Damage identification utilizing harmonic excitation in static-like frequency-based inverse analysis

A. Świercz, P. Kołakowski & J. Holnicki-Szulc
Smart-Tech Centre, Institute of Fundamental Technological Research, Warsaw, Poland

ABSTRACT: The presented approach to damage identification is a continuation of research done within the PiezoDiagnostics (PD) project (FP5 EC Project 2001). The general purpose of the PD project was identification of corrosion (or damage of considerable extent) in pipelines. Generation and detection of a global structural mode by piezo-actuators and sensors was tested in the PD project. Perturbations of the mode due to various damage scenarios were investigated. A software tool, based on the Virtual Distortion Method (VDM), was developed (Kołakowski, Zieliński, and Holnicki-Szulc 2004). The tool is able to perform damage identification via the solution of an inverse, dynamic problem in time domain thanks to employing gradient-based optimization. A well-calibrated FE model is required for the approach in order to produce meaningful results with experimental data. In this paper, the possibility of carrying out the damage identification in frequency domain will be explored. A dynamic problem with no damping will be considered first. A number of selected excitation frequencies will be the subject of analysis. Steady-state dynamic responses will be provoked and static-like influence matrices in the framework of VDM will be built accordingly. As a consequence, the optimization process in frequency domain is expected to be significantly faster compared to the one analyzed previously in time domain. The newly formulated approach mainly reduces the vast consumption of computational time, observed in the previous approach.

1 INTRODUCTION

The damage detection systems based on array of piezoelectric transducers sending and receiving strain waves have been intensively discussed by researchers recently. The signal-processing problem is the crucial point in this concept and a neural network based method is one of the most often suggested approaches to develop a numerically efficient solver for this problem.

An alternative approach to the inverse dynamic analysis problem is based on the dynamic VDM (Virtual Distortion Method) concept, making use of a dynamic influence matrix $\mathbf{D}$. Pre-computation of the time-dependent matrix $\mathbf{D}$ allows for decomposition of the dynamic structural response into components caused by external excitation in undamaged structure (the linear part) and components describing perturbations caused by the internal defects (the non-linear part). As a consequence, analytical formulas for calculation of these perturbations and the corresponding gradients can be derived. The physical meaning of the virtual distortions used in this paper are externally induced strains (non-compatible in general, e.g. caused by piezoelectric transducers, similarly to the effect of non-homogeneous heating). The compatible strains and self-equilibrated stresses are structural responses to these distortions.

Assuming possible locations of all potential defects in advance, an optimization technique with analytically calculated gradients can be applied to solve the problem of the most probable defect locations. The considered damage can affect the local stiffness as well as the mass distribution modification. It is possible to identify the position as well as intensity of several, simultaneously generated defects.

The proposed methodology can be applied e.g. to corrosion detection (reduction of material thickness), and identification of its location in steel pipelines, using long-distance transmissions of impulses. This time-domain-based methodology of data processing for damage identification (VDM based PD-software, cf. Ref. (Kołakowski, Zieliński, and Holnicki-Szulc 2004; Świercz and Zieliński 2004) fits well to the following technique of measurements (PD-hardware):

i) wave generator produces a low frequency impulse of flexural wave with long-distance prop-

agation,

ii) few well located, distanced sensors collect measurements of frontal section of the transferred wave,

iii) if the received structural response differs significantly from the reference response (for undamaged structure), the collected measurements are transmitted to a computer centre for further data processing (damage identification).

There is a class of problems where a concept similar to the above-mentioned VDM approach, but based on frequency-domain rather than time-domain response can be applied. This numerically economical method can be addressed to problems, where steady-state response can be the basis of dynamic analysis. For example, the following tasks can be solved on the basis of the VDM-F (*Virtual Distortion Method in Frequency Domain*) method:

- remodelling of vibrating system with harmonic excitation in order to reduce vibrations in a selected area,
- identification of material/structural properties on the basis of monitored structural responses for samples of harmonic excitations,
- detection and identification of damages (via inverse dynamic problem) on the basis of monitored structural responses for samples of harmonic excitations.

