$$

where, ε_{sh} = total hydric shrinkage, ε_{AS} = autogenous shrinkage and ε_{DS} = drying shrinkage.

The previous experiments (Torrenti et al. 1999, Granger et al. 1997) showed that concrete shrinkage can be assumed to be proportional to the loss of concrete moisture, autogenous shrinkage and drying shrinkage can be written:

$$\begin{aligned}\varepsilon_{AS}(t) &= \beta_{AS} \cdot \Delta h_{AS} = \beta_{AS} \cdot (h_0 - h_{AS}(t)) \\ \varepsilon_{DS}(x,y,t) &= \beta_{DS} \cdot \Delta h_{DS} = \beta_{DS} \cdot (h_0 - h_{DS}(x,y,t))\end{aligned} \tag{16}$$

where, β_{AS} and β_{DS} = autogenous and drying shrinkage coefficient, respectively, h_0 = initial humidity (100%). Both autogenous and drying shrinkage are caused by the loss of pore humidity and its microscopic mechanism to exert the deformation is identical, therefore, a single value of the shrinkage coefficient (β_{sh}) can represent the two coefficients (β_{AS} and β_{DS}). The shrinkage coefficient should be experimentally determined and calibrated for each specific case.

3 RESTRAINED STRESS DUE TO SHRINKAGE OF CONCRETE

Concrete deformations due to early-age shrinkage can develop to significant stress and lead to cracking in restrained conditions. It is well known that early-age concrete exhibits creep or relaxation behavior that is a typical response of viscoelastic materials. A viscoelastic constitutive model is proposed to analyze stress development of the restrained concrete. The generalized Maxwell model is selected to discuss the viscoelastic mechanical properties of concrete at early age, as shown in Figure 2.

According to Boltzmann superposition principle (Tingqing 1990), the viscoelastic constitutive relations with relaxation modulus can be written:

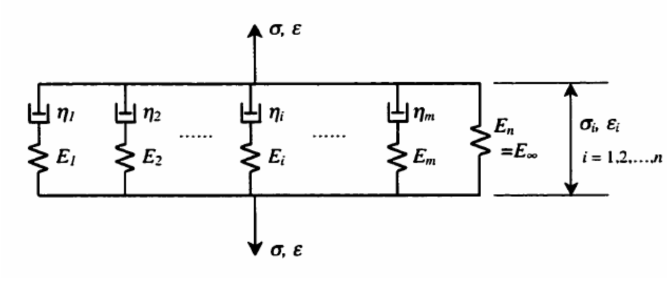

Figure 2. The generalized Maxwell model

$$\sigma_{ij}(t) = \int_0^t Y_{ijkl}(t-\xi)\frac{\partial \varepsilon_{kl}(\xi)}{\partial \xi} d\xi \qquad (17)$$

where, σ_{ij} = the stress tensor, ε_{kl} = the strain tensor, ξ = the initial loading time, Y_{ijkl} = the fourth order tensor of relaxation modulus. Y_{ijkl} can be represented as:

$$Y_{ijkl}(t) = \frac{1}{3}[Y_2(t) - Y_1(t)]\delta_{ij}\delta_{kl} + \frac{1}{2}Y_1(t)(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}) \qquad (18)$$

$$Y_1(t) = 2G(t), \qquad Y_2(t) = 3K(t) \qquad (19)$$

where, $Y_1(t)$, $Y_2(t)$ = independent relaxation modulus functions, δ_{ij} = the Kronecker delta. G(t) = the shear relaxation function, K(t) = the volumetric relaxation function, $K(t) = \lambda(t) + 2G(t)/3$.

The constitutive relation can be written:

$$\sigma_{ij}(t) = \int_0^t \left[\lambda(t-\xi)\delta_{ij}\frac{\partial \varepsilon_{kk}(\xi)}{\partial \xi} + 2G(t-\xi)\delta_{ij}\frac{\partial \varepsilon_{ij}(\xi)}{\partial \xi} \right] d\xi \qquad (20)$$

Taking into account early-age shrinkage of concrete due to humidity reduction, if the coupled effects are not considered, the constitutive equations will involve hygro-mechanical relations as follows:

$$\sigma_{ij}(t) = \int_0^t Y_{ijkl}(t-\xi)\frac{\partial \varepsilon_{kl}(\xi)}{\partial \xi} d\xi + \int_0^t \varphi_{ij}(t-\xi)\frac{\partial \omega(\xi)}{\partial \xi} d\xi$$
$$(21)$$

where, $\lambda(t)$, $G(t)$, $\varphi(t)$ should be experimentally defined properly.

4 EVALUATION OF CRACKING DUE TO CONCRETE SHRINKAGE AT EARLY AGE

The hydration reaction in early-age concrete causes the development of mechanical properties after casting. At the same time, the restrained shrinkage of concrete will result in the development of restrained stresses. Therefore, cracking due to the restrained shrinkage stress may be observed at early-age concrete when the stress is higher than the resistance to cracking, as shown in Figure 3.

