INTERNATIONAL WORKSHOP "Civil Structural Health Monitoring 2"

WIM (Weigh In Motion), LOAD CAPACITY, AND BRIDGE PERFORMANCE
In the Context of Risk Assessment, Maintenance and Life Cost Based Design

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DYNAMIC RESPONSE OF A FOOTBRIDGE TO WALKING PEOPLE

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1 The Forchheim Footbridge

The Footbridge over the Regnitz River in Forchheim, Germany is a cable stayed bridge composed of a main span of 88.1 *m* and one side span of 29.4 *m*. Two truss beams compose the cross-section. They also compose the parapets and are connected at their intrados by means of a reticular horizontal structure, which also supports the wooden deck, which is about 4.0 *m* width. In the main span the girder is suspended to three couples of fan-shaped stays, starting at the top of a steel tower and connected to the deck by means of transversal beams.

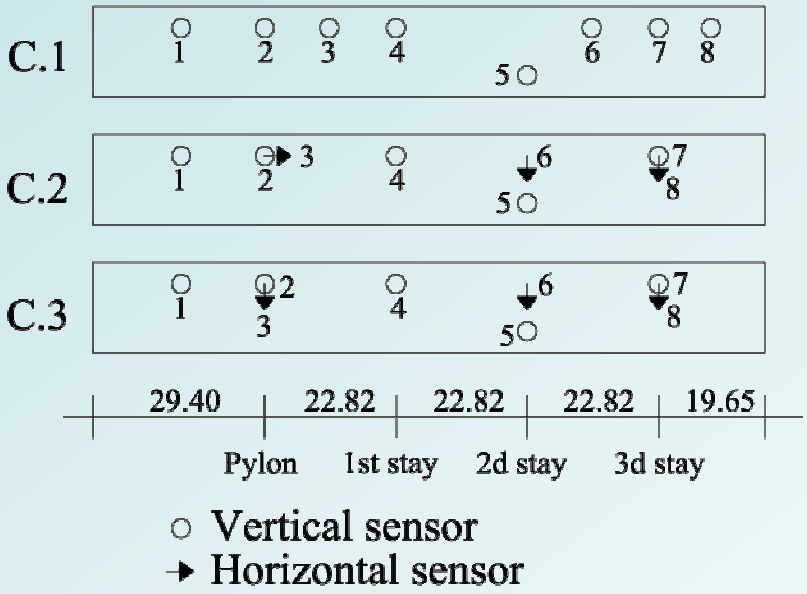
2 The Forchheim Footbridge

Cables are almost regularly spread along the deck. Two anchor cables are constrained to the abutment, i.e., to an external gravity anchoring. The girder can be supposed to be hinged to the A-shaped pylon. This is composed by two steel circular pillars (diameter $\approx$ 900 *mm*) and has a height of about 31.0 *m* from the ground. Each pillar is hinged at its base.

3 The Experimental set-up

n. 8 seismometer Kinemetrix SS1
HP3566A signal conditioner

Sensors layout in three configurations:
n. 4 recordings in configuration C.1
n. 2 recordings in configuration C.2
n. 2 recordings in configuration C.3



- Output-only measurement
- Ambient excitation
- Sampling time step: $\Delta t = 7{,}8125 \cdot 10^{-3}$ sec
- Records lenght: 64 sec

4 The Finite Element Model

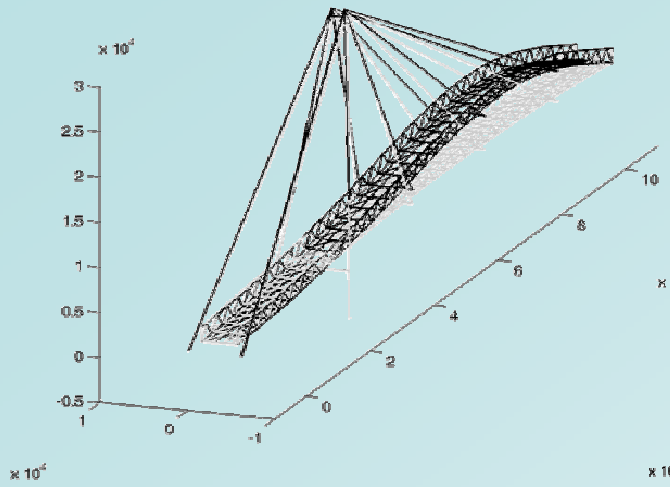
A finite element model has been developed in Matlab using 3D frame elements.

