An improved vibration method in the applications of arch bridge suspender tension tests

D.S. Li & J.P. Ou
School of Civil Engineering, Harbin Institute of Technology, Harbin, Heilongjiang, China

ABSTRACT: If the arch bridge suspenders are idealized as taut strings, this idealization simplifies the analysis that may introduce unacceptable errors in many applications. Analyzed the errors' cause, a simple and advantage application formula for the vibration method are proposed herein taking the effects of bending stiffness and terminal condition of the suspenders into account. Utilizing the testing natural frequency, the suspender tensions are easily estimated by this formula. At last, the capabilities of this formula are demonstrated through examples, the results show that the computing values of using this formula is well coincided with the theory values by the finite element method.

1 INTRODUCTION

A prominent application of structural cables is in arch bridges, where suspenders are arguably the most crucial elements of the entire structure (Mehrabi,1998). Its using lives and states will decide safeties of the whole bridge. It is very necessary to monitor its stress. The accurate measurement of suspender tensions is the key of the arch bridge trouble-free construction and successful repair (Zhi Fang 1997). The testing value of suspender tensions will exist some deviations through comparison of the theory values, because they suffer some effects of the testing instruments, computing pattern, anchoring ways, valid length computing, and inclination of dual suspender. Therefore, accurate measurement of suspender tensions has practical importance and a simple, speedy, and reliable method of measured is needed by the field engineers (Zui,1996)

The common suspender tension testing methods have load sensor measurement, stretching jack, oil pressure jack, and frequency method and so on (Jun Xu, 2001). The front three ways belong to direct testing tools. Due to easily operated, they are commonly applied in suspender tensions control during construction, but they exit two shortcomings, (1) the instrument is too expensive, (2) they can only test terminal suspender tensions, in fact, during the construction, if some positions of suspenders are locked while stretching, that will appear this phenomena that terminal suspender forces are very great, while the middle suspender forces are very small. These cases often occur in arch bridges construction. As the miniaturized electron devices appearing, the testing precision improving, facilitating mounting, the testing instrument being repeated used, this makes vibration method to test suspender tension becomes reality. Especially to in-service suspender, vibration method to test suspender tension probably is a unique way. But vibration method to test suspender tension have some defects, if the arch bridge suspenders are idealized as taut strings, this idealization simplifies the analysis that may introduce acceptable errors in many applications by ignoring the bending stiffness and choosing different boundary condition, the reason of causing errors is because suspenders' length is shorter than inclined cables, its boundary condition can't be simplified hinge, bending stiffness can't be ignored. If considered the suspender clamped at both ends and bending stiffness, the computing formula will become very complex, it isn't facility using in practical engineering, so it is necessary to seek a simple formula of computing suspender tension.

2 FREQUENCY METHOD OF CABLE TENSION FORCE TESTS

It is assumed that it isn't considered disturbed forces, the sag-to-span ratio is sufficiently small, and the cables vibrates only within the *xy*-plane and its motion in the *x*-direction is ignored, the condition of equilibrium of vertical forces leads to

$$\rho \frac{\partial^2 y}{\partial^2 t} + EI \frac{\partial^2 y}{\partial x^4} - T \frac{\partial^2 y}{\partial^2 x^2} = 0 \qquad (1)$$

where EI =bending stiffness; ρ =mass per unit length; T = cable tension force; y = lateral in-plane displacement due to vibration.

The free vibration equation's solution of two special boundary conditions is given in the following.

2.1 *The suspenders are hinged at both ends*

By means of variable separation and solution, the general solution of equation (1) is

$$T = \frac{4\rho L^2 f_n^2}{n^2} - \frac{n^2\pi^2 EI}{L^2} \tag{2}$$

where f_n = values of the nth order natural frequency of suspenders; L = the length of suspenders, the remain signs is the same of equation (1).

As long as knowing the nth natural frequency, the suspender tensions can be easily solved by equation (2). The distance of adjacent frequency is a equal gap by ignoring the bending stiffness. Its value equals to the first frequency. Although considering bending stiffness, using equation (2) to compute suspender tension isn't still accurate, sometimes the computing values may introduce unacceptable errors. The most main cause is because the boundary condition can't be simplified hinge.

2.2 *The suspenders are clamped at both ends*

According to the Irvine's solving result, the equation (1) of free vibration of obtained:

$$2n\pi\eta_n(1-\cos\alpha L\cosh\beta L)+\xi\sin\alpha L\sinh\beta L=0 \tag{3}$$

where

$$\alpha L=\frac{\xi}{\sqrt{2}}\sqrt{\sqrt{1+(\frac{2n\pi\eta_n}{\xi})^2}-1}$$

$$\beta L=\frac{\xi}{\sqrt{2}}\sqrt{\sqrt{1+(\frac{2n\pi\eta_n}{\xi})^2}+1}$$

$$\xi=\sqrt{\frac{T}{EI}}L \qquad \eta_n=\frac{f_n}{f_n^s}$$

where $f_n^s=\frac{\omega_n}{2\pi}$; f_n^s is the nth order natural frequency of taut string.