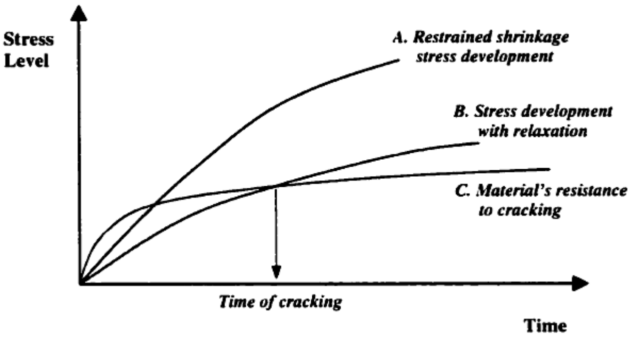

Figure 3. Restrained stress and concrete cracking at early age

The resistance to cracking of concrete is offered by the tensile strength which is developing at early age. Several studies has indicated that the cracking stress was always lower than the nominal tensile strength of concrete, the ratio of the cracking stress to the tensile strength is approximately 50% to 75% (Bentur 2001) or 80% (Altoubat & Lange 2001). Therefore, a failure analysis based on strength criterion isn't accurate to estimate time of first crack in restrained concrete. A strength reduction factor must be applied for prediction of concrete shrinkage cracking at early age.

5 CASE STUDY

A cylinder specimen of concrete is studied which has a low w/c (0.35). The concrete specimen is allowed to dry only through the top surface. At the drying surface, a convection type boundary condition is assumed with a film coefficient (5mm/day). The environmental humidity is 50% RH. The humidity reduction due to self-desiccation and external drying is shown in Figure 4.

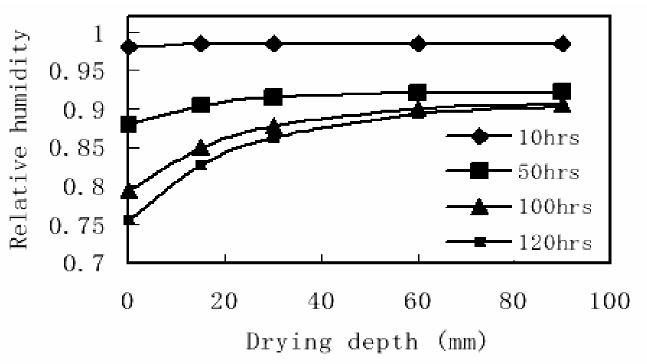

Figure 4. Relative humidity reduction of early-age concrete

Figure 5. shows the comparison between the measured shrinkage deformation (See 2003) and calculated shrinkage of concrete. It seems that the

estimation of the deformation has similar tendency and is close to the measured value.

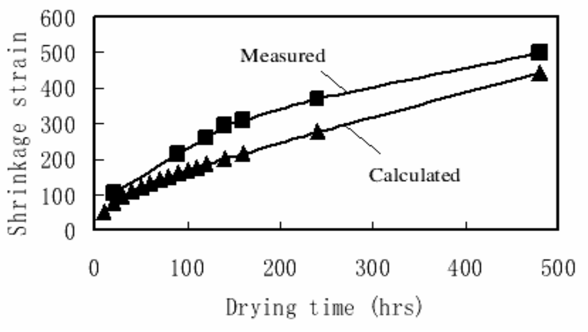

Figure 5. Measured and calculated shrinkage of the concrete

The shrinkage stress development of concrete upon a constant strain rate at early age is calculated using the viscoelastic model in this paper and the elastic model respectively, as shown in Figure 6. It is indicated that the stress development decreased considerable value with time due to concrete creep and relaxation properties at early age. The stress relaxation occurs at approximately 50% when it is compared to their elastic responses.

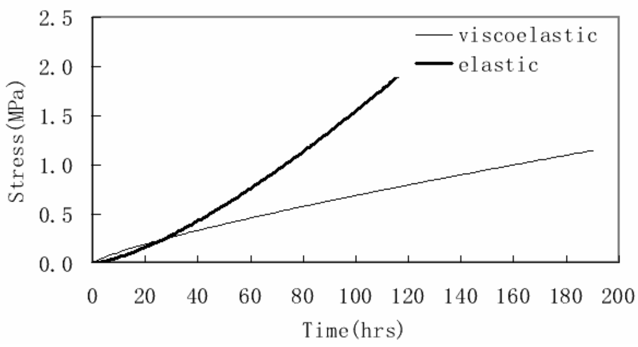

Figure 6. Elastic and viscoelastic stress development of the concrete specimen

6 CONCLUSIONS

Concrete deformations due to combination of autogenous and drying shrinkage can develop and lead to early age sensitivity to cracking in restrained conditions. For the purpose of stress analysis, a linear viscoelastic material model is implemented that represents realistic constitutive behavior of early age concrete. The assessment of the risk of cracking can be based on the model with the input of the estimated humidity changes in the concrete.

In this paper temperature effects on the overall behavior of early-age concrete is not be included, as for follow up work, its effects to the restrained stress development and cracking should be considered. A much more comprehensive approach is required in modern concrete structures to control the stress and predict risks of cracking, such as the development of mechanical properties at early age, curing practices, restraining conditions, viscoelastic responses and the role of reinforcing bars

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