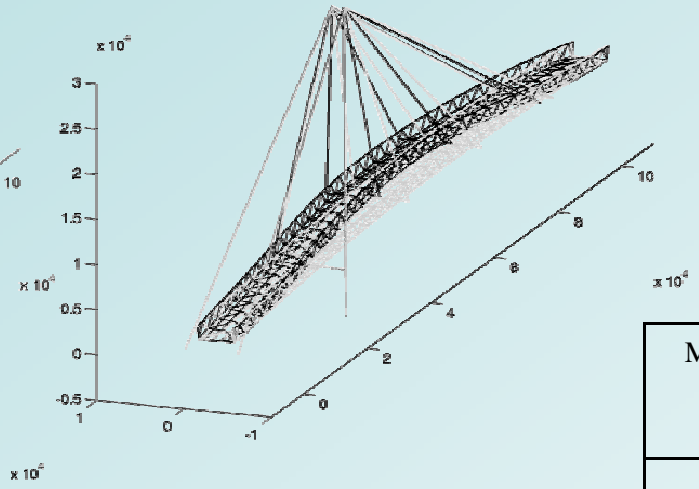
The sag effect of cables is modelled by means of the Ernst reduced modulus.

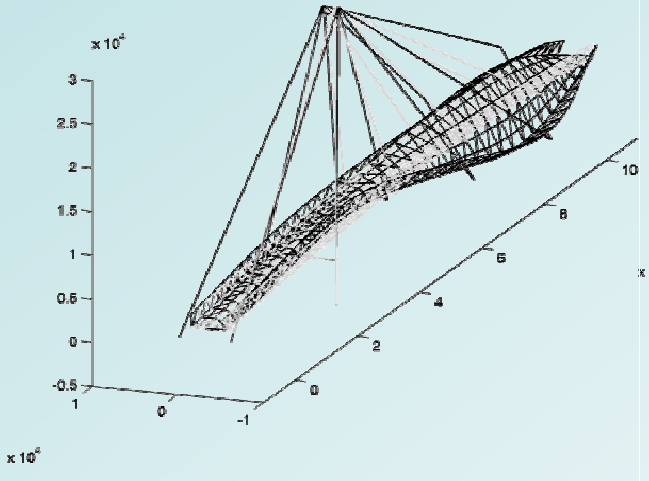
The first model was not suitable to fit the measured values of certain frequencies correctly. In particular, the numerical frequency relative to the first lateral mode was much higher than the experimental one. So the model has been manually updated, by introducing elastic connections between the pylon and the girder. The stiffness of these connection spring elements has been changed in order to obtain a good match between experimental and numerical results.

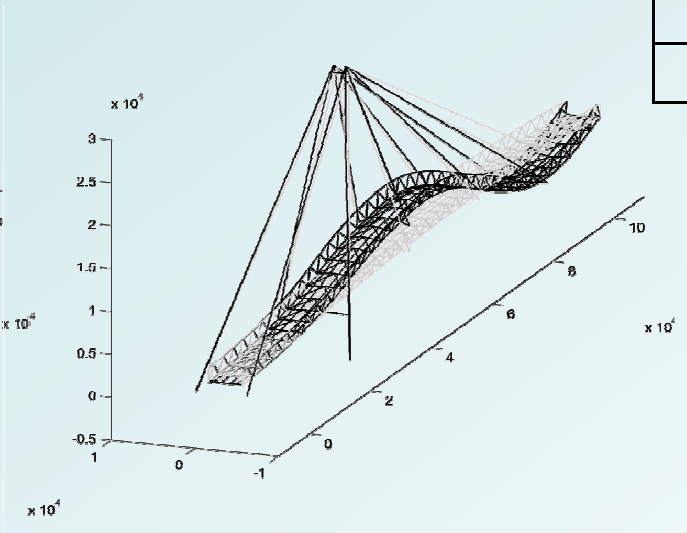
The model is composed by means of 912 frame elements, and the total number of degrees of freedom is equal to 1524, after condensation of the rotational degrees of freedom.