The solution η_n for a given value of ξ can be obtained by iteration such as the Newton Rapson method. The value of ξ can be reflected the effect of bending stiffness versus the natural frequency though its expression. The figure 1 shows the relationship of the value of ξ versus the no dimensional natural frequency (the ratio value of considering bending stiffness obtained the natural frequency and the theoretical value of the natural frequency of a beam clamped at both ends, $\overline{f}=\frac{f_1}{f_s^1}$).

From the figure1, it was found that when ξ becomes very small, the no dimensional natural frequency $\overline{f}$ increases very rapidly, and it becomes difficult to obtain exact solution. According to practice engineering experience, the ξ value of arch suspenders is generally than less 50, by computing, when ξ equals 50, $\overline{f}$ equals percent 4.6.Therefore,if adopted the formula of taut strings, this idealization simplifies the analysis may introduce unacceptable error, this is the reason that computing values of using formula(2) is not coincided with the theory values by the finite element method. It is complex to compute suspenders tension using equation(3), so accurate measurement of the suspender tensions has practical importance and a simple, speedy, and reliable method is needed.

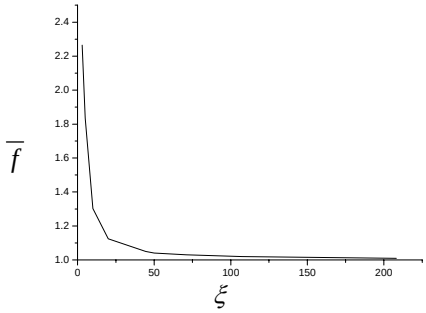

Fig 1 The relationship of ξ value and the no dimensional natural frequency

3 A SIMPLIFIED COMPUTATION FORMULA OF SUSPENDER TENSION FORCE

Through the above analysis, if considering the suspender practical boundary conditions and bending stiffness, the computing formula is very complex, if not considering the cases, the computing result may introduce unacceptable errors, so it is very necessary to look for an accurate, simple method. According to reference (Wenfeng Liu, 2003), the natural frequency can be expressed in the following type:

$$\omega^2=\frac{EI\int_0^l\varphi''(x)dx+T\int_0^l\varphi'^2(x)dx}{\rho\int_0^l\varphi^2(x)dx} \tag{4}$$

where $\varphi(x)$ is the mode function.

The mode function of the beam clamped at both ends can be written:

$$\varphi(x)=\frac{\cos\alpha_n l-\cosh\alpha_n l}{\sinh\alpha_n l-\sin\alpha_n l}(\sinh\alpha_n x-\sin\alpha_n x)+\cosh\alpha_n x-\cos\alpha_n l \tag{5}$$

The frequency equation of the beam clamped at both ends can be expressed:

$$1-\cosh\alpha_n l\cos\alpha_n l=0 \tag{6}$$

$\alpha_1 l$ can be get by (6): $\alpha_1 l$ =4.73

By $\alpha_1 l$ substituting into (5), the first-order mode of suspenders can be given as follows:

$$\varphi(x) = 0.9825 * (\sin\frac{4.73}{l}x - \sinh\frac{4.73}{l}x) + \cosh\frac{4.73}{l}x - \cos\frac{4.73}{l}l \quad (7)$$

Substitution of equation (7) into (4), by integrating, the natural frequency can be written:

$$\omega^2 = \frac{12.14T}{\rho l^2} + \frac{502.15EI}{\rho l^4} \quad (8)$$

The simplified suspender tension computing formula can be expressed taking the effects of bending stiffness and terminal condition of the suspenders into account:

$$T = 3.25\rho l^2 f_1^2 - \frac{41.36EI}{l^2} \quad (9)$$

In practical testing, for the natural frequency amplitude is too small, it is very difficult to identify it from frequency spectrum figure. In order to use equation (9) computing suspender tension, it is necessary to how to determine natural frequency.