The objective of this paper is to investigate the third of the above-mentioned problems.

2 PROBLEM FORMULATION

In order to present basic formulas of the VDM-F method, let us focus on quick remodelling of vibrating truss structures under harmonic excitation. Having an existing structure and its parameters we could introduce some modifications to those parameters and then calculate its response, i.e. displacements and internal forces of the modified structure. The general form of equations of motion for a multi-degree of freedom case is given as follows:

$$\mathbf{M}\ddot{\mathbf{u}}(t) + \mathbf{C}\dot{\mathbf{u}}(t) + \mathbf{K}\mathbf{u}(t) = \mathbf{f}(t), \tag{1}$$

where $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are mass, damping and stiffness matrices, respectively and $\mathbf{f(t)}$ is the vector of external forces. Each of the above-mentioned matrices represents a set of parameters, which can be modified in the following way:

$$\hat{\mathbf{M}}\,\ddot{\mathbf{u}}(t) + \hat{\mathbf{C}}\,\dot{\mathbf{u}}(t) + \hat{\mathbf{K}}\,\mathbf{u}(t) = \mathbf{f}(t), \tag{2}$$

where $\hat{\mathbf{M}} = \mathbf{M} + \Delta\mathbf{M}$, $\hat{\mathbf{C}} = \mathbf{C} + \Delta\mathbf{C}$, $\hat{\mathbf{K}} = \mathbf{K} + \Delta\mathbf{K}$ describe modifications of the mass, damping and stiffness matrices, respectively. The modification parameters cause non-linear variations of the mass as well as stiffness matrix, which influence the structural response $\mathbf{u}$. The VDM-F based formulation allows to calculate this response quickly (for modified structure) for given (modified) structural parameters. Knowing the responses for original and modified structure, the damage identification process leads to multiple re-computations of dynamic responses with imposed modifications on original structure. This paper is concentrated on the remodelling problem neglecting the damping component.

3 VIRTUAL DISTORTION IN FREQUENCY DOMAIN

From now on, it is assumed that the structure is subjected to a harmonic excitation. Substituting Equation:

$$\mathbf{f}(t) = \mathbf{f}\sin(\omega t), \tag{3}$$

to Equation 1 and 2, the expected response $\mathbf{u}$ can be written in the following form:

$$\mathbf{u}(t) = \mathbf{u}\sin(\omega t). \tag{4}$$

Modifications of stiffness and mass distribution are modelled by *virtual distortions* denoting initial strains in structural elements and virtual forces in structural nodes, oscillating with the same frequency as external excitation:

$$\boldsymbol{\varepsilon}^{\mathbf{0}}(t) = \boldsymbol{\varepsilon}^{\mathbf{0}}\sin(\omega t), \qquad \mathbf{p}^{\mathbf{0}}(t) = \mathbf{p}^{\mathbf{0}}\sin(\omega t), \tag{5}$$

where the first quantity models stiffness, while the second one the mass redistributions, respectively.
Let us call the *modified structure* — a structure in which changes were introduced to the mass and stiffness matrix and the *modelled structure* — a structure in which changes are modelled by virtual distortions, without changing mass and stiffness matrices. The equations of motion for the modified and modelled structures can be obtained introducing virtual distortion component(s) and substituting quantities for steady-state problem (Equations 3, 4, 5) (cf. (Holnicki-Szulc, Pawłowski, and Wikło 2003)) to Equations 1 and 2.

$$-\omega^2 \hat{M}_{ij}u_j + G_{\alpha i}\hat{S}_{\alpha\beta}G_{\beta j}u_j = f_i, \tag{6}$$

$$-\omega^2 M_{ij}u_j + G_{\alpha i}S_{\alpha\beta}L_{\underline{\beta}}(\varepsilon_{\underline{\beta}} - \varepsilon^0_{\underline{\beta}}) = f_i + p^0_i. \tag{7}$$

where, $\hat{S}_{\alpha\beta}$ and $S_{\alpha\beta}$ are diagonal matrices with $\hat{S}_{\alpha\alpha} = E_{\underline{\alpha}}\hat{A}_{\underline{\alpha}}/l_{\underline{\alpha}}$ and $S_{\alpha\alpha} = E_{\underline{\alpha}}A_{\underline{\alpha}}/l_{\underline{\alpha}}$, respectively ($E_\alpha$ —