5 The Finite Element Model









Mode	Prevalent Displ.	Freq. (Hz)
1	Vertical	1.22
2	Lateral	1.24
3	Torsional	2.06
4	Vertical	2.77

Preliminary analysis

Time responses were multiplied by a Hanning window

$$w(t)=\begin{cases}\frac{1}{2}\left(1+\cos(\frac{2\pi}{T}t)\right) & |t|<T/2\\ 0 & |t|>T/2\end{cases}$$

Fourier Transform of the time histories and evaluation of PSD and CPSD by Welch Periodogram method. If the input can be supposed a random white process,

$$\boldsymbol{S}_{uu}(\omega)=\tilde{\boldsymbol{H}}_C(\omega)\boldsymbol{S}_{WW}\boldsymbol{H}_C^T(\omega)$$

and:

$$\boldsymbol{H}_C(\omega)=\sum_{j=1}^{2N}\frac{\boldsymbol{\varphi}_j\boldsymbol{\varphi}_j^T}{i\omega-\alpha_j^*}$$

In pole-residue form:

$$\boldsymbol{S}_{uu}(\omega)=\sum_{j=1}^{N}\frac{A_j}{i\omega-\alpha_j^*}+\sum_{j=1}^{N}\frac{\tilde{A}_j}{i\omega-\tilde{\alpha}_j^*}+\sum_{j=1}^{N}\frac{B_j}{i\omega-\alpha_j^*}+\sum_{j=1}^{N}\frac{\tilde{B}_j}{i\omega-\tilde{\alpha}_j^*}$$

7 Modal Identification by FDD

The PSD matrix can be decomposed by Singular Value Decomposition at each frequency value (Brincker et al. 2000):

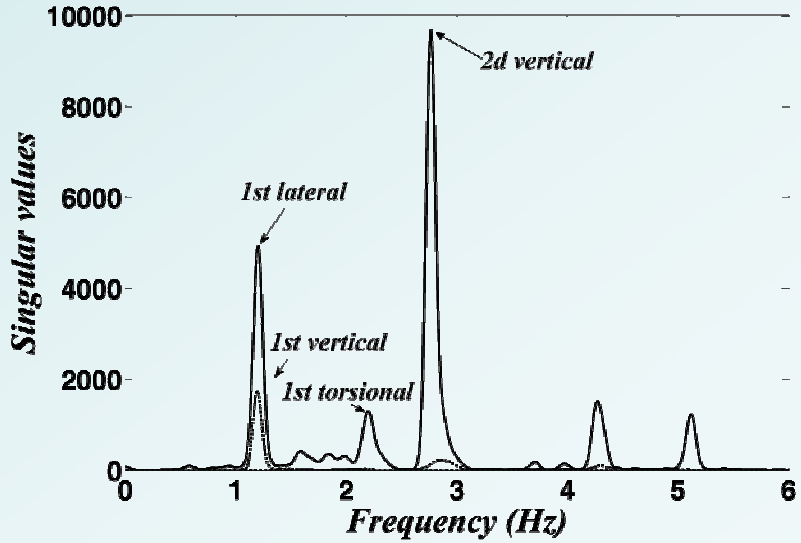
$$S_{uu}(\omega_i) = U_i S_i V_i$$

The first singular vector is the first mode

If there are close frequency modes the second singular vector is the second mode

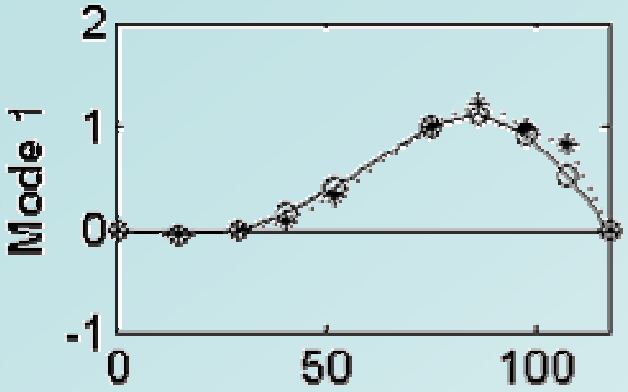
Singular values allow the evaluation of the structure frequencies by traditional peak-picking technique.