4 NATURAL FREQUENCY DETERMINATION OF SHORT CABLES

Because the sensors are placed the position in practical testing which the hands can reach, the natural frequency signals aren't apparent, the high frequency signals are prominent among pick-off signals. Adopting equation (9) becomes inconvenient, due to can't rightly determine natural frequency. According to past documentation, they determine the natural frequency using the method that the adjacent resonance frequency value subtracts, this method is feasible to long cables. From short cables frequency spectrum figure, we can find the phenomenon that the spacing of adjacent frequency becomes big with frequency order increasing, so to short cables, this method is not precise. Based on these reasons, the natural frequency is decided using the following method (Haijun Wu, 2001): firstly it is found the maximum peak value from the frequency spectrum figure, secondly it is identified it that belongs to which order frequency, finally using its frequency value f_n divides its order, the ratio is considered as its natural frequency. Adopting the method to define natural frequency, it brings two merits: (1) because the amplitude of choosing frequency f_n value is the maximum in the frequency spectrum figure, its interference is much less than others. (2) the error is also much less than the direct fundamental frequency method, this is the reason that even if f_n has ΔF error compared with the practical n multiple fundamental frequency, by dividing, the error becomes $\Delta F/n$. The equation (9) can be written:

$$T = \frac{3.25\rho l^2 f_n^2}{n^2} - \frac{41.36EI}{l^2} \quad (10)$$

5 ENGINEERING APPLICATIONS

Ebian Dadu river is a through steel pipe concrete arch bridge, its main span length is 138 meters. Suspenders are composed of twelve high-strength cables, in order to evaluate its safety, in 2005 year we detect all suspenders.

The acceleration transducers are placed the position which the hands can reach (Fig 2).

Fig 2 transducers placed and acquisition system

The excitation ways utilize ambient excitation, every times testing time is ten minutes, and every suspender tests three times. Sample frequency is 100Hz, sample length is 1K. Pick-off signals are used FFT transformation to get its each frequency. Assuming the numbering is increased from Chengdu to Ebian orientation. A few representative suspenders are chosen to verify this article method validity. The numbering is the shortest No.1, No.3, No.5, No.7, No.9, No.11, the longest No.13 No.15 suspenders. Table 1 shows the suspender properties used in verification.

Table 1 suspender parameters

suspender numbering	suspender Length(*m*)	unit length Mass(*kg.m*)	bending Stiffness(*kN.m*)
No.1	8.89	22.92	600.27
No.3	13.19	25.29	600.27
No.5	20.21	28.39	600.27
No.7	24.01	28.93	600.27
No.9	26.72	29.24	600.27
No.11	28.31	29.39	600.27
No.13	28.84	29.43	600.27
No.15	27.64	28.58	600.27

Pick-off signals are used FFT transformation. The figures of the shortest and the longest suspenders time domain and frequency-domain are given in following:

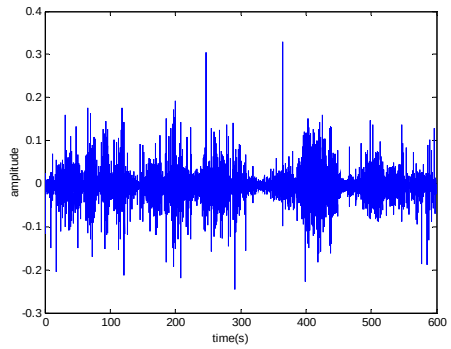

Fig3 the shortest suspender time domain signal

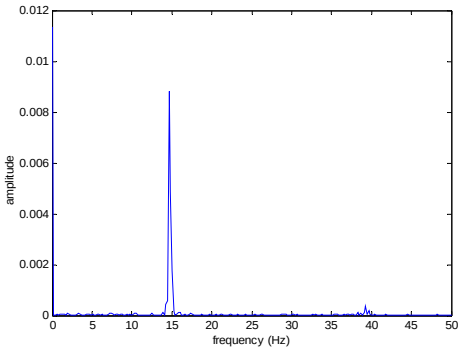

Fig4 the shortest suspender frequency domain signal

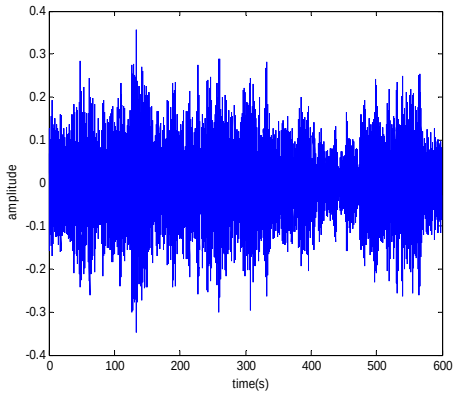

Fig5 the longest suspender time domain signal

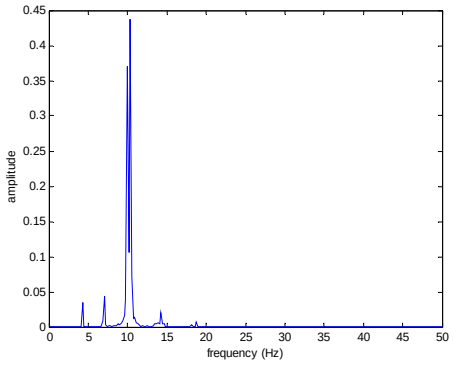

Fig6 the longest suspender frequency domain signal

From above the two figures it is found that the shortest suspender frequency domain signals appear signal peak value, this because the acceleration transducers are placed in center of it, the main mode is the first order, others modes isn't predominant. The longest suspenders frequency domain figure appears multiple peak value, theirs spacing distance isn't uniformly-spaced; it is increasing with mode order.