Young's modulus, A_α — cross section ($\hat{A}_\alpha$ modified) and l_α length of an element α), L_β — is a vector of lengths of structural elements. In the Equation 6 (and next formulas) there is no summation over underlined indices. The Greek letters run over structural elements and the Latin ones are related to degrees of freedom of a considered structure. The matrix $G_{\alpha i}$ is a transformation matrix, whose elements are related to cosines of angles between elements and directions of degrees of freedom. In the Equation 6, 7, the time-dependent components have been eliminated. The displacement depends only on the frequency and the amplitude can be decomposed as follows:

$$u_i = u_i^L + D_{i\alpha}^{\varepsilon}\varepsilon_\alpha^0 + D_{ik}^{p}\, p_k^0, \tag{8}$$

where: $D_{i\alpha}^{\varepsilon}$ — influence matrix denoting amplitude of displacement u_i generated by unit, harmonic strain distortion with amplitude $\varepsilon_\alpha^0 = 1$ of frequency ω applied in element α. Matrix D_{ij}^{p} — influence matrix denoting amplitude of displacement u_i generated by unit, harmonic force with amplitude $p_i^0 = 1$ of frequency ω applied in j-th degree of freedom.

It is postulated that response of the structure modelled by virtual distortions has to be identical with the response of the modified structure. Therefore, for each element, which is modified, the compatibility of strains and stresses is required:

$$P_\alpha = E_{\underline{\alpha}}\hat{A}_{\underline{\alpha}}\,\varepsilon_{\underline{\alpha}} = E_{\underline{\alpha}}A_{\underline{\alpha}}\left(\varepsilon_{\underline{\alpha}} - \varepsilon_{\underline{\alpha}}^0\right). \tag{9}$$

Assuming that the cross sections of structural elements are modified, the vector of stiffness modification can be expressed as follows:

$$\mu_\alpha = \frac{\hat{A}_{\underline{\alpha}}}{A_{\underline{\alpha}}} = \frac{\varepsilon_{\underline{\alpha}} - \varepsilon_{\underline{\alpha}}^0}{\varepsilon_{\underline{\alpha}}}. \tag{10}$$

The vector μ_α is the vector of structural modification, which involves modification of the mass as well as stiffness matrix. The updated vector of strain ε_α can be obtained through multiplying Eqn (8) by $\frac{1}{L_{\underline{\alpha}}}G_{\underline{\alpha}i}$:

$$\varepsilon_\alpha = \varepsilon_\alpha^L + B_{\alpha\beta}^{\varepsilon}\varepsilon_\beta^0 + B_{\alpha k}^{p}\, p_k^0. \tag{11}$$

where:

$$B_{\alpha\beta}^{\varepsilon} = \frac{1}{L_{\underline{\alpha}}}\,G_{\underline{\alpha}i}D_{i\beta}^{\varepsilon}, \qquad B_{\alpha k}^{p} = \frac{1}{L_{\underline{\alpha}}}\,G_{\underline{\alpha}i}D_{ik}^{p}. \tag{12}$$

Substituting Equation 11 to Equation 10, the first (of two) relation between the vector of stiffness modification μ_α and virtual distortions ε_α^0 and p_i^0 can be determined:

$$\left[(\mu_{\underline{\alpha}} - 1)B_{\underline{\alpha}\beta}^{\varepsilon} + \delta_{\alpha\beta}\right]\varepsilon_\beta^0 + (\mu_{\underline{\alpha}} - 1)B_{\underline{\alpha}k}^{p}\, p_k^0 = (1 - \mu_{\underline{\alpha}})\,\varepsilon_{\underline{\alpha}}^L. \tag{13}$$