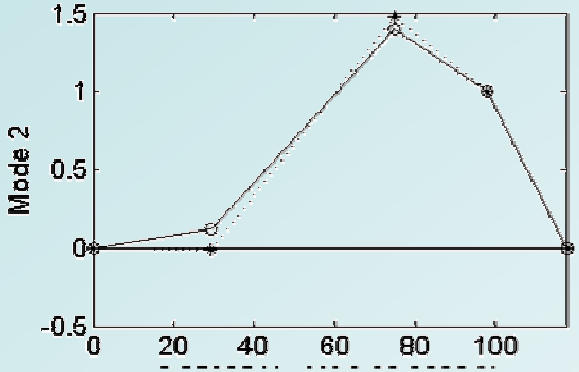
Mode	Prevalent Displ.	Freq. (Hz)	Exp. Freq. (Hz)
1	Vertical	1.22	1.19
2	Lateral	1.24	1.20
3	Torsional	2.06	2.19
4	Vertical	2.77	2.76

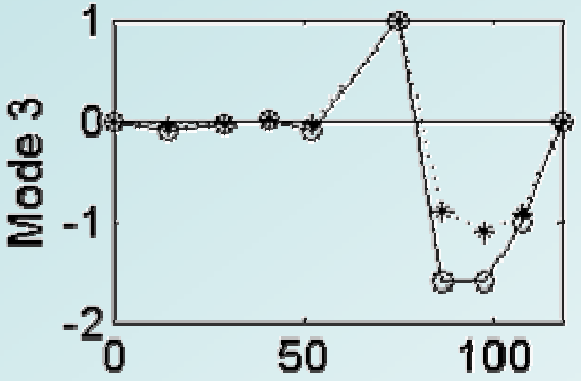


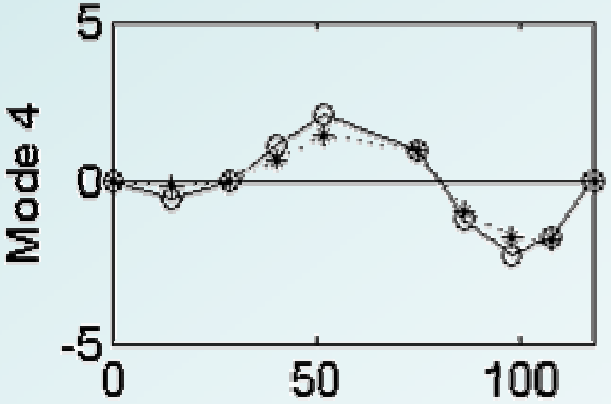
Modal Identification by FDD

Vertical FEM and experimental modes have been compared by means of MAC







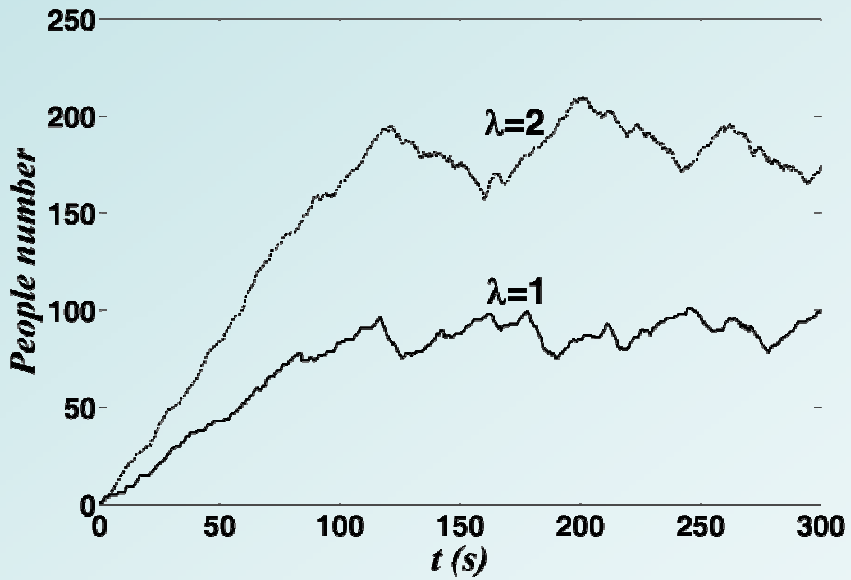


Mode	Prevalent Displ.	MAC (%)
1	Vertical	97.68
2	Lateral	99.38
3	Torsional	95.23
4	Vertical	96.69