Three methods are introduced to computer suspender tension. The first method doesn't consider bending stiffness in hinged boundary condition. The second method considers bending stiffness in hinged boundary condition. The third method is used in article method. The theory value is the result by calculating finite element program ANSYS. Table 2 shows different computing result:

From lower the table it is found that the computing result is coincided with the theory value using this article's method, except for the shortest and No.15 suspenders. The rest two methods can't well reflect the truth result, the errors are too much great, the main reason is the bending stiffness isn't neither considered, nor the boundary condition isn't ignored. The shortest suspender tension isn't accurate cause that because its length is too short, the measuring ambient has a great effect on the testing frequency than others suspenders, so it isn't suit to use frequency method measure its tension. From equation (2), (10), it is found even if the testing frequency compared with the truth value has a little discrepancy, the suspenders tension will produce greatly errors. It is advised adopting others method to computing its tension. To No.15 suspender, the testing value is much less than theory value , by checking, it was found the under anchor's nut doesn't contact cross beam, the suspenders tension is mainly accepted by the hole friction, and the anchor appeared rap. The anchor case is shown in figure 6.

Fig7 anchor assemble case

In a word, when using frequency method test suspender tension, not only its bending stiffness is

considered, but also its boundary condition can't be ignored. Using this article method can successfully solve these problems.

Table 2 different method computing result

suspenders numbering	method 1 T1(*kN*)	method 2 T2(*kN*)	method 3 T3(*kN*)	theory T0(*kN*)	error (T1-T0)/T0	error (T2-T0)/T0	error (T3-T0)/T0
No.1	1432.51	1424.93	849.82	301.20	375.60%	373.08%	182.14%
No.3	647.84	645.25	418.71	409.14	58.34%	57.71%	2.34%
No.5	591.13	589.72	419.51	400.20	47.71%	47.36%	4.83%
No.7	549.95	548.45	403.43	399.19	37.77%	37.39%	1.06%
No.9	547.33	546.47	409.91	399.20	37.11%	36.89%	2.68%
No.11	526.36	525.52	396.69	399.30	31.82%	31.61%	0.65%
No.13	557.31	556.64	422.93	400.10	39.29%	39.12%	5.71%
No.15	455.07	454.23	338.76	399.15	14.01%	13.80%	15.22%

6 CONCLUSION AND DISCUSSION

Because the arch suspenders difference with inclined cables, if they are idealized as taut strings, this idealization simplifies the analysis may introduce acceptable errors. To solve this problem, based on the principle of frequency method measuring cables tension, firstly the effect of each parameter versus natural frequency was analyzed, it was found it was found that when ξ becomes very small, the no dimensional natural frequency $\overline{f}$ increases very rapidly, and it becomes difficult to obtain exact solution. Secondly the simplified suspenders tension computing formula was deduced, and introduced the natural frequency determine method. Finally a practical engineering was checked using this article method, the results show that the computing values of using this formula is coincided with the theory values by the finite element method, have a very high precision.

AKNOWLEDGEMENTS

This study was financially supported by NSFC under grant No.50420120133 and Ministry of Transportation under WTST grant No.200331882010.

REFRENCE

Armin B.Mehrabi, Habib Tabatabai. 1998. Unified Finite Difference Formulation for Free Vibration of Cables, Journal of structure engineering: 1313-1322.

Zhi Fang, Zhiyong Zhang. 1997. Test of cable tension in cable-stayed bridges. China Journal of Highway and Transport, Vol.10(1):51-58.

Hiroshi Zui et al. 1996. Practical Formulas for Estimation of Cable Tension by Vibration Method, Journal of structure engineering: 651-656.

Jun Xu.2001.Precision Analysis of Calculating Tension Force of Cable, Journal of Tongji University29(5): 611-615.

Wenfeng Liu et al. 2003. Precise Solution of Cable Tensile Force in Consideration of Cable Stiffness and Boundary Condition, Journal of Vibration and Shock 22(4):12-14.

Haijun Wu et al. 2001. Research on the Monitoring Method of Cable Force, Journal of Chongqing Jiaotong University 20(4):23-25.