Equation 13 contains two kinds of virtual distortions ε_α^0 and p_k^0. In order to determine those distortions, let us determine the second relationship from Equation 6 and 7:

$$p_i^0 = \omega^2(\hat{M}_{ij} - M_{ij})\,u_j = \omega^2\Delta M_{ij}\,u_j, \tag{14}$$

where u_i is described by Equation 8. Let us assume (for simplicity) the diagonal mass matrices M_{ij} and $\hat{M}_{ij}$. Their difference can be determined as follows:

$$\Delta M_{ij} = \frac{1}{2}\sum_\alpha(\mu_{\underline{\alpha}} - 1)\rho_{\underline{\alpha}}A_{\underline{\alpha}}l_{\underline{\alpha}}\,M_{ij}^{(\alpha)}, \tag{15}$$

where a_{ir}^{α} determines relation between degrees of freedom of a finite element α (indices r, s) and degrees of freedom of the whole structure and $\frac{1}{2}\rho_{\underline{\alpha}}A_{\underline{\alpha}}l_{\underline{\alpha}}\,M_{ij}^{(\alpha)}$ is the global mass matrix calculated for a structural element α. Thus, let us substitute the Equation 15 into the Equation 14:

$$-\omega^2\Delta M_{ij}(\mu_\alpha)D_{j\beta}^{\varepsilon}\varepsilon_\beta^0 + \left(-\omega^2\Delta M_{ij}(\mu_\alpha)D_{jk} + \delta_{ik}\right)p_k^0 = \omega^2\Delta M_{ij}(\mu_\alpha)u_j^L. \tag{16}$$

Finally, the formula for determining virtual distortions ε_β^0 and p_k^0, taking into account Equation 13 and 16, can be written:

$$\mathbf{A}\,\mathbf{d^0} = \mathbf{b^L}, \tag{17}$$

where: $A_{11} = (\mu_{\underline{\alpha}} - 1)B_{\underline{\alpha}\beta}^{\varepsilon} + \delta_{\alpha\beta}$,
$A_{12} = (\mu_{\underline{\alpha}} - 1)B_{\underline{\alpha}k}^{p}$, $A_{21} = -\omega^2\Delta M_{ij}(\mu_\alpha)D_{j\beta}^{\varepsilon}$,
$A_{22} = -\omega^2\Delta M_{ij}(\mu_\alpha)D_{jk}^{p} + \delta_{ik}$,
$b_1^L = (1 - \mu_{\underline{\alpha}})\varepsilon_{\underline{\alpha}}^L$, $b_2^L = \omega^2\Delta M_{ij}(\mu_\alpha)u_j^L$.
The virtual distortions ε_α^0 and p_k^0 obtained from the above set of equations model modification of cross section area A_α (see Equation 10) of structural elements. Using Equation 8 (or Equation 11) the displacements field (strain field) can be quickly calculated. It was presented in (Świercz, Kołakowski, and Holnicki-Szulc 2005) how the VDM models response of the modified structure.

4 DAMAGE IDENTIFICATION TECHNIQUE

The result of the damage identification indicate the severity of damaged structural element(s) and their location. This inverse problem leads to minimization of a suitable objective function. The objective function has to depend on structural modification parameters to be detected. The accuracy of the result is related

with the number of measured responses (sensors). Let us propose the objective function as follows:

$$f = (\varepsilon_\psi - \varepsilon_\psi^M)(\varepsilon_\psi - \varepsilon_\psi^M), \tag{18}$$

where $\varepsilon_\psi = \varepsilon_\psi(\mu_\vartheta)$ is the vector of strain in the considered structure with internal (unknown) defects μ_ϑ modelled by virtual distortions (ε_α^0, p_i^0) and ε_ψ^M is the measured response — in this case obtained numerically — of the damaged structure. In the above formula index ψ runs over selected structural elements. To minimize the objective function (Equation 18), the steepest descent method can be used. To this end, let us calculate the gradient of the objective function $\frac{\partial f}{\partial \mu_\vartheta}$:

$$\frac{\partial f}{\partial \mu_\vartheta} = \nabla_\vartheta f = \frac{\partial f}{\partial \varepsilon_\psi}\frac{\partial \varepsilon_\psi}{\partial \varepsilon_\alpha^0}\frac{\partial \varepsilon_\alpha^0}{\partial \mu_\vartheta} + \frac{\partial f}{\partial \varepsilon_\psi}\frac{\partial \varepsilon_\psi}{\partial p_i^0}\frac{\partial p_i^0}{\partial \mu_\vartheta}. \tag{19}$$

In the Equation 19 the distortion gradients can be determined through differentiation from Equation 23. The partial derivatives $\frac{\partial \varepsilon_\psi}{\partial \varepsilon_\alpha^0}$ and $\frac{\partial \varepsilon_\psi}{\partial p_i^0}$ can be obtained through differentiation of Equation 11, accounting for Equation 12:

$$\frac{\partial \varepsilon_\psi}{\partial \varepsilon_\alpha^0} = B^\varepsilon_{\psi\alpha}, \qquad \frac{\partial \varepsilon_\psi}{\partial p_i^0} = B^p_{\psi i}. \tag{20}$$

In order to determine gradients $\frac{\partial \varepsilon_\beta^0}{\partial \mu_\vartheta}$ and $\frac{\partial p_i^0}{\partial \mu_\vartheta}$ let us differentiate Equation 13 and 16, respectively:

$$\left[(\mu_{\underline{\alpha}} - 1)B^\varepsilon_{\underline{\alpha}\beta} + \delta_{\underline{\alpha}\beta}\right]\frac{\partial \varepsilon_\beta^0}{\partial \mu_\vartheta} + (\mu_{\underline{\alpha}} - 1)B^p_{\underline{\alpha}k}\frac{\partial p_k^0}{\partial \mu_\vartheta} = -\delta_{\underline{\alpha}\vartheta}\varepsilon_{\underline{\alpha}}, \tag{21}$$

$$-\omega^2 \Delta M_{ij} D^\varepsilon_{j\beta}\frac{\partial \varepsilon_\beta^0}{\partial \mu_\vartheta} + \left[-\omega^2 \Delta M_{ij} D_{jk} + \delta_{ik}\right]\frac{\partial p_k^0}{\partial \mu_\vartheta} = \omega^2 M_{ij}^{(\vartheta)} u_j. \tag{22}$$

Similarly to the Eqn (17), the set of equations concerning distortion gradients with respect to the vector of modification parameters is given as follows:

$$\mathbf{A}\,\mathbf{g^0} = \mathbf{b}, \tag{23}$$

where the matrix $\mathbf{A}$ is the same as in Eqn (17), moreover: $g_1^0 = \frac{\partial \varepsilon_\beta^0}{\partial \mu_\vartheta}$, $g_2^0 = \frac{\partial p_k^0}{\partial \mu_\vartheta}$, $b_1 = \delta_{\underline{\alpha}\vartheta}\left(\varepsilon^L_{\underline{\alpha}} + B^\varepsilon_{\underline{\alpha}\beta}\varepsilon_\beta^0 + B^p_{\underline{\alpha}j}p_j^0\right)$, $b_2 = \frac{1}{2}\omega^2 \rho_{\underline{\vartheta}} A_{\underline{\vartheta}} l_{\underline{\vartheta}} M_{ij}^{(\vartheta)}\left(u_j^L + D^\varepsilon_{j\beta}\varepsilon_\beta^0 + D^p_{jk}p_k^0\right)$.
Having calculated distortion gradients, the gradient-based formulation for damage identification can be applied.
Now, the gradient of the objective function can be expressed as follows:

$$\nabla_\vartheta f = 2\left(\varepsilon_\psi - \varepsilon_\psi^M\right)\left[B^\varepsilon_{\psi\beta}\frac{\partial \varepsilon_\beta^0}{\partial \mu_\vartheta} + B^p_{\psi k}\frac{\partial p_k^0}{\partial \mu_\vartheta}\right]. \tag{24}$$