9 Numerical evaluation of dynamic response under walking people

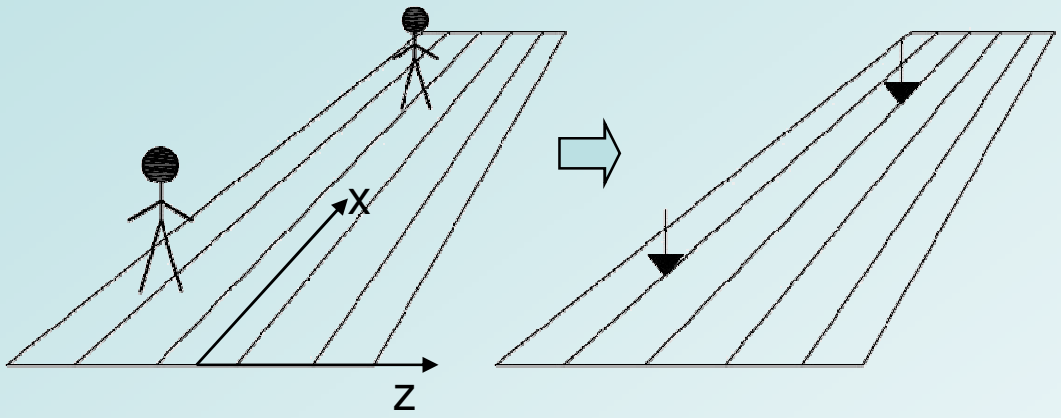
The simulation of walking people is based on the following consideration. For simplicity, it is supposed that people move in one direction only. The arrival times are probabilistically defined through a Poisson distribution with parameter λ

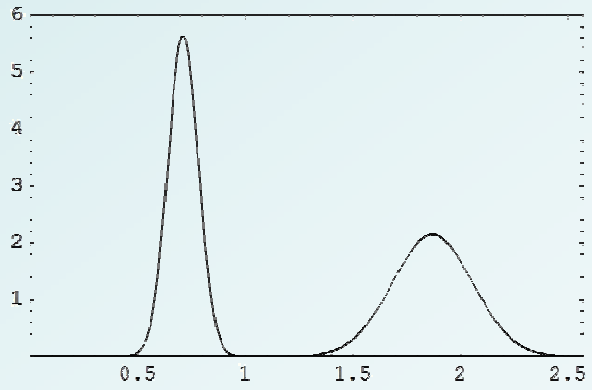
$$p(N,t) = \frac{e^{-\lambda t}(\lambda t)^N}{N!}$$



The human weight has been defined as a random variable with uniform distribution between 500 and 1000 *N*. The step frequency and length have been supposed to be random parameters with Gaussian distribution; mean values and standard deviation are (Živanovic 2007 et al.):

$$\mu_s = 0.71m; \qquad \sigma_s = 0.071m \qquad \mu_f = 1.87\,Hz; \qquad \sigma_f = 0.186\,Hz;$$





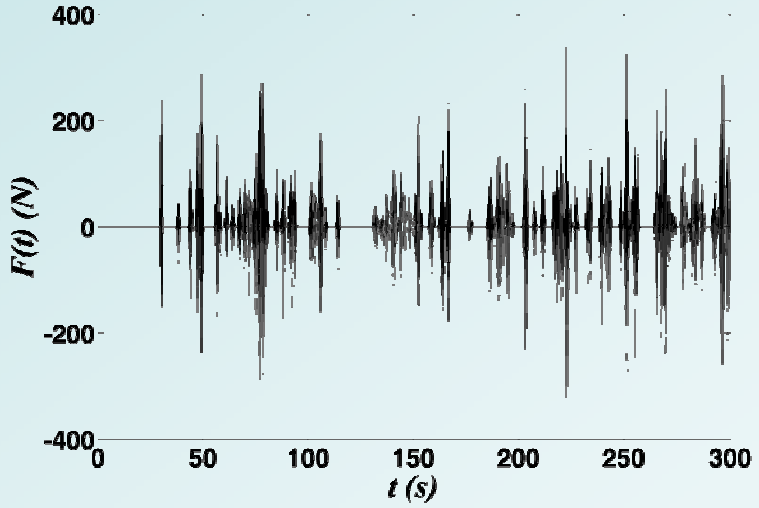
Numerical evaluation of dynamic response under walking people