The vector of modification parameters is determined iteratively according to the formula:

$$\mu_\alpha^{(s+1)} = \mu_\alpha^{(s)} - \Delta f^{(s)}\frac{\nabla_\alpha f^{(s)}}{\left[\nabla_\alpha f^{(s)}\right]^T \nabla_\alpha f^{(s)}}, \tag{25}$$

with *a priori* assumed original vector of modification parameters $\mu_\vartheta^{(0)}$ (eg. for undamaged structure), where s denotes the current iteration, $s+1$ the next one and the step length Δ is appropriately selected from the interval $(0, 1)$.

5 NUMERICAL EXAMPLES

In this section some results will be presented illustrating the methodology for the damage identification. Let us consider the simple truss-structure shown in Figure 1. It is subjected to harmonic load $F = 200\sin(\omega t)$ in nodal point 9 with applied fraquency $\omega = 1000\,[\frac{rad}{s}]$. All elements of the original structure have the same parameters: Young's modulus $E = 210\,GPa$, cross section area $A = 10^{-5}\,m^2$, density $7800\,\frac{kg}{m^3}$. Width and height of the single section is equal $1\,m$. For the inverse analysis it is assumed, that measurements (strain response) are collected from selected or all elements. The damage is defined according to Equation 10 and the dimensionless cross section areas (μ_i) are iteratively calculated. The results are presented after 100 and 500 iterations. In our investigation 3 cases of the inverse analysis are considered:

1. measurements are colected from sections III and IV (elements $11 - 20$) — one defect to detect in the whole structure, thus the index ψ in Equation 18 run over elements in sections III and IV, and the index ϑ in Equation 24 run over all structural elements ($1 - 20$). The collected data for original structure and for the modified one are given in the Table 2. The results of the inverse analysis is shown in the Figure 2.

2. measurements are collected from all sections — three defects to detect in all sections. The identified cross section areas μ_i are presented in the Figure 3.

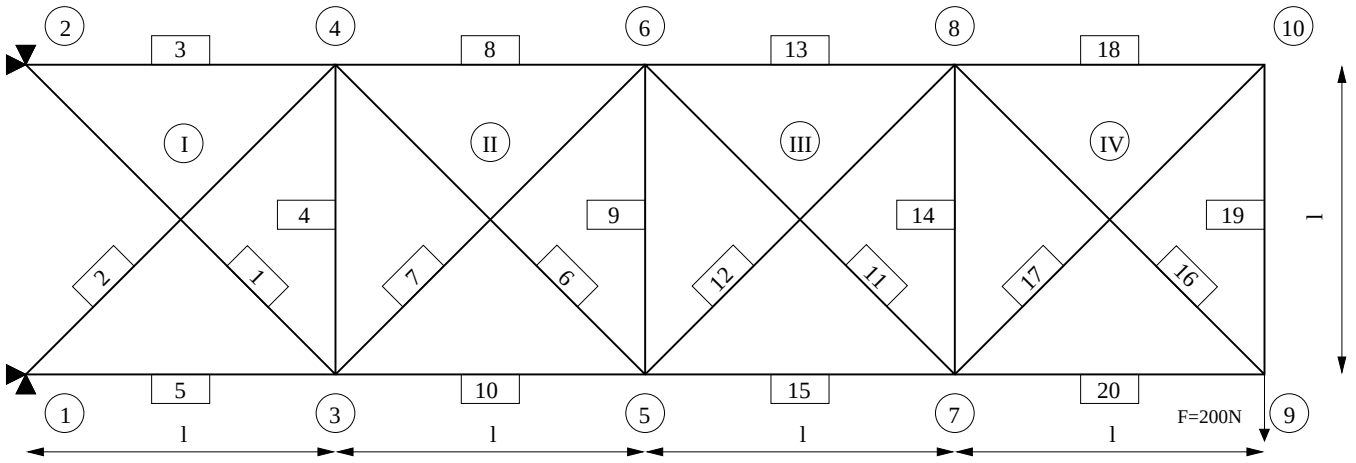

Figure 1: Tested truss structure.