The human load have been modeled in the time domain by a superposition of harmonic and sub-harmonic components (Živanovic et al.. 2007). The dynamic part of the load is:

$$F(t) = \sum_{i=1}^{5} F_i(t) + \sum_{i=1}^{5} F_i^s(t)$$

$$F_i(t) = W\, DLF_i \sum_{\bar{f}_j = i-0.25}^{i+0.25} \overline{DLF}_i(\bar{f}_j) \cos(2\pi\, \bar{f}_j f_s\, t - \theta(\bar{f}_j))$$

$$F_i^s(t) = W\, DLF_i^s \sum_{\bar{f}_j = i-0.75}^{i-0.25} \overline{DLF}_i^s(\bar{f}_j^s) \cos(2\pi\, \bar{f}_j^s f_s\, t - \theta(\bar{f}_j^s))$$



The equation of motion is a linear differential equation with variable coefficient:

$$\boldsymbol{M}(t)\ddot{\boldsymbol{u}}(t)+\boldsymbol{C}\dot{\boldsymbol{u}}(t)+\boldsymbol{K}(t)\boldsymbol{u}(t)=\boldsymbol{F}(t)$$

$\boldsymbol{M}$(t) is the time-variable mass matrix, $\boldsymbol{C}$ is the damping matrix, $\boldsymbol{K}$(t) is the stiffness matrix, $\boldsymbol{u}$(t) the displacements vector, $\boldsymbol{F}$(t) the load vector dependent from human walking. The damping matrix is determined assuming a modal damping ratio equal to 1% for all modes; this modal damping ratio has been estimated as an averaged value by means of half-power bandwidth method for the identified modes:

$$\boldsymbol{C}=\boldsymbol{M}_0\boldsymbol{\Phi}\boldsymbol{\Xi}\boldsymbol{\Phi}^T\boldsymbol{M}_0$$

In state space form:

$$\dot{\boldsymbol{Y}}(t)=\boldsymbol{A}(t)\boldsymbol{Y}(t)+\boldsymbol{H}(t)$$

$$\boldsymbol{Y}(t)=\begin{Bmatrix}\boldsymbol{u}(t)\\ \dot{\boldsymbol{u}}(t)\end{Bmatrix} \qquad \boldsymbol{A}(t)=\begin{Bmatrix}\boldsymbol{0} & \boldsymbol{I}\\ -\boldsymbol{M}^{-1}(t)\boldsymbol{K}(t) & -\boldsymbol{M}^{-1}(t)\boldsymbol{C}\end{Bmatrix} \qquad \boldsymbol{H}(t)=\begin{Bmatrix}\boldsymbol{0}\\ \boldsymbol{F}(t)\end{Bmatrix}$$

Numerical evaluation of dynamic response under walking people

The equation of motion can be integrated by means of a time-step method:

$$\boldsymbol{Y}(t_{k+1}) = e^{\int_{t_k}^{t_{k+1}} \boldsymbol{A}(t)dt} \boldsymbol{Y}(t_k) + \int_{t_k}^{t_{k+1}} e^{\int_{\tau}^{t_{k+1}} \boldsymbol{A}(t)dt} \boldsymbol{H}(\tau)d\tau$$

or:

$$\boldsymbol{Y}(t_{k+1}) = \boldsymbol{\Theta}_k(\Delta t)\boldsymbol{Y}(t_k) + \int_{t_k}^{t_{k+1}} \boldsymbol{\Theta}_k(t_{k+1}-\tau)\boldsymbol{H}(\tau)d\tau$$

The matrix $\boldsymbol{A}$ must redefined at every time step:

$$\boldsymbol{\Theta}_k(\Delta t) = e^{\boldsymbol{A}(t_k)\Delta t}$$

Assuming the excitation as linear variable in the time step:

$$\boldsymbol{Y}(t_{k+1}) = \boldsymbol{\Theta}_k(\Delta t)\boldsymbol{Y}(t_k) + \boldsymbol{\gamma}_{0,k}(\Delta t)\boldsymbol{H}(t_k) + \boldsymbol{\gamma}_{1,k}(\Delta t)\boldsymbol{H}(t_{k+1})$$

$$\boldsymbol{\gamma}_{0,k}(\Delta t) = \left[\boldsymbol{\Theta}_k(\Delta t) - \frac{1}{\Delta t}\boldsymbol{L}_k(\Delta t)\right]\boldsymbol{A}^{-1}(t_k); \quad \boldsymbol{\gamma}_{1,k}(\Delta t) = \left[\frac{1}{\Delta t}\boldsymbol{L}_k(\Delta t) - \boldsymbol{I}\right]\boldsymbol{A}^{-1}(t_k)$$