Table 1: The collected data: damage case 3.

Strain's amplitude	Element's number				
	16	17	18	19	20
$\varepsilon_\alpha[\times 10^{-5}]$ (orginal)	8.086	-6.663	6.004	4.398	-7.092
$\varepsilon_\alpha^M[\times 10^{-5}]$ (modified)	12.729	-7.826	9.521	11.596	-10.393

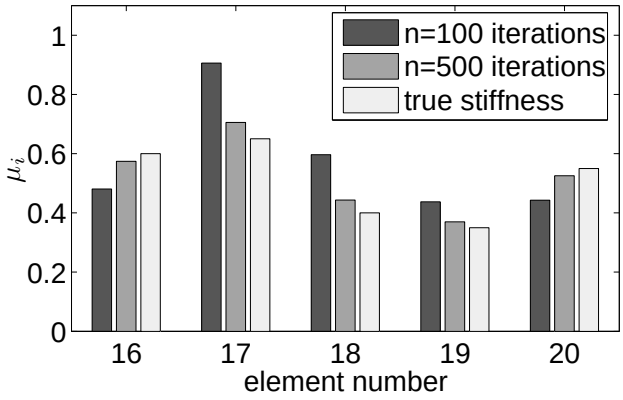

Figure 4: Results of the inverse analysis: damage case 3.

3. measurements are colected from section IV (elements $16-20$). In this case the distribution of defects (μ_i) in the IV section have to be determined based on responses from this section. The collected responses are shown in the Table 1.

The convergence of the objective function (cf. Equation 18) during the inverse analysis depends on the number of sensors and the number of cross section areas modifications μ_i to be determined. Concluding, the more sensors is used the less iterations have to be done to solve the problem.

It is important to notice that the presented results were obtained for excitation acting in one selected frequency ω. An extention of the set of excitations allows to expect, that the presented methogology will be more efficient.

6 SUMMARY AND CONCLUSION

The Virtual Distortion Method in frequency domain (VDM-F) is an useful tool to investigate steady-state problems. Static-like influence matrices are built only once for each frequency. This allows to identify multiple defects.

The presented damage identification methodology requires comparison of the reference response(s) (referring to the undamaged structure) and the damaged one. The optimization process in frequency domain based on VDM-F is expected to be significantly faster compared to the one analyzed in time domain.

ACKNOWLEDGEMENT

The authors would like to gratefully acknowledge the financial support through the 5FP EU project Research Training Networks "SMART SYSTEMS" HPRN-CT-2002-00284 and through the grant No. KBN 5T07A05222 founded by the State Committee for Scientific Research in Poland. The work presents a part of the Ph.D. thesis of the first author, supervised by the third author.

REFERENCES

FP5 EC Project (2001). Smart Structural Diagnostics using Piezo-Generated Elastic Waves, PiezoDiagnostics, No. G1RD-CT-2001-00659.

Holnicki-Szulc, J., P. Pawłowski, and M. Wikło (2003, 2–5 September). Design of adaptive structures under random impact conditions. In *AMAS/ECCOMAS/STC Workshop on Smart Materials and Structures*, Jadwisin, pp. 45–67.

Kołakowski, P., T. G. Zieliński, and J. Holnicki-Szulc (2004). Damage identification by the dynamic virtual distortion method. *15(6)*, 479–493.

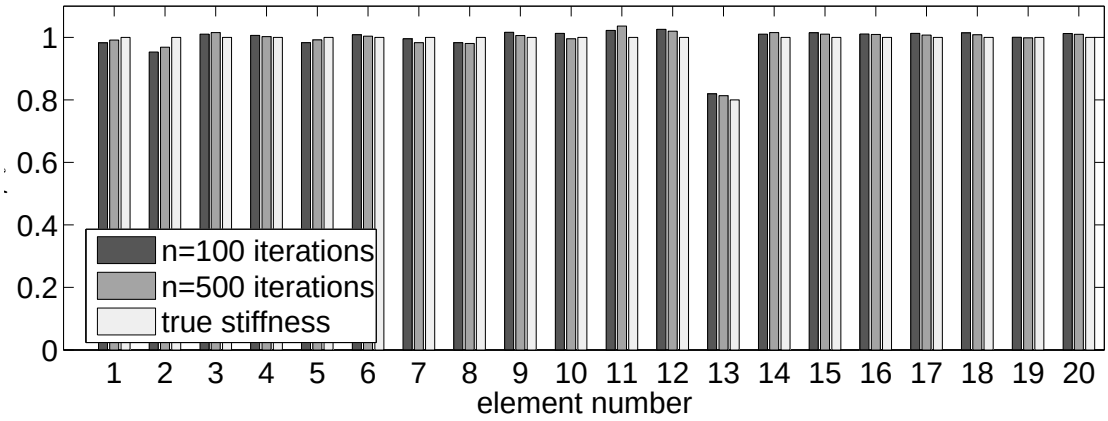

Figure 2: Results of the inverse analysis: damage case 1.

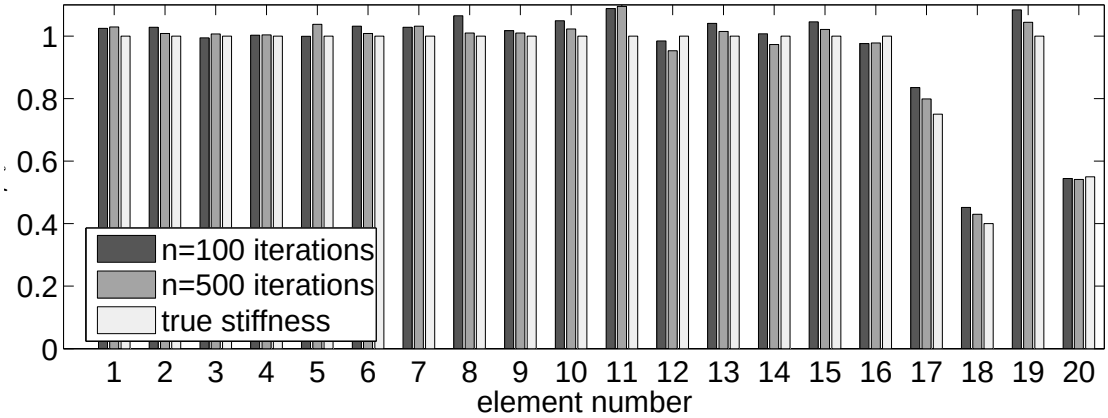

Figure 3: Results of the inverse analysis: damage case 2.

Table 2: The collected data for original and single damaged structure.

Strain's amplitude	Element's number									
	11	12	13	14	15	16	17	18	19	20
$\varepsilon_\alpha[\times 10^{-5}]$ (original)	0.685	−0.867	13.891	−0.464	−13.866	8.086	−6.663	6.004	4.398	−7.092
$\varepsilon_\alpha^M[\times 10^{-5}]$ (modified)	1.322	−0.241	18.330	−1.006	−14.591	8.326	−6.951	6.570	4.416	−7.231

Świercz, A., P. Kołakowski, and J. Holnicki-Szulc (2005, 18–21 July). Damage identification through frequency-based analysis for steady-state dynamic responses. In *II ECCOMAS Thematic Conference on Smart Stuctures and Materials*, Lisboa.

Świercz, A. and T. G. Zieliński (2004, 24-28 July). Software tools for structural damage identification. In *4th European Congress on Computational Methods in Applied Sciences and Engineering*, Volume II, Jyvaskyla, pp. 121 (abstract).