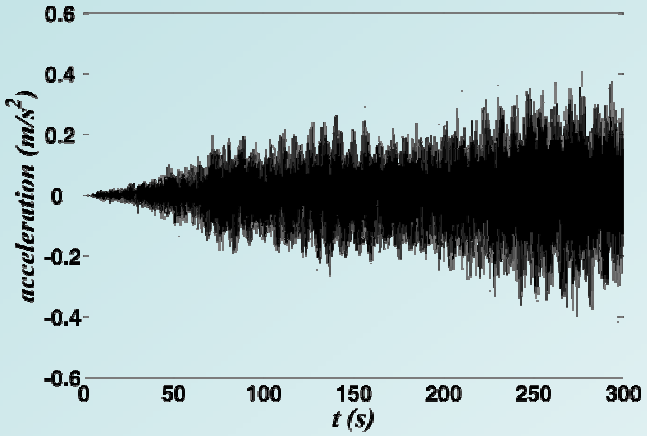
$$\boldsymbol{L}_k(\Delta t) = \left[\boldsymbol{\Theta}_k(\Delta t) - \boldsymbol{I}\right]\boldsymbol{A}^{-1}(t_k)$$

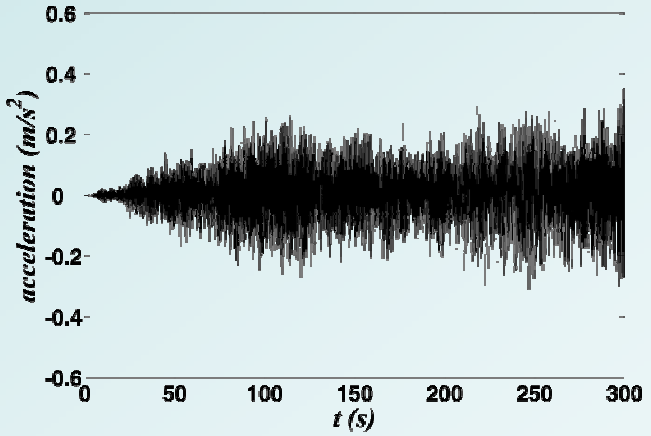
Numerical evaluation of dynamic response under walking people

Vertical response in terms of acceleration in a node located at first stay anchor point: constant and variable mass matrix

$\lambda = 1$

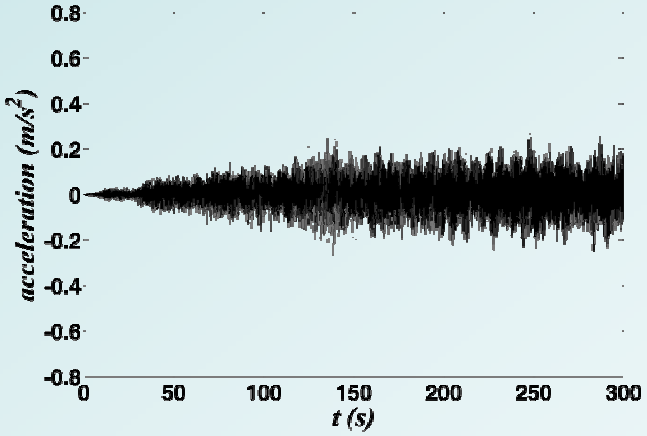
$0.19\ people/m^2$

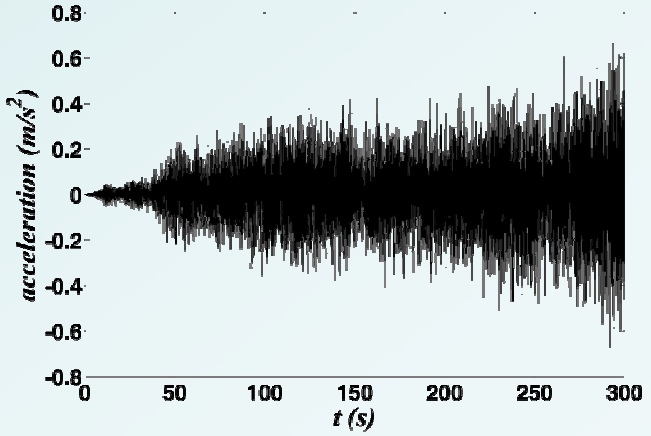




$\lambda = 2$

$0.42\ people/m^2$





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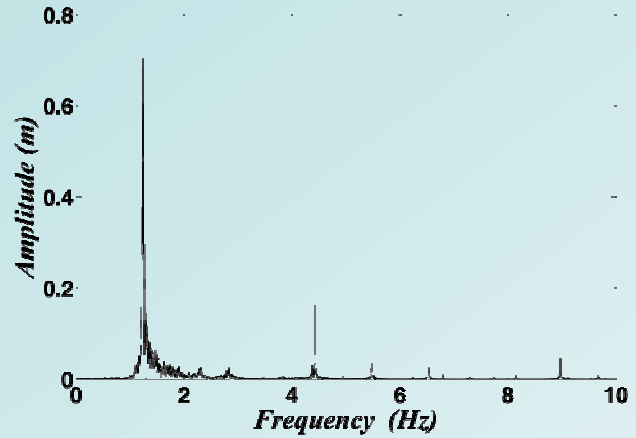
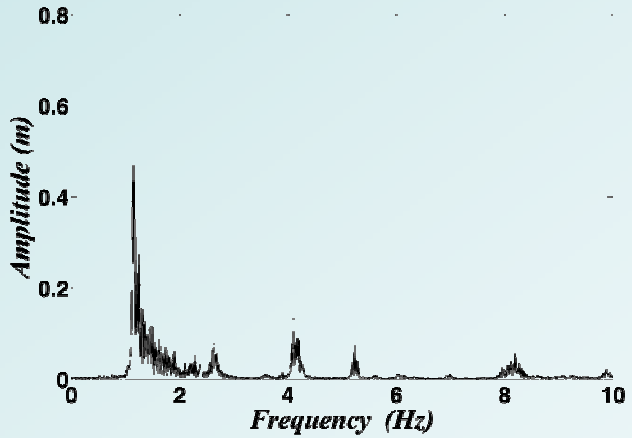
Numerical evaluation of dynamic response under walking people

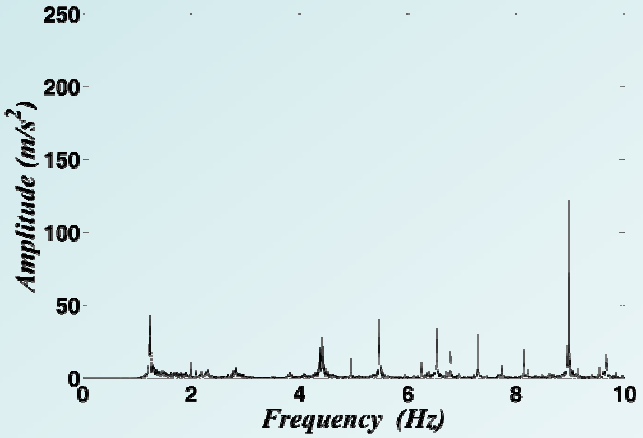
Fourier amplitude for displacement and acceleration: constant and variable mass-matrix

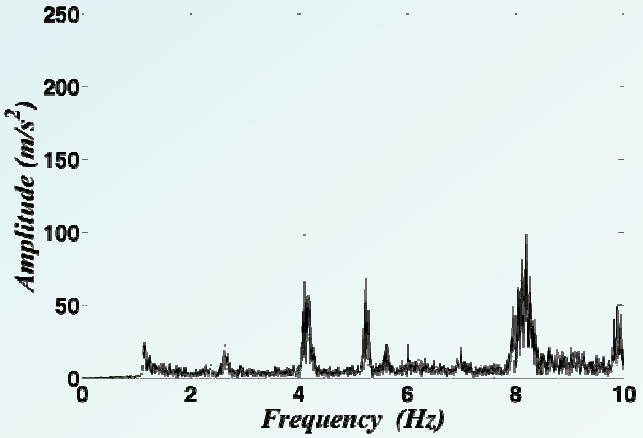
$\lambda = 2$

$0.42\ people/m^2$









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Conclusions

- The modal properties of the Forchheim cable stayed footbridge have been identified by means of Frequency Domain Decomposition technique;

- A finite element model of the bridge has developed and modified to match the experimental frequencies;

- A numerical analysis has been performed in order to evaluate the vertical dynamic response to walking people, taking into account human moving mass and weight;

- Results in terms of acceleration show differences among the analysis based on time variable mass matrix and analysis with constant mass matrix, in the case of crowd on the bridge;

- A next step would be the stochastic characterization of